

**INTEGRAL TRANSFORMS OF H-FUNCTION
OF TWO VARIABLES INVOLVING GENERALIZED M-SERIES**

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ABSTRACT

The object of this paper is to establish some integral transforms involving product of generalized M-series and H-function of two variables. Some special cases have also been derived.

Key words: H-function of two variables, Mellin transform, Laplace transform and Generalized M-series.

1. INTRODUCTION

Recently, the Mellin-Barnes type contour integral of H-function of two variables evaluated by P.C.Srinivas [9]. In the present paper we establish the Mellin transform and Laplace transform of H-function of two variables with generalized M-series.

We shall utilized following formulae in present investigation. The H-function of one variable given by Charles Fox [2]

$$\begin{aligned}
 H[x] &= H_{p,q}^{m,n}[x] = H_{p,q}^{m,n} \left[x \left| \begin{matrix} (a_j, A_j)_{1,p} \\ (b_j, B_j)_{1,q} \end{matrix} \right. \right] \\
 &= \frac{1}{2\pi i} \int_L F(s) x^s ds, \quad i = \sqrt{-1}, x \neq 0
 \end{aligned} \tag{1.1}$$

Where

$$F(s) = \frac{\prod_{j=1}^m \Gamma(b_j - B_j s) \prod_{j=1}^n \Gamma(1 - a_j + A_j s)}{\prod_{j=m+1}^q \Gamma(1 - b_j + B_j s) \prod_{j=n+1}^p \Gamma(a_j - A_j s)}$$

An empty product is interpreted as unity; m,n,p and q are integers satisfying $0 \leq m \leq q, 0 \leq n \leq p$; $A_j(j = 1, \dots, p)$, $B_j(j = 1, \dots, q)$ are positive numbers and $a_j(j = 1, \dots, p)$, $b_j(j = 1, \dots, q)$ are complex numbers such that no poles of $\Gamma(b_j - B_j s)$, $j = 1, \dots, m$ coincide with any pole of $\Gamma(1 - a_j + A_j s)$, $j = 1, \dots, n$ i.e $A_j(b_k + N) \neq B_k(a_j - M - 1)$, where $k = 1, \dots, m; j = 1, \dots, n; M = 0, 1, 2, \dots$

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The contour L runs from $\sigma-i\infty$ to $\sigma+i\infty$, σ be a positive constant such that the points $s = \frac{b_h + N}{B_h}, h = 1, \dots, m; N = 1, \dots, n;$ which are the poles of $\Gamma(b_j - B_j s), j = 1, \dots, m$ lie to the right and the points $s = \frac{a_j - M - 1}{a_j}, j = 1, \dots, mn; M = 0, 1, \dots$ which are the poles of $\Gamma(1 - a_j + A_j s), j = 1, \dots, n$ lie to the left of L.

The H-function of two variables given by Prasad and Gupta[6]

$$H[x, y] = H_{P, Q}^{M, N : m, n ; g, h} \left[\begin{matrix} \gamma x^\sigma (a_j; \alpha_j, A_j)_{1, P} : (c_j, C_j)_{1, p} ; (e_j, E_j)_{1, u} \\ \eta x^\delta (b_j; \beta_j, B_j)_{1, Q} : (d_j, D_j)_{1, q} ; (f_j, F_j)_{1, v} \end{matrix} \right]$$

$$= \frac{1}{(2\pi i)^2} \int_{L_1} \int_{L_2} \phi_1(s) \phi_2(t) \psi(s, t) x^s y^t ds dt, i = \sqrt{-1}$$
(1.2)

where $x, y \neq 0$,

$$\phi_1(s) = \frac{\prod_{j=1}^m \Gamma(d_j - D_j s) \prod_{j=1}^n \Gamma(1 - c_j + C_j s)}{\prod_{j=m+1}^q \Gamma(1 - d_j + D_j s) \prod_{j=n+1}^p \Gamma(c_j - C_j s)}$$

$$\phi_2(t) = \frac{\prod_{j=1}^g \Gamma(f_j - F_j t) \prod_{j=1}^h \Gamma(1 - e_j + E_j t)}{\prod_{j=g+1}^v \Gamma(1 - f_j + F_j t) \prod_{j=h+1}^u \Gamma(e_j - E_j t)}$$

$$\psi(s, t) = \frac{\prod_{j=1}^M \Gamma(b_j - \beta_j s - B_j t) \prod_{j=1}^N \Gamma(1 - a_j + \alpha_j s + A_j t)}{\prod_{j=M+1}^Q \Gamma(1 - b_j + \beta_j s + B_j t) \prod_{j=N+1}^P \Gamma(a_j - \alpha_j s - A_j t)}$$

where $M, N, P, Q, m, n, p, q, g, h, u, v$ are all non negative integers such that $0 \leq N \leq P, Q \geq 0, 0 \leq m \leq q, 0 \leq n \leq q, 0 \leq g \leq v, 0 \leq h \leq u$ and $\alpha_j, \beta_j, A_j, B_j, C_j, D_j, E_j, F_j$ are all positive. The sequence of parameters $(a_p), (b_q), (c_p), (d_q), (e_u)$ and (f_v) are so restricted that none of the poles of the integrand coincide.

The contour L_1 lies in the complex s-plane and runs from $-i\infty$ to $+i\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(d_j - D_j s), (j = 1, 2, \dots, m)$, lie to the right of the path; and those of $\Gamma(1 - c_j + C_j s), (j = 1, 2, \dots, n)$ and $\Gamma(1 - a_j + \alpha_j s + A_j t), (j = 1, 2, \dots, N)$ lie to the left of the path.

Also the contour L_2 lies in the complex t-plane running from $-i\infty$ to $+i\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(f_j - F_j t), (j = 1, 2, \dots, g)$, lie to the right of the path; and those of $\Gamma(1 - e_j + E_j t), (j = 1, 2, \dots, h)$ and $\Gamma(1 - a_j + \alpha_j s + A_j t), (j = 1, 2, \dots, N)$ lie to the left of the path. All poles of the integrand are simple poles.

The Mellin transform of the function $f(x)$ is defined as

$$M \{ f(x); s \} = \int_0^\infty x^{s-1} f(x) dx, \text{Re}(s) > 0$$
(1.3)

If Laplace transform of $f(t)$ is $F(p)$ and $G(s)$ is Mellin transform of $f(t)$, then

$$F(p) = \sum_{s=0}^\infty \frac{(-p)^s}{s!} G(s+1)$$
(1.4)

According Erdelyi [1, p.307]

$$\int_0^{\infty} x^s - 1 \left[\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(s)x^{-s} ds \right] dx = g(s) \tag{1.5}$$

The Generalized M-Series is defined by Sharma and Renu[8] as

$$M_{p,q}^{\alpha,\beta} = M_{p,q}^{\alpha,\beta}(a_1, \dots, a_p; b_1, \dots, b_q; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \dots (a_p)_k}{(b_1)_k \dots (b_q)_k} \frac{z^k}{\Gamma(\alpha k + \beta)} \quad z, \alpha, \beta \in \mathbb{C}, \Re(\alpha) > 0 \tag{1.6}$$

Series is convergent for all z if q ≥ p, it is convergent for |z| < 1 if p = q+1 and divergent if p > q+1. Where p = q+1 and |z| = 1, the series convergent in some case. Let $\beta = \sum_{j=1}^p a_j - \sum_{j=1}^q b_j$

It can be shown that when p = q+1 the series is absolutely convergent for |z| = 1 if $\Re(\beta) < 0$. Conditionally convergent for z = -1 if $0 \leq \Re(\beta) \leq 1$ and divergent for |z|=1 if $1 \leq \Re(\beta)$.

2. MAIN RESULTS

Theorem 1: Prove that

$$M \left\{ M_{l,m}^{\alpha,\beta}(a_1, \dots, a_l; b_1, \dots, b_m; ax^\lambda) H_{P,Q}^{M,N:m_1,n_1;m_2,n_2} \left[\begin{matrix} \gamma x^\sigma (a_j; \alpha_j, A_j)_{1,P} : (c_j, C_j)_{1,p_1}; (e_j, E_j)_{1,p_2} \\ \eta x^\delta (b_j; \beta_j, B_j)_{1,Q} : (d_j, D_j)_{1,q_1}; (f_j, F_j)_{1,q_2} \end{matrix} \right]; s \right\}$$

$$= \frac{1}{\delta} \frac{\prod_{j=1}^m b_j}{\prod_{j=1}^l a_j} \sum_{k=0}^{\infty} \frac{\prod_{j=1}^l (a_j + k)}{\prod_{j=1}^m (b_j + k)} \frac{a^k}{\Gamma(\alpha k + \beta)} \eta^{-\frac{(s+\lambda k)}{\delta}} H_{P+P_1+Q_2, Q+Q_1+P_2}^{M+m_1+n_2, N+n_1+m_2} \left[\begin{matrix} -\frac{\sigma}{\delta} \gamma (a_j + \frac{(s+\lambda k)}{\delta} A_j, \alpha_j - (\sigma/\delta) A_j)_{1,P} \\ (b_j + \frac{(s+\lambda k)}{\delta} B_j, \beta_j - (\sigma/\delta) B_j)_{1,Q} \end{matrix} \right]$$

$$\left[(1-f_j - \frac{(s+\lambda k)}{\delta} F_j, (\sigma/\delta) F_j)_{1,n_2}, (c_j, C_j)_{1,n_1}, (1-f_j - \frac{(s+\lambda k)}{\delta} F_j, (\sigma/\delta) F_j)_{n_2+1, p_2}, (c_j, C_j)_{n_1+1, p_1} \right]$$

$$\left[(1-e_j - \frac{(s+\lambda k)}{\delta} E_j, (\sigma/\delta) E_j)_{1,m_2}, (d_j, D_j)_{1,m_1}, (1-e_j - \frac{(s+\lambda k)}{\delta} E_j, (\sigma/\delta) E_j)_{m_2+1, q_2}, (d_j, D_j)_{m_1+1, q_1} \right]$$

Provided $\sigma > 0, \delta > 0, \lambda$ is complex number

$\alpha_j - (\sigma/\delta) A_j > 0$ for j = 1, 2, ---, P

$\beta_j - (\sigma/\delta) B_j > 0$ for j = 1, 2, ---, Q

$|\arg \gamma| < (1/2) \pi \Delta_1, |\arg \eta| < (1/2) \pi \Delta_2$

where $\Delta_1 = \sum_{j=1}^N \alpha_j - \sum_{j=N+1}^P \alpha_j + \sum_{j=1}^M \beta_j - \sum_{j=M+1}^Q \beta_j + \sum_{j=1}^{m_1} D_j - \sum_{j=m_1+1}^{q_1} D_j + \sum_{j=1}^{n_1} C_j - \sum_{j=n_1+1}^{p_1} C_j$

and $\Delta_2 = \sum_{j=1}^N A_j - \sum_{j=N+1}^P A_j + \sum_{j=1}^M B_j - \sum_{j=M+1}^Q B_j + \sum_{j=1}^{m_2} F_j - \sum_{j=m_2+1}^{q_2} F_j + \sum_{j=1}^{n_2} E_j - \sum_{j=n_2+1}^{p_2} E_j$

and $\text{Re} \left\{ s + \frac{\delta(a_j - 1)}{A_j} \right\} < 0$ for j = 1, 2, ..., N

$$\operatorname{Re}\left\{s + \frac{\delta b_j}{B_j}\right\} > 0 \quad \text{for } j = 1, 2, \dots, M$$

$$\operatorname{Re}\left\{s + \frac{\delta b_j}{B_j} + \frac{\sigma d_i}{D_i}\right\} > 0 \quad \text{for } i = 1, 2, \dots, m_1; \text{ for } j = 1, 2, \dots, m_2$$

$$\operatorname{Re}\left\{s + \frac{\delta(e_j - 1)}{E_j} + \frac{\sigma(c_i - 1)}{C_i}\right\} > 0 \quad \text{for } i = 1, 2, \dots, n_1; \text{ for } j = 1, 2, \dots, n_2$$

Proof: To prove this theorem, take f(x) as

$${}_{l, m}^{\alpha, \beta} M(a_1, \dots, a_l; b_1, \dots, b_m; ax^\lambda) H_{P, Q: p_1, q_1; p_2, q_2}^{M, N: m_1, n_1; m_2, n_2} \left[\gamma x^\sigma \left((a_j; \alpha_j, A_j)_{1, P} : (c_j, C_j)_{1, p_1} ; (e_j, E_j)_{1, p_2} \right) \right. \\ \left. \left[\eta x^\delta \left((b_j; \beta_j, B_j)_{1, Q} : (d_j, D_j)_{1, q_1} ; (f_j, F_j)_{1, q_2} \right) \right] \right] \text{ in (1.3),}$$

then expression becomes

$$M \left\{ {}_{l, m}^{\alpha, \beta} M(a_1, \dots, a_l; b_1, \dots, b_m; ax^\lambda) H_{P, Q: p_1, q_1; p_2, q_2}^{M, N: m_1, n_1; m_2, n_2} \left[\gamma x^\sigma \left((a_j; \alpha_j, A_j)_{1, P} : (c_j, C_j)_{1, p_1} ; (e_j, E_j)_{1, p_2} \right) \right. \right. \\ \left. \left. \left[\eta x^\delta \left((b_j; \beta_j, B_j)_{1, Q} : (d_j, D_j)_{1, q_1} ; (f_j, F_j)_{1, q_2} \right) \right] \right] \right\} \quad (2.2)$$

$$= \int_0^\infty x^{s-1} M \left\{ {}_{l, m}^{\alpha, \beta} M(a_1, \dots, a_l; b_1, \dots, b_m; ax^\lambda) H_{P, Q: p_1, q_1; p_2, q_2}^{M, N: m_1, n_1; m_2, n_2} \left[\gamma x^\sigma \left((a_j; \alpha_j, A_j)_{1, P} : (c_j, C_j)_{1, p_1} ; (e_j, E_j)_{1, p_2} \right) \right. \right. \\ \left. \left. \left[\eta x^\delta \left((b_j; \beta_j, B_j)_{1, Q} : (d_j, D_j)_{1, q_1} ; (f_j, F_j)_{1, q_2} \right) \right] \right] \right\} dx$$

By using (1.2) and (1.6) represent H-function in integral form and M-series as series form. Put $\delta t_2 = -u$, we get

$$\sum_{k=0}^\infty \frac{(a_1)_k \dots (a_l)_k}{(b_1)_k \dots (b_m)_k} \frac{a^k}{\Gamma(\alpha k + \beta)} \frac{1}{(2\pi i)^2} \int_{L_1} \int_{L_2} \phi_1(t_1) \phi_2\left(\frac{-u}{\delta}\right) \psi\left(t_1, \frac{-u}{\delta}\right) \gamma t_1 \eta \frac{-u}{\delta} x^{-u} x^{\sigma t_1 + \lambda k + s - 1} \left(\frac{du}{\delta}\right) dt_1 dx$$

Interchange the order of integration, we get

$$= \sum_{k=0}^\infty \frac{(a_1)_k \dots (a_l)_k}{(b_1)_k \dots (b_m)_k} \frac{a^k}{\Gamma(\alpha k + \beta)} \frac{1}{\delta} \frac{1}{2\pi i} \int_{L_1} \phi_1(t_1) \gamma^{t_1} \left\{ \int_0^\infty x^{\sigma t_1 + \lambda k + s - 1} \left[\frac{1}{2\pi i} \int_{L_2} \phi_2\left(\frac{-u}{\delta}\right) \psi\left(t_1, \frac{-u}{\delta}\right) \eta \frac{-u}{\delta} x^{-u} du \right] dx \right\} dt_1$$

Use result (1.5) and (1.1) to get the result. Change of order of integration is justifiable due to convergence of integrals.

Theorem 2:

$$L \left\{ {}_{l, m}^{\alpha, \beta} M(a_1, \dots, a_l; b_1, \dots, b_m; ax^\lambda) H_{P, Q: p_1, q_1; p_2, q_2}^{M, N: m_1, n_1; m_2, n_2} \left[\gamma x^\sigma \left((a_j; \alpha_j, A_j)_{1, P} : (c_j, C_j)_{1, p_1} ; (e_j, E_j)_{1, p_2} \right) \right. \right. \\ \left. \left. \left[\eta x^\delta \left((b_j; \beta_j, B_j)_{1, Q} : (d_j, D_j)_{1, q_1} ; (f_j, F_j)_{1, q_2} \right) \right] \right] ; s \right\}$$

$$= \frac{1}{\delta} \frac{\prod_{j=1}^m b_j}{\prod_{j=1}^l a_j} \sum_{s=0}^\infty \frac{(-p)^s}{s!} \sum_{k=0}^\infty \frac{\prod_{j=1}^l (a_j + k)}{\prod_{j=1}^m (b_j + k)} \frac{a^k}{\Gamma(\alpha k + \beta)} \eta^{\frac{(s+\lambda k+1)}{\delta}} H_{P+p_1+q_2, Q+q_1+p_2}^{M+m_1+n_2, N+n_1+m_2} \left[\eta \frac{-\sigma}{\delta} \gamma \left((a_j + \frac{(s+\lambda k+1)}{\delta}) A_j, \alpha_j - (\sigma/\delta) A_j \right)_{1, P}, \right. \\ \left. \left(b_j + \frac{(s+\lambda k+1)}{\delta} \right) B_j, \beta_j - (\sigma/\delta) B_j \right]_{1, Q},$$

$$\left[(1-f_j - \frac{(s+\lambda k+1)}{\delta}) F_j, (\sigma/\delta) F_j \right]_{1, n_2}, (c_j, C_j)_{1, n_1}, (1-f_j - \frac{(s+\lambda k+1)}{\delta}) F_j, (\sigma/\delta) F_j \right]_{n_2+1, p_2}, (c_j, C_j)_{n_1+1, p_1}$$

$$\left[(1-e_j - \frac{(s+\lambda k+1)}{\delta}) E_j, (\sigma/\delta) E_j \right]_{1, m_2}, (d_j, D_j)_{1, m_1}, (1-e_j - \frac{(s+\lambda k+1)}{\delta}) E_j, (\sigma/\delta) E_j \right]_{m_2+1, q_2}, (d_j, D_j)_{m_1+1, q_1}$$

Proof: To prove this theorem use (1.4) to get the result

3. SPECIAL CASES

(i) Take

$$(\alpha_P) = (A_P) = (B_Q) = (b_Q) = (C_{P_1}) = (D_{Q_1}) = (E_{P_2}) = (F_{Q_2}) = 1$$

We get Mellin and Laplace transforms of G-function of two variables with Generalized M-series

(ii) Put M = N = P = Q = 0 in (2.1) and (2.3), we get

$$M \left\{ \begin{matrix} \alpha, \beta \\ l, m \end{matrix} M(a_1, \dots, a_l; b_1, \dots, b_m; ax^\lambda) H_{P_1, Q_1}^{m_1, n_1} \left[\gamma x^\sigma \begin{matrix} (c_j, C_j)_{1, P_1} \\ (d_j, D_j)_{1, Q_1} \end{matrix} \right] H_{P_2, Q_2}^{m_2, n_2} \left[\eta x^\delta \begin{matrix} (e_j, E_j)_{1, P_2} \\ (f_j, F_j)_{1, Q_2} \end{matrix} \right] ; s \right\} \quad (3.1)$$

$$M \left\{ \begin{matrix} \alpha, \beta \\ l, m \end{matrix} M(a_1, \dots, a_l; b_1, \dots, b_m; ax^\lambda) H_{P_1, Q_1}^{m_1, n_1} \left[\gamma x^\sigma \begin{matrix} (c_j, C_j)_{1, P_1} \\ (d_j, D_j)_{1, Q_1} \end{matrix} \right] H_{P_2, Q_2}^{m_2, n_2} \left[\eta x^\delta \begin{matrix} (e_j, E_j)_{1, P_2} \\ (f_j, F_j)_{1, Q_2} \end{matrix} \right] ; s \right\}$$

$$= \frac{1}{\delta} \frac{\prod_{j=1}^m b_j}{\prod_{j=1}^l a_j} \sum_{k=0}^{\infty} \frac{\prod_{j=1}^l (a_j + k)}{\prod_{j=1}^m (b_j + k)} \frac{a^k}{\Gamma(\alpha k + \beta)} \eta^{-\frac{(s+\lambda k)}{\delta}} H_{P_1+Q_2, Q_1+P_2}^{m_1+n_2, n_1+m_2} \left[\eta^{-\frac{\sigma}{\delta}} \gamma \begin{matrix} (1-f_j - (\frac{s+\lambda k}{\delta})F_j, (\sigma/\delta)F_j)_{1, P_2}, (c_j, C_j)_{1, n_1}, \\ (1-e_j - (\frac{s+\lambda k}{\delta})E_j, (\sigma/\delta)E_j)_{1, Q_2}, (d_j, D_j)_{1, m_1}, \\ (c_j, C_j)_{n_1+1, P_1}, \\ (d_j, D_j)_{m_1+1, Q_1} \end{matrix} \right]$$

$$L \left\{ \begin{matrix} \alpha, \beta \\ l, m \end{matrix} M(a_1, \dots, a_l; b_1, \dots, b_m; ax^\lambda) H_{P_1, Q_1}^{m_1, n_1} \left[\gamma x^\sigma \begin{matrix} (c_j, C_j)_{1, P_1} \\ (d_j, D_j)_{1, Q_1} \end{matrix} \right] H_{P_2, Q_2}^{m_2, n_2} \left[\eta x^\delta \begin{matrix} (e_j, E_j)_{1, P_2} \\ (f_j, F_j)_{1, Q_2} \end{matrix} \right] \right\} \quad (3.2)$$

$$= \frac{1}{\delta} \frac{\prod_{j=1}^m b_j}{\prod_{j=1}^l a_j} \sum_{s=0}^{\infty} \frac{(-p)^s}{s!} \sum_{k=0}^{\infty} \frac{\prod_{j=1}^l (a_j + k)}{\prod_{j=1}^m (b_j + k)} \frac{a^k}{\Gamma(\alpha k + \beta)} \eta^{-\frac{(s+\lambda k+1)}{\delta}} H_{P_1+Q_2, Q_1+P_2}^{m_1+n_2, n_1+m_2} \left[\eta^{-\frac{\sigma}{\delta}} \gamma \begin{matrix} (1-f_j - (\frac{s+\lambda k+1}{\delta})F_j, (\sigma/\delta)F_j)_{1, P_2}, \\ (1-e_j - (\frac{s+\lambda k+1}{\delta})E_j, (\sigma/\delta)E_j)_{1, Q_2}, \\ (c_j, C_j)_{1, n_1}, (c_j, C_j)_{n_1+1, P_1}, \\ (d_j, D_j)_{1, m_1}, (d_j, D_j)_{m_1+1, Q_1} \end{matrix} \right]$$

(iii) For $\beta = 1$ in (2.1) and (2.3), we get Mellin and Laplace transform of H-function of two variables with generalized M-series by M. Sharma [7]

(iv) For $l = m = 0$ in (2.1) and (2.3), we get Mellin and Laplace transform of H-function of two variables involving Mittag-Leffler function $E_{\alpha, \beta}(ax^\lambda)$ [4]

(v) Take $\alpha = \beta = 1$ in (2.1) and (2.3), we get Mellin the Laplace transform of H-function of two variables involving generalized hypergeometric function [3]

(vi) Put $l = 0, m = 1$ and $b_1 = 1$ in (2.1) and (2.3), we get Mellin and Laplace transform of H-function of two variables involving Wright function $W(ax^\lambda; \alpha, \beta)$ [5].

CONCLUSION

Thus we proved Mellin and Laplace transforms of H-function of two variables involving generalized M-series and special cases, which gives some interesting applications in fractional calculus.

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