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SOME STAR AND BISTAR RELATED SIGNED PRODUCT CORDIAL GRAPHS

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ABSTRACT

A vertex labeling f: V(G)→{ -1, 1} of a graph G with induced edge labeling f^*:E(G)→ {-1, 1} defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$ where v_f (-1) is the number of vertices labeled with '-1' and v_f (1) is the number of vertices labeled with '+1', $e_f(-1)$ is the number of edges labeled with '-1' and $e_f(1)$ is the number of edges labeled with '+1'.A graph with a signed product cordial labeling is called a signed product cordial graph. In this paper, we prove that splitting graphs of star $K_{1,n}$ and Bistar $B_{n,n}$ are signed product cordial graphs.

Keywords: Signed product cordial labeling, Signed product cordial graph.

1. INTRODUCTION

Let G = (V(G), E(G)) be a simple, finite and connected graph.

Now we give some useful terms such as labeling, cordial labeling **(CL)**, Product cordial labeling **(PCL)**, Total product cordial labeling**(TPCL)** , Signed product cordial labeling**(SPCL)** and total signed product cordial labeling **(TSPCL)**. Also, We derive few theorems in signed product cordial labeling.

Definition 1.1: A mapping f: V(G)→{0, 1} is called binary vertex labeling of G under f(v),called label of vertex v of G under f. The induced edge labeling f^*: E(G)→{0, 1} is given by $f^*(e=uv) = |f(u) - f(v)|$.Let $V_f(0)$, $V_f(1)$ be the number of vertices labeled with 0 and 1 under f. Let $e_f(0)$, $e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*. A binary vertex labeling of graph G is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition 1.2:A vertex labeling f:V(G)→{ -1, 1} of a graph G with induced edge labeling f^* : E(G)→{-1, 1} defined by $f^*(uv) = f(u)f(v)$ is called signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$ Where $V_f(-1)$ is the number of vertices labeled with -1, $V_f(1)$ is the number of vertices labeled with 1.$e_f(-1)$ is the number of edges labeled with -1 and $e_f(1)$ is the number of edges labeled with 1. A graph G is signed product cordial if it admits signed product cordial labeling.

Definition 1.3: Let f be a function from V(G) to { 0,1,2….k-1} where k is an integer, $2 \leq k \leq |V(G)|$. For each edge uv, assign the label f(u)f(v) (mod k). Then f is called a k-Total product cordial labeling of G if $|f(i) - f(j)| \leq 1$,i,j ∈ {0,1,2……….k-1} where f(x) denotes the total number of vertices and edges labeled with x (x=0,1,2,…k-1).A graph with a k-total product cordial labeling is called a k-Total product cordial labeling.

Definition 1.4: Let f be a function from V(G) to {-1, 1} with induced edge labeling f^*: E(G)→{-1, 1} defined by $f^*(uv)$ =f(u)f(v). Then f is called a total signed product cordial labeling if $|f(i) - f(j)| \leq 1$ where i, j ∈{-1, 1} and f(x) denotes the total number of vertices and edges with x (x = -1, 1). A graph with a k-Total signed product cordial labeling is called a k-total signed product cordial graph.

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Definition 1.5: For a graph G the splitting graph S'(G) of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that N(V) = N(V').

Definition 1.6: Let G= (V(G),E(G)) be a graph with V= $S_1 \cup S_2 \cup \ldots \cup S_i \cup T$ where each S_i is a set of vertices having atleast two vertices of the same degree and T= $V \setminus \cup S_i$. The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, \ldots, w_i$ and joining to each vertex of S_i for $1 \le i \le n$.

Definition 1.7: The Shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G '', join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

Definition 1.8: The graph G^2 is a graph with the vertex set V as in G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

2. MAIN RESULTS

Theorem 2.1: $S'(K_{1,n})$ is a signed product cordial graph.

Proof: Let $v_1, v_2, \ldots, v_n$ be pendant vertices and Let v be the apex vertex of $K_{1,n}$ and $u, u_1, u_2, \ldots, u_n$ are added vertices corresponding to $v_1, v_2, \ldots, v_n$ to obtain $S'(K_{1,n})$.

Let G be the graph $S'(K_{1,n})$. Then $|V(G)| = 2n + 2$ and $|E(G)| = 3n$.

Define f: V(G) →{-1, 1} as follows,

Case (i): When n is odd
f(u) =1, f(v)=-1
$f(u_i) = 1$ $\quad i \equiv 1, 3 \pmod 4$
$f(u_i) = -1$ $\quad i \equiv 0, 2 \pmod 4$
$f(v_i) = 1$ $\quad i \equiv 0, 2 \pmod 4$
$f(v_i) = -1$ $\quad i \equiv 1, 3 \pmod 4$

Case (ii): When n is even
f(u) =1, f(v)=-1
$f(u_i) = -1$ $\quad i \equiv 1, 3 \pmod 4$
$f(u_i) = 1$ $\quad i \equiv 0, 2 \pmod 4$
$f(v_i) = -1$ $\quad i \equiv 1, 3 \pmod 4$
$f(v_i) = 1$ $\quad i \equiv 0, 2 \pmod 4$

In view of the above labeling pattern, we have
$V_f(1) = V_f(-1) = n+1$, $e_f(1) = e_f(-1) = 3n / 2$. Hence, $S'(K_{1,n})$ is a **SPCL**.

Example 2.2: $S'(K_{1,3})$ is a signed product cordial graph.

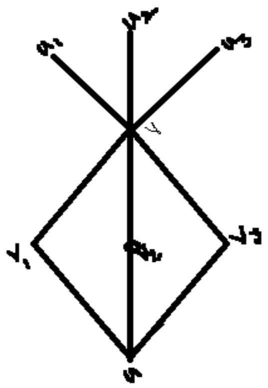

Fig.1

Theorem 2.3: S' $(B_{n,n})$ is a signed product cordial graph.

Proof: Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i; 1 \le i \le n\}$ where u_i, v_i are pendant vertices. In order to obtain $S'(B_{n,n})$ add the vertices u', v' u_i', v_i' corresponding to u, v, u_i, v_i for $1 \le i \le n$.

If G= S' $(B_{n,n})$ then $|V(G)|$= 4(n+1) and $|E(G)|$= 6n+3.

Define vertex labeling f: v (G) →{-1, 1} as follows,

Case (i): When n is odd
$f(u) = -1$, $f(u') = 1$
$f(u_i) = 1$ $\quad i \equiv 0, 2 \pmod 4$
$f(u_i) = -1$ $\quad i \equiv 1, 3 \pmod 4$
$f(u_i') = 1$ $\quad i \equiv 1, 3 \pmod 4$
$f(u_i') = -1$ $\quad i \equiv 0, 2 \pmod 4$

$f(v) = -1$, $f(v') = 1$
$f(v_i) = -1$ $\quad i \equiv 1, 3 \pmod 4$
$f(v_i) = 1$ $\quad i \equiv 0, 2 \pmod 4$
$f(v_i') = 1$ $\quad i \equiv 1, 3 \pmod 4$
$f(v_i') = -1$ $\quad i \equiv 0, 2 \pmod 4$

In view of the above labeling pattern, we have $v_f(1) = v_f(-1) = n+1$ and $e_f(-1) - 1 = e_f(1) = 3n + 1$.

Case (ii): When n is even
$f(u) = -1$, $f(u') = 1$
$f(u_i) = 1$ $\quad i \equiv 1, 3 \pmod 4$
$f(u_i) = -1$ $\quad i \equiv 0, 2 \pmod 4$
$f(u_i') = 1$ $\quad i \equiv 1, 3 \pmod 4$
$f(u_i') = -1$ $\quad i \equiv 0, 2 \pmod 4$

$f(v) = -1$, $f(v') = 1$
$f(v_i) = 1$ $\quad i \equiv 1, 3 \pmod 4$
$f(v_i) = -1$ $\quad i \equiv 0, 2 \pmod 4$
$f(v_i') = 1$ $\quad i \equiv 1, 3 \pmod 4$
$f(v_i') = -1$ $\quad i \equiv 0, 2 \pmod 4$

In view of the above labeling pattern, we have $v_f(1) = v_f(-1) = n+1$ and $e_f(-1)-1 = e_f(1) = 3n +1$.

Hence $S'(B_{n,n})$ is a signed product cordial.

Example 2.4: $S'(B_{6,6})$ is a signed product cordial.

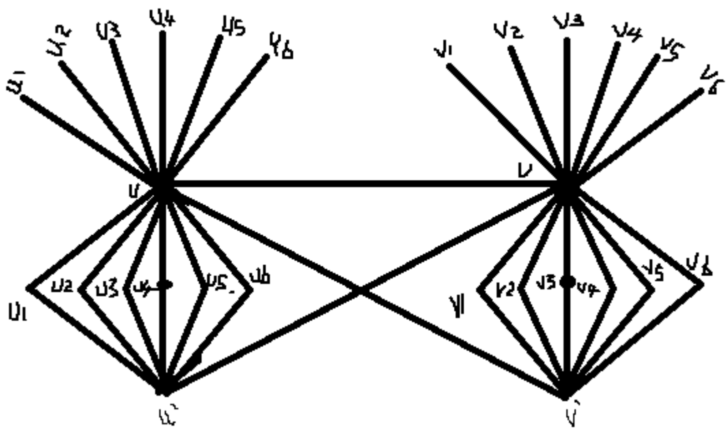

Fig.2

Theorem 2.5: $D_2(B_{n,n})$ is a signed product cordial

Proof: Consider two copies of $B_{n,n}$.

Let $\{u, v, u_i, v_i \; ; 1 \le i \le n\}$ and $\{u', v', u_i', v_i'; 1 \le i \le n\}$ be the corresponding vertex sets of each copy of $B_{n,n}$.

Let G be the graph $D_2(B_{n,n})$ then $|V(G)| = 4(n+1)$ and $|E(G)| = 4(2n+1)$

Define vertex labeling f: v(G) →{-1, 1} as follows,

Case (i): When n is odd
f(u)= -1, f(v) = -1

f(u') =1, f(v') = 1
$f(u_i) = -1 \qquad i \equiv 1, 3 \pmod 4$
$f(u_i) = 1 \qquad i \equiv 0, 2 \pmod 4$
$f(u_i') = 1 \qquad i \equiv 1, 3 \pmod 4$
$f(u_i') = -1 \qquad i \equiv 0, 2 \pmod 4$
$f(v_i) = -1 \qquad i \equiv 1, 3 \pmod 4$
$f(v_i) = 1 \qquad i \equiv 0, 2 \pmod 4$
$f(v_i') = 1 \qquad i \equiv 1, 3 \pmod 4$
$f(v_i') = -1 \qquad i \equiv 0, 2 \pmod 4$

Case (ii): When n is even
f(u)= -1, f(v) = -1

f(u') =1, f(v') = 1
$f(u_i) = 1 \qquad i \equiv 1, 3 \pmod 4$
$f(u_i) = -1 \qquad i \equiv 0, 2 \pmod 4$
$f(u_i') = 1 \qquad i \equiv 1, 3 \pmod 4$
$f(u_i') = -1 \qquad i \equiv 0, 2 \pmod 4$
$f(v_i) = 1 \qquad i \equiv 1, 3 \pmod 4$
$f(v_i) = -1 \qquad i \equiv 0, 2 \pmod 4$
$f(v_i') = 1 \qquad i \equiv 1, 3 \pmod 4$
$f(v_i') = -1 \qquad i \equiv 0, 2 \pmod 4$

Hence, $D_2(B_{n,n})$ is a signed product cordial

Example 2.6: $D_2(B_{5,5})$ is a signed product cordial

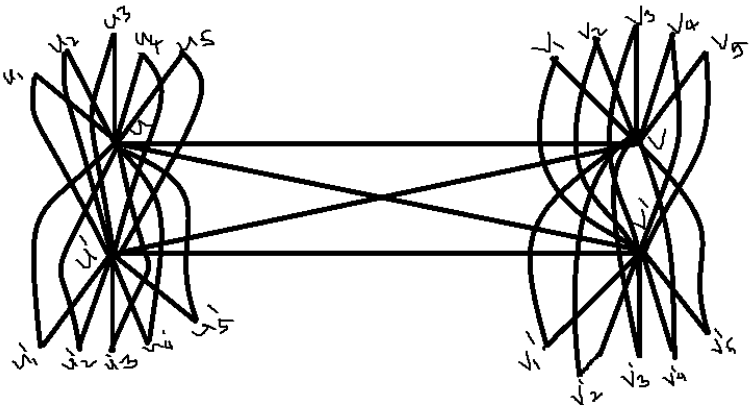

Fig.3

Theorem 2.7: $B^2_{n,n}$ is a signed product cordial.

Proof: Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i; 1 \le i \le n\}$ where u, v are pendant vertices.

Let G be the graph $B^2_{n,n}$ then $|V(G)|$= 2n+2 and $|E(G)|$= 4n+1.

Define vertex labeling f: v(G) →{-1,1} as follows,
f(u)= -1
f(v) = 1
$f(u_i) = -1 \qquad i \equiv 1, 3 \pmod 4$
$f(u_i) = 1 \qquad i \equiv 0, 2 \pmod 4$
$f(v_i) = 1 \qquad i \equiv 1, 3 \pmod 4$
$f(v_i) = -1 \qquad i \equiv 0, 2 \pmod 4$

In view of the above labeling pattern, we have $v_f(1) = v_f(-1) = n+1$ and $e_f(-1)-1 = e_f(1) = 2n$.

Hence, $B^2_{n,n}$ is a signed product cordial.

Example 2.8: $B^2_{5,5}$ is a signed product cordial.

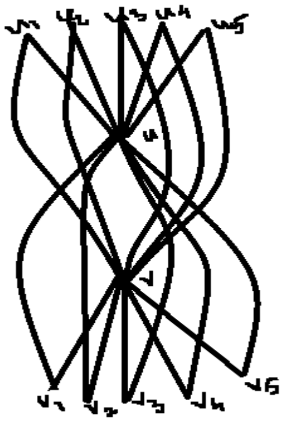

Fig.4

CONCLUSION

As all the graphs are not signed product cordial graphs it is very interesting to investigate signed product cordial labeling for the graph or graph families which admit ` signed product cordial labeling. Here we have contributed some new results by investigating signed product cordial labeling for some star and bistar related graphs.

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