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# ON NEIGHBOURLY EDGE IRREGULAR FUZZY GRAPHS

N. R. SANTHI MAHESWAR[1]*, C. SEKAR[2]

[1]Department of Mathematics,
G. Venkataswamy Naidu College, Kovilpatti-628502, Tamil Nadu, India.

[2]Department of Mathematics,
Aditanar College of Arts and Science, Tiruchendur, Tamil Nadu, India.

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***ABSTRACT***

*In this paper, neighbourly edge irregular fuzzy graphs, neighbourly edge totally irregular fuzzy graphs are introduced. Comparative study between neighbourly edge irregular fuzzy graph and neighbourly edge totally irregular fuzzy graph is done. Some properties of neighbourly edge irregular fuzzy graphs are studied and they are examined for neighbourly edge totally irregular fuzzy graphs.*

***Key Words:*** *degree and total degree of a vertex in fuzzy graph, neighbourly irregular fuzzy graph, neighbourly totally irregular fuzzy graph, edge degree in fuzzy graph, total edge degree in fuzzy graph, edge regular fuzzy graphs, strongly edge irregular fuzzy graph.*

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## 1. INTRODUCTION

Euler first introduced the concept of graph theory in 1736. In 1965, Lofti A.Zadeh[10] introduced the concept of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situation. Azriel Rosenfeld introduced fuzzy graphs in 1975[7]. It has been growing fast and has numerous applications in various fields. The relation between fuzzy sets were also considered by Rosenfeld, and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts [2]. M.Akram and A.Dudek introduced the cocept of regular bipolar fuzzy graphs.

A. Nagoorgani and K.Radha [4] introduced the concept of regular fuzzy graphs and defined degree of a vertex in fuzzy graphs. A. Nagoorgani and S.R.Latha [3] introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008. S.P.Nandhini and E.Nandhini introduced the concept of strongly irregular fuzzy graphs and discussed about its properties [5].

K. Radha and N. Kumaravel [6] introduced the concept of edge degree, total edge degree in fuzzy graph and edge regular fuzzy graphs and discussed about the degree of an edge in some fuzzy graphs. N.R. Santhi Maheswari and C. Sekar introduced the concept of strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs and discussed about its properties [8]. Also, N.R.Santhi Maheswari and C. Sekar introduced the concept of edge irregular fuzzy graphs and edge totally irregular fuzzy graphs and discussed about its properties [9].

This is the background to introduce neighbourly edge irregular fuzzy graphs, neighbourly edge totally irregular fuzzy graphs and comparative study between neighbourly edge irregular fuzzy graphs and neighbourly edge totally irregular fuzzy graphs is done and neighbourly edge irregularity on some fuzzy graphs whose underlying crisp graphs are a path, a cycle, a star and a barbell graph is also studied.

*Corresponding Author: N. R. Santhi Maheswari[1]*, Department of Mathematics,*
*G. Venkataswamy Naidu College, Kovilpatti-628502, Tamil Nadu, India.*

## 2. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper.

By graph, we mean a pair G= (V, E), where V is the set and E is a relation on V. The elements of V are vertices of G and the elements of E are edges of G.

**Definition 2.1:** A Fuzzy graph denoted by $G : (\sigma, \mu)$ on the graph $G^* : (V,E)$: is a pair of functions $(\sigma, \mu)$ where $\sigma : V \to [0; 1]$ is a fuzzy subset of a set $V$ and $\mu : V X V \to [0; 1]$ is a symmetric fuzzy relation on σ such that for all $u, v$ in $V$ the relation $\mu(u, v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph $G$ is complete if $\mu(u, v) = \mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where $uv$ denotes the line joining $u$ and $v$. $G^* : (V,E)$ is called the underlying crisp graph of the fuzzy graph $G : (\sigma, \mu)$, where σ and μ are called membership function[12].

**Definition 2.2:** Let $G$: $(\sigma, \mu)$ be a fuzzy graph on $G^*$: (V, E). The degree of a vertex $u$ is $d_G(u) = \Sigma\mu(uv)$, for $uv \in E$ and $\mu(uv) = 0$, for $uv$ not in $E$, this is equivalent to $d_G(u) = \Sigma\, \mu(uv)$, $u \neq v \;\; uv \in E$ [6].

**Definition 2.3:** Let $G$: $(\sigma, \mu)$ be a fuzzy graph on $G^*$: (V, E). *If* $d(v) = k$ for all $v \in V$, then $G$ is said to be a regular fuzzy graph of degree $k$[6].

**Definition 2.4:** Let $G$: $(\sigma, \mu)$ be a fuzzy graph on $G^*$: $(V; E)$. The total degree of a vertex $u$ is defined as $td(u) = \Sigma\mu(uv) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$ [6]. If each vertex of $G$ has the same total degree $k$, then $G$ is said to be a totally regular fuzzy graph of degree $k$ or $k$-totally regular fuzzy graph [6].

**Definition 2.5:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G$: (V, E). Then G is said to be an irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct degrees [5].

**Definition 2.6:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E). Then G is said to be a totally irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct total degrees [5].

**Definition 2.7:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E). Then G is said to be a strongly irregular fuzzy graph if every pair of vertices have distinct degrees [9].

**Definition 2.8:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E). Then G is said to be a strongly totally irregular fuzzy graph if every pair of vertices have distinct total degrees [9].

**Definition 2.9:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E). Then G is said to be a neighbourly irregular fuzzy graph if every pair of adjacent vertices have distinct degrees [5].

**Definition 2.10:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E). Then G is said to be a neighbourly totally irregular fuzzy graph if every pair of adjacent vertices have distinct total degrees [5].

**Definition 2.11:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E).
The degree of an edge $uv$ is defined as $d_G(uv) = d_G(u) + d_G(v) - 2\,\mu(uv)$.
The minimum degree of an edge is $\delta_E(G) = \Lambda\{d_G(uv) : uv \in E\}$
The maximum degree of an edge is $\Delta_E(G) = V\{d_G(uv) : uv \in E\}$[10]

**Definition 2.12:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E).
The total degree of an edge $uv$ is defined as $td_G(uv) = d_G(u) + d_G(v) - \mu(uv)$.
The minimum total degree of an edge is $\delta_{tE}(G) = \Lambda\{td_G(uv) : uv \in E\}$
The maximum total degree of an edge is $\Delta_{tE}(G) = V\{td_G(uv) : uv \in E\}$[10]

**Definition 2.13:** The degree of an edge $uv$ in the underlying graph is defined as $d_G(uv) = d_G(u) + d_G(v) - 2$[1].

## 3. NEIGHBOURLY EDGE IRREGULAR FUZZY GRAPHS AND NEIGHBOURLY EDGE TOTALLY IRREGULAR FUZZY GRAPHS.

In this section, neighbourly edge irregular graphs and neighbourly edge totally irregular graphs are introduced.

**Definition 3.1:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E). Then G is said to be a neighbourly edge irregular fuzzy graph if every pair of adjacent edges have distinct degrees.

**Definition 3.2:** Let $G: (\sigma, \mu)$ be a connected fuzzy graph on $G^*: (V, E)$. Then G is said to be a neighbourly edge totally irregular fuzzy graph if every pair of adjacent edges have distinct total degrees.

**Example 3.3:** Graph which is both neighbourly edge irregular fuzzy graph and neighbourly edge totally irregular fuzzy graph.

Consider $G^*: (V, E)$ where $V = \{u; v; w; x\}$ and $E = \{uv; vw; wx; xu\}$

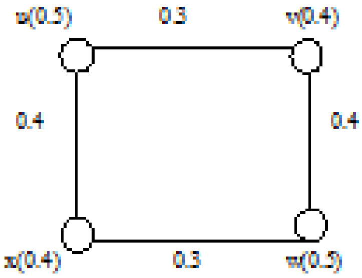


***Figure.1***

From Figure 1, $d_G(u) = 0.7; d_G(v) = 0.7; d_G(w) = 0.7; d_G(x) = 0.7$, Degrees of the edges are calculated below.

$d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv) = 0.7 + 0.7 - 2(0.3) = 0.8.$

$d_G(vw) = d_G(v) + d_G(w) - 2\mu(vw) = 0.7 + 0.7 - 2(0.4) = 0.6.$

$d_G(wx) = d_G(w) + d_G(x) - 2\mu(wx) = 0.7 + 0.7 - 2(0.3) = 0.8.$

$d_G(xu) = d_G(x) + d_G(u) - 2\mu(xu) = 0.7 + 0.7 - 2(0.4) = 0.6.$

It is noted that every pair of adjacent edges have distinct degrees. Hence *G* is a neighbourly edge irregular fuzzy graph.

Total degrees of the edges are calculated below.

$td_G(uv) = d_G(uv) + \mu(uv) = 0.8 + 0.3 = 1.1.$

$td_G(vw) = d_G(vw) + \mu(vw) = 0.6 + 0.4 = 1.$

$td_G(wx) = d_G(wx) + \mu(wx) = 0.8 + 0.3 = 1.1.$

$td_G(uv) = d_G(xu) + \mu(xu) = 0.6 + 0.4 = 1.$

It is observed that every pair of adjacent edges having distinct total degrees. So, *G* is a neighbourly edge totally irregular fuzzy graph. Hence *G* is both neighbourly edge irregular fuzzy graph and neighbourly edge totally irregular fuzzy graph.

**Example 3.4:** Neighbourly edge irregular fuzzy graph need not be neighbourly edge totally irregular fuzzy graph.

Consider $G: (\sigma, \mu)$ be a fuzzy graph such that $G^*: (V, E)$, a star on four vertices.

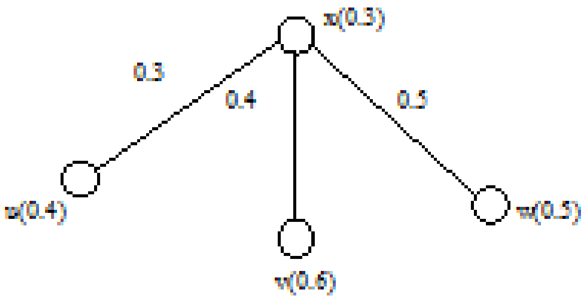


***Figure.2***

From Figure 2, $d_G(u) = 0.3; d_G(v) = 0.4; d_G(w) = 0.5; d_G(x) = 1.2$

$d_G(ux) = 0.9; d_G(vx) = 0.8; d_G(wx) = 0.7.$

$td_G(ux) = 1.2; td_G(vx) = 1.2; td_G(wx) = 1.2.$

Here, $d_G(ux) \neq dG(vx) \neq d_G(wx)$. Hence *G* is a neighbourly edge irregular fuzzy graph. But *G* is not neighbourly edge totally irregular fuzzy graph, since all edges have same total degree.

 

**Example 3.5:** Neighbourly edge totally irregular fuzzy graphs need not be neighbourly edge irregular fuzzy graphs.

Consider $G$: (σ, μ) be a fuzzy graph such that $G^*$: (V, E), a path on four vertices.


***Figure.3***

From Figure 3, $d_G(u) = 0.3$; $d_G(v) = 0.9$; $d_G(w) = 0.9$; $d_G(x) = 0.3$
$d_G(uv) = 0.6$; $d_G(vw) = 0.6$; $d_G(wx) = 0.6$.
$td_G(uv) = 0.9$; $td_G(vw) = 1.2$; $td_G(wx) = 0.9$.

Here, $d_G(uv) = d_G(vw) = d_G(wx)$. Hence $G$ is not neighbourly edge irregular fuzzy graph. But $G$ is neighbourly edge totally irregular fuzzy graph, since $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$.

**Theorem 3.6:** Let $G$: (σ, μ) be a connected fuzzy graph on $G^*$:(V, E) and μ is a constant function. If $G$ is neighbourly edge irregular fuzzy graph, then $G$ is neighbourly edge totally irregular fuzzy graph.

**Proof:** Assume that μ is a constant function, let $\mu(uv) = c$, for all $uv$ in $E$, where $c$ is constant.

Suppose that $G$ is neighbourly edge irregular fuzzy graph⟹every pair of adjacent edges have distinct degrees
$\Rightarrow d_G(uv) \neq d_G(vw)$, where $uv$ and $vw$ are pair of adjacent edges in $E$.

Now, $d_G(uv) \neq d_G(vw) \Rightarrow d_G(uv) + c \neq d_G(vw) + c \Rightarrow d_G(uv) + \mu(uv) \neq d_G(vw) + \mu(vw) \Rightarrow td_G(uv) \neq td_G(vw)$, where $uv$ and $vw$ are pair of adjacent edges in $E$. Hence $G$ is a neighbourly edge totally irregular fuzzy graph.

**Theorem 3.7:** Let $G$: (σ, μ) be a connected fuzzy graph on $G^*$(V, E) and $\mu$ is constant function. If $G$ is neighbourly edge totally irregular fuzzy graph, then $G$ is neighbourly edge irregular fuzzy graph.

**Proof:** Assume that $\mu$ is a constant function, let $\mu(uv) = c$ for all $uv$ *in* $E$, where $c$ is constant. Suppose that $G$ is neighbourly edge totally irregular fuzzy graph ⟹every pair of adjacent edges have distinct total degrees
$\Rightarrow td_G(uv) \neq td_G(vw)$, where $uv$ and $vw$ are pair of adjacent edges *in* E

*Now;* $td_G(uv) \neq td_G(vw) \Rightarrow d_G(uv) + \mu(uv) \neq d_G(vw) + \mu(vw) \Rightarrow d_G(uv) + c \neq d_G(vw) + c \Rightarrow d_G(uv) \neq d_G(vw)$, where $uv$ and $vw$ are pair of adjacent edges in $E$. Hence $G$ is neighbourly edge irregular fuzzy graph.

**Remark 3.8:** Let $G$: (σ, μ) be a connected fuzzy graph on $G^*$(V, E): If G is both neighbourly edge irregular fuzzy graph and neighbourly edge totally irregular fuzzy graph. Then σ need not be constant function.

**Example 3.9:** Consider $G$ : (σ, μ) be a fuzzy graph such that $G^*$ : (V,E) is a path on four vertices.


***Figure.4***

From Figure 4, $d_G(u) = 0.2$; $d_G(v) = 0.5$; $d_G(w) = 0.5$; $d_G(x) = 0.2$
$d_G(uv) = 0.3$; $d_G(vw) = 0.4$; $d_G(wx) = 0.3$.
$td_G(uv) = 0.5$; $td_G(vw) = 0.7$; $td_G(wx) = 0.5$.

Here, $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence $G$ is a neighbourly edge irregular fuzzy graph. Also, $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence $G$ is a neighbourly edge totally irregular fuzzy graph. But $\mu$ is not constant.

**Theorem 3.10:** Let $G$: (σ, μ) be a connected fuzzy graph on $G^*$(V, E) and μ is constant function. If $G$ is strongly irregular fuzzy graph, then $G$ is neighbourly edge irregular fuzzy graph.

**Proof:** Let $G$ : (σ, μ) be a connected fuzzy graph on $G^*$(V,E). Assume that μ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where $c$ is constant. Let $uv$ and $vw$ are any two adjacent edges in $G$. Let us suppose that $G$ is strongly irregular fuzzy graph⟹ every pair of vertices in $G$ having distinct degrees

$\Rightarrow d_G(u) \neq d_G(v) \neq d_G(w) \Rightarrow d_G(u) + d_G(v) - 2c \neq d_G(v) + d_G(w) - 2c$

$\Rightarrow d_G(u)+d_G(v)-2\mu(uv) \neq d_G(v)+d_G(w)-2\mu(uv)$
$\Rightarrow d_G(uv) \neq d_G(vw)$
$\Rightarrow$ every pair of adjacent edges have distinct degrees. Hence $G$ is a neighbourly edge irregular fuzzy graph.

**Theorem 3.11:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and $\mu$ is constant function. If $G$ is strongly irregular fuzzy graph, then $G$ is neighbourly edge totally irregular fuzzy graph.

**Proof:** Proof is similar to the above Theorem 3.10.

**Remark 3.12:** Converse of the above theorems need not be true.

**Example 3.13:** Consider $G$ : $(\sigma, \mu)$ be a fuzzy graph such that $G^*$: (V,E) is a path on four vertices.


***Figure.5***

From Figure 5, $d_G(u) = 0.2$; $d_G(v) = 0.4$; $d_G(w) = 0.4$; $d_G(x) = 0.2$. Here, $G$ is not strongly irregular fuzzy graph.
$d_G(uv) = 0.2$; $d_G(vw) = 0.4$; $d_G(wx) = 0.2$.
$td_G(uv) = 0.4$; $td_G(vw) = 0.8$; $td_G(wx) = 0.4$.

It is noted that $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence $G$ is neighbourly edge irregular fuzzy graph. Also, $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence $G$ is both $G$ is neighbourly edge irregular fuzzy graph neighbourly edge totally irregular fuzzy graph. But $G$ is not strongly irregular fuzzy graph.

**Theorem 3.14:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and $\mu$ is a constant function. Then $G$ is highly irregular fuzzy graph if and only if $G$ is a neighbourly edge irregular fuzzy graph.

**Proof:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Assume that $\mu$ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where $c$ is constant.

Let $uv$ and $vw$ are any two adjacent edges in $G$. Let us suppose that $G$ is a highly irregular fuzzy graph
$\Rightarrow$ every vertex adjacent to the vertices in $G$ having distinct degrees
$\Rightarrow d_G(u) \neq d_G(w)$
$\Rightarrow d_G(u)+d_G(v) \neq d_G(v)+d_G(w)$
$\Rightarrow d_G(u) + d_G(v) - 2c \neq d_G(v) + d_G(w) - 2c$
$\Rightarrow d_G(u) + d_G(v) - 2\mu(uv) \neq d_G(v) + d_G(w) - 2\mu(uv)$
$\Rightarrow d_G(uv) \neq d_G(vw) \Rightarrow$ every pair of adjacent edges have distinct degrees.

Hence $G$ is a neighbourly edge irregular fuzzy graph.

Conversely, Let $v$ be any vertex adjacent with $u$, $w$ and $x$. Then $uv$, $vw$ and $vx$ are adjacent edges in $G$. Let us suppose that $G$ is a neighbourly edge irregular fuzzy graph$\Rightarrow$ every pair of adjacent edges in $G$ having distinct degrees
$\Rightarrow d_G(uv) \neq d_G(vw) \neq d_G(vx)$
$\Rightarrow d_G(v)+d_G(u)-2\mu(uv) \neq d_G(v)+d_G(w)-2\mu(vw) \neq d_G(v)+d_G(x)-2\mu(vx)$
$\Rightarrow d_G(v)+d_G(u)-2c \neq d_G(v)+ d_G(w) - 2c \neq d_G(v) + d(x) - 2c$
$\Rightarrow d_G(v) + d_G(u) \neq d_G(v) + d_G(w) \neq d_G(v) + d_G(x)$
$\Rightarrow d_G(u) \neq d_G(w) \neq d_G(x)$
$\Rightarrow$the vertex $v$ is adjacent to the vertices with distinct degrees. Hence $G$ is highly irregular fuzzy graph.

**Theorem 3.15:** Let $G$ : $(\sigma, \mu)$ be a connected fuzzy graph on $G^*(V,E)$ and $\mu$ is a constant function. Then $G$ is highly irregular fuzzy graph if and only if $G$ is neighbourly edge totally irregular fuzzy graph.

**Proof:** Proof is similar to the above theorem 3.14.

Some known definitions and theorems in strongly edge irregular fuzzy graph[13].

**Definition 3.20:** Let $G$: $(\sigma, \mu)$ be a connected fuzzy graph on $G^*$: (V, E). Then G is said to be a strongly edge irregular fuzzy graph if every pair of edges have distinct degrees (or) no two edges have same degree [13].

**Definition 3.21:** Let $G$: (σ, μ) be a connected fuzzy graph on $G^*$: (V, E): Then G is said to be a strongly edge totally irregular fuzzy graph if every pair of edges have distinct total degrees (or) no two edges have same total degree [13].

**Theorem 3.22:** Every strongly edge irregular fuzzy graph is neighbourly edge irregular fuzzy graph [13].

**Theorem 3.23:** Every strongly edge totally irregular fuzzy graph is neighbourly edge totally irregular fuzzy graph [13].

**Theorem 3.24:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E); a path on $2m$ ($m > 1$) vertices. If the membership value of the edges $e_1, e_2, e_3,\ldots, e_{2m+1}$ are respectively $c_1, c_2, c_3,\ldots, c_{2m+1}$ such that $c_1 < c_2 < c_3 <\ldots< c_{2m+1}$, then $G$ is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph.

**Theorem 3.25:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E), *a* cycle on $n$ ($n \geq 4$) vertices. If the membership value of the edges $e_1, e_2, e_3,\ldots, e_n$ are respectively $c_1, c_2, c_3,\ldots, c_n$ such that $c_1 < c_2 < c_3 < \ldots < c_n$, then $G$ is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph.

**Theorem 3.26:** Let $G$: (σ, μ)be a fuzzy graph on $G^*$: (V, E), a star $K_{1,n}$. If the membership values of no two edges are same, then $G$ is strongly edge irregular fuzzy graph and $G$ is totally edge regular fuzzy graph.

**Theorem 3.27:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E), a Barbell graph $B_{n,m}$. If the membership values of no two edges are same, then $G$ is strongly edge irregular fuzzy graph.

## 4. NEIGHBOURLY EDGE IRREGULARITY ON A PATH, A CYCLE, A STAR, A BARBELL GRAPH WITH SOME SPECIFIC MEMBERSHIP FUNCTIONS

**Theorem 4.1:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E), a star $K_{1,n}$. If the membership values of no two edges are same, then $G$ is neighbourly edge irregular fuzzy graph.

**Proof:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E), a star $K_{1,n}$. If the membership values of no two edges are same, then by theorems 3.22 and 3.26, $G$ is neighbourly edge irregular fuzzy graph.

**Theorem 4.2:** Let $G$: (σ, μ) be a fuzzy graph such that $G^*$: (V, E), a path on $2m$ ($m > 1$) vertices. If the alternate edges have the same membership value, then $G$ is both neighbourly edge irregular fuzzy graph and neighbourly edge totally irregular fuzzy graph.

**Proof:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E); a path on $2m$ ($m >1$) vertices. If the alternate edges have the same membership values, then

$$\mu(e_i)= \begin{cases} c_1 & \text{if } i \text{ is odd} \\ c_2 & \text{if } i \text{ is even.} \end{cases}$$

where $c_1 \neq c_2$ and $c_2 \neq 2c_1$

$d_G(e_1) = c_1 + c_1 + c_2 - 2c_1 = c_2$

For $i = 3, 5, 7,\ldots, 2m- 3$:

$d_G(e_i) = c_1 + c_2 + c_1 + c_2 - 2c_1 = 2c_2$

For $i = 2, 4, 6,\ldots, 2m - 2$

$d_G(e_i) = c_1 + c_2 + c_1 + c_2 - 2c_2 = 2c_1$

$d_G(e_{2m-1}) = c_1 + c_2 + c_1 - 2c_1 = c_2$

We observed that the adjacent edges have distinct degrees. Hence $G$ is a neighbourly edge irregular fuzzy graphs.

$d_G(e_1) = c_1 + c_1 + c_2 - c_1 = c_1 + c_2$

For $i = 3, 5, 7,\ldots, 2m- 3$

$d_G(e_i) = c_1 + c_2 + c_1 + c_2 - c_1 = c_1 + 2c_2$

For $i = 2, 4, 6,\ldots, 2m - 2$

$td_G(e_i)= c_1 + c_2 + c_1 + c_2 - c_2 = c_2 + 2c_1$

$td_G(e_{2m-1}) = c_1 + c_2 + c_1 - c_1 = c_1 + c_2$

We noted that the adjacent edges have distinct degrees. Hence $G$ is a neighbourly edge totally irregular fuzzy graph.

**Theorem 4.3:** Let $G$: (σ, μ) be a fuzzy graph such that $G^*$: (V, E) is an even cycle of length $2m + 2$. If the alternate edges have the same membership value, then $G$ is both neighbourly edge irregular fuzzy graph and neighbourly edge totally irregular fuzzy graph.

**Proof:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E), *an* even cycle of length $2m+2$. If the alternate edges have the same membership values, then

$$\mu(e_i) = \begin{cases} c_1 & \text{if } i \text{ is odd} \\ c_2 & \text{if } i \text{ is even.} \end{cases}$$

where $c_1 \neq c_2$.

For $i = 2, 4, 6, \ldots, 2m - 2$

$d_G(e_i) = c_1 + c_2 + c_1 + c_2 - 2c_2 = 2c_1$

For $i = 1, 3, 5, \ldots, 2m - 3, 2m - 1$

$d_G(e_i) = c_1 + c_2 + c_1 + c_2 - 2c_1 = 2c_2$

We observed that the adjacent edges have distinct degrees. Hence $G$ is a neighbourly edge irregular fuzzy graphs.

For $i = 2, 4, 6, \ldots, 2m - 2$

$td_G(e_i) = c_1 + c_2 + c_1 + c_2 - c_2 = 2c_1 + c_2$

For $i = 1, 3, 5, 7, \ldots, 2m - 3, 2m - 1$

$td_G(e_i) = c_1 + c_2 + c_1 + c_2 - c_1 = 2c_2 + c_1$

We noted that the adjacent edges have distinct degrees. Hence $G$ is a neighbourly edge totally irregular fuzzy graph.

**Theorem 4.4:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E), a cycle on $n$ ($n \geq 4$) vertices. If the membership value of the edges $e_1, e_2, e_3, \ldots, e_n$ are respectively $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3 < \ldots < c_n$, then $G$ is both neighbourly edge irregular fuzzy graph and neighbourly edge totally irregular fuzzy graph.

**Proof:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E), a cycle on $n$ ($n \geq 4$) vertices. If the membership value of the edges $e_1, e_2, e_3, \ldots, e_n$ are respectively $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3 < \ldots < c_n$, then by theorems 3.22, 3.23 and 3.25 ,$G$ is neighbourly edge irregular fuzzy graph and neighbourly edge totally irregular fuzzy graph.

**Theorem 4.5:** Let $G$: (σ, μ) be a fuzzy graph on $G^*$: (V, E), a Barbell graph $B_{n,m}$. If the membership values of no two edges are the same, then $G$ is neighbourly edge irregular fuzzy graph.

**Proof:** We proved that,let $G$: (σ, μ) be a fuzzy graph on $G$ : (V, E), a Barbell graph $B_{n,m}$. If the membership values of no two edges are the same, then by theorems 3.22 and 3.27, $G$ is neighbourly edge irregular fuzzy graph.

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## REFERENCES

1. S. Arumugam and S. Velammal, Edge domination in graphs, *Taiwanese Journal of Mathematics* Volume 2, Number 2, June 1998, 173-179.
2. M.Akram and Wieslaw A.Dudek, Regular bipolar fuzzy graphs, Neural Computer & Applic (2012) 21 (Suppl):S197-S205.
3. A.Nagoor Gani and S.R.Latha, On Irregular Fuzzy graphs, Applied Mathematical Sciences, 6 (2012), 517-523.
4. A. Nagoorgani and K. Radha, The degree of a vertex in some fuzzy graphs, *International Journal of Algorithms, Computing and Mathematics*, Volume 2, Number 3, August 2009, 107-116.
5. S.P.Nandhini and E.Nandhini Strongly Irregular Fuzzy graphs, *International Journal of Mathematical Archive*, Vol5 (5), 2014, 110-114.
6. K. Radha and N. Kumaravel, Some Properties of edge regular fuzzy graphs, *Jamal Academic Research Journal*, Special issue, 2014, 121- 127.
7. A.Rosenfeld, fuzzy graphs, In:L.A.Zadeh,K.S.Fu, M.Shimura,EDs., Fuzzy sets and Their Applications, *Academic press* (1975),77-95.

 

8. N. R. Santhi Maheswari, Strongly edge irregular fuzzy graphs, accepted to Kragujevac journal of Mahematics.
9. N. R. Santhi Maheswari and C.Sekar, edge irregular fuzzy graphs, accepted to International Journal of Mahematics and soft Computing.
10. L.A. Zadeh, Fuzzy Sets, Information and control 8, 1965, 338-353.

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