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CONNECTED TOTAL DOMINATION IN FUZZY GRAPHS

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ABSTRACT

A total dominating set D of a fuzzy graph G is a connected total dominating set if the induced subgraph <D> is connected. The minimum cardinality of a connected total dominating set of G is the fuzzy connected total domination number $\gamma_{fct}(G)$. Since any nontrivial connected dominating set is also a total dominating set. In this paper we study a connected total domination in fuzzy graphs and investigate the relationship of with other known parameters.

Keywords: *Fuzzy Graph, Total Dominating set, Connected Total Dominating set, Fuzzy Connected Total Domination Number.*

1. INTRODUCTION

The study of dominating sets in graphs was begun by Orge and Berge, V.R.Kulli wrote on theory of domination in graphs. The domination number is introduced by cockayne and Hedetniemi Resenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A.Somasundaram and S.Somasundaram discussed domination fuzzy graph NagoorGani and Chandrasekaran discussed domination in fuzzy graph using strong arc. NagoorGani and Vadivel discussed domination, independent domination and irredundant in fuzzy using strong arcs. The notation used in this Paper follows. Among the various applications to the theory of domination in fuzzy graph, here we consider the fault tolerant property in communication network, that is, even if any communication link to a station is failed, still it can communicate the message to that station.

2. PRELIMINARIES

A fuzzy subset of a non empty set V is a mapping σ: V→[0,1].A fuzzy relation on V is a fuzzy subset of V×V. A *fuzzy graph* G = (σ, μ) is a pair of function σ: V→[0,1] and μ: V × V →[0,1], where μ (u, v) ≤ σ(u)Λ σ (v) for all u, v ∈V. The fuzzy graph H=(τ, ρ) is called a fuzzy *subgraph* of G = (σ, μ) if τ(u) ≤ σ(u) for all u ∈ V and ρ (u, v) ≤ μ (u, v) for all u, v ∈ V. A fuzzy subgraph of H=(τ, ρ) is said to be a *Spanning fuzzy subgraph* of G = (σ, μ) if τ(u) ≤ σ(u) for all u. Let G = (σ, μ) be a fuzzy graph and τ be any fuzzy subset of σ, (ie) τ(u) ≤ σ(u) for all u. Then the fuzzy subgraph of G = (σ, μ) *induced* by τ is the maximal fuzzy subgraph of G = (σ, μ) that has fuzzy nodes set τ. A set D ⊆ V of a fuzzy graph G is said to be independent ifμ (u, v) ≤ σ(u)Λ σ (v) for all u, v in D. The *order p* and *size q* of the fuzzy graph G = (σ, μ) are define by p = $\sum_{v\in V}\sigma(v)$ and q = $\sum_{u,v\in E}\mu(u,v)$. The *complement* of a fuzzy graph G = (σ, μ) is a fuzzy graph G = (σ, μ) where σ = σ and μ^c (u, v) = σ(u) Λ σ (v) -μ (u, v) for all u, v in V. The *strength* of connectedness between two nodes u, v in a fuzzy graph G is μ^∞(u, v) = sup{μ^k(u, v): k=1, 2, 3….} where μ^k(u, v) = sup {μ(u, u_1) Λ μ(u_1, u_2)Λ….Λμ(u_{k-1}, v)}, An arc (u, v) is said to be a *strong arc* if μ(u, v) ≥ μ^∞(u, v) and the node V is said to be a strong neighbor of u. If μ(u, v) = 0 for every v∈V, then u is called *isolated node*. A node cover of a graph G is a nodes that covers all the sizes and an size cover of G is a set of sizes that covers all the nodes. The node (size) covering number (α_0(G), α_1(G)) of G is minimum cardinality of a node (size) cover. A set S of nodes of G is independent if no two nodes in S are adjacent. The independence number β_0(G) of G is the maximum cardinality of an independent set. A β_0 set is a maximum independent set. A set F of nodes of G is independent if no two sizes in F are adjacent. The size independence number β_1(G) of G is the maximum cardinality among the independent sets of sizes. Let G be a fuzzy graph and u be a node in G then there exists node v such that (u, v) is a strong arc then *u dominates v*. A fuzzy graph G = (σ, μ) is said to be *connected* if there exists a strongest path between any two nodes of G.

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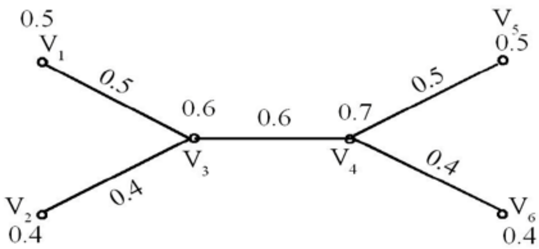
A subset S of V is called a *dominating set* in G if for every v $\notin$S, there exists u$\in$S such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the *domination number* of G and is denoted by γ(G). Let G = (σ, μ) be a fuzzy graph. A subset D of V is said to be *fuzzy dominating set* of G if for every v $\in$ V-D, there exists u$\in$D such that (u, v) is a Strong arc. A fuzzy dominating set D of a fuzzy graph G is called *minimal dominating set* of G, if for every node v $\in$ D, D-{v}is not a dominating set. The *fuzzy domination number* γ_f(G) is the minimum cardinalities taken over all minimal dominating sets nodes of G. A set S $\subseteq$ V is a *total dominating set* if every node in V is adjacent to some node of S. Alternatively, we may define a dominating set D to be a total dominating set if G[D] has no isolated vertices. The *total domination number* of G, denoted γ_t(G) is the cardinality of a smallest total dominating set, and we refer to such a set as a γ_t-set. A *connected dominating set* S to be a dominating set S whose induced subgraph <S> is connected. Since a dominating set must contain atleast one node from each component of G. The minimum cardinality of a connected dominating set is the *connected domination number* γ_c(G). Let G: (σ, μ) be a fuzzy graph. The *degree* of a node u is $d_G(u)= \sum_{u\neq v} \mu(uv)$. Since μ(uv) > 0 for uv $\in$ E and μ(uv) = 0 for uv$\notin$E, This is equivalent to $d_G(u)= \sum_{uv\in E} \mu(uv)$. The *minimum degree* of G is $\delta (G) = \wedge \{d(v) / v\in V\}$. The *maximum degree* of G is $\Delta (G) = \vee \{d(v) / v\in V\}$.

3. MAIN RESULTS

Definition: 3.1 A total dominating set D of a fuzzy graph G is a connected total dominating set if the induced subgraph <D> is connected. The minimum cardinality of a connected total dominating set of G is the fuzzy connected total domination number γ_{fct}(G). Since any nontrivial connected dominating set is also a total dominating set, $\gamma_f(G) \leq \gamma_{ft}(G) \leq \gamma_{fc}(G)$ for any connected graph G with $\Delta(G) < n-1$.

Example: 3.2



D ={v_3,v_4} =1.3,
γ_{fc} = 1.3, γ_{fct} = 1.3, γ_{ft} = 1.3
p = 3.1, q = 2.4, Δ(G) = 1.5, $\in_T$ =1.8

Theorem: 3.3 For any connected fuzzy graph of G with maximum degree Δ, $\frac{p}{\Delta+1} \leq \gamma_{fct}(G) \leq 2q - p$.

Theorem: 3.4 For any connected fuzzy graph G $\gamma_{ft}(G) \leq \gamma_{fc}(G) \leq \gamma_{fct}(G)$.

Proposition: 3.5 Let G be a connected fuzzy graph of order n. Then

(i) Diam (G) $- 1 \leq \gamma_{fct}(G) \leq n - \Delta$
(ii) $\gamma_{fct}(G) \leq 2\beta_1$

Theorem: 3.6 For any connected fuzzy graph,

(i) $\gamma_{fct}(G) \leq 2\beta_0 - 1$
(ii) $\gamma_{fct}(G) \leq 3\gamma - 2$

Theorem: 3.7 For any connected fuzzy graph of order n $\geq$ 2 if G has no isolated node, Then $\gamma_{fct}(G) \leq n - \Delta(G) + 1$.

Proposition: 3.8 For any connected fuzzy graph of G, γ_f G) $\leq \gamma_{fct}$(G) .

Theorem: 3.9 Let G be connected fuzzy graph and let x, y be any two nodes in v. Then there exists a strong path from x to y.

Proof: Since G is connected fuzzy graph, there exists a path from x to y such that $\mu(x_{i-1}, x_i) > 0$ for all $1\leq I \leq n$. If (x_{j-1}, x_j) is not strong for some $1\leq j\leq n$. we must have $\mu(x_{j-1}, x_j) < \mu^{\infty}(x_{j-1}, x_j)$. Hence there exists a path ρ_j from x_{j-1} to x_j Whose strength is greater than $\mu(x_{j-1}, x_j)$, so that all its arcs have weights greater than $\mu(x_{j-1}, x_j)$. If some arc on ρ_j is not strong, this argument can be repeated. Evidently, the argument cannot be repeated arbitrarily often. Thus eventually we can find a path from x to y on which all the arcs are strong.

Theorem: 3.10 If H is a connected spanning subgraph of a fuzzy graph G, then $\gamma_{fct}(G) \leq \gamma_{fct}(H)$.

Since every connected dominating set of H is also a connected dominating set of G.

Notation: Let $\in_T$ denote the maximum number of pendant vertices in any spanning tree of G.

Hedetniemi and Laskar determined the connected domination number of a graph G in terms of $\in_T$.

Theorem: 3.11 For any connected fuzzy graph G with p vertices $\gamma_{fc} + \in_T = p$.

Proof: Let T be any spanning tree of G with $\in_T$ end nodes and let F denote the set of end nodes. Then T-F is a connected dominating set with $p - \in_T$ nodes. Hence $\gamma_{fc} \leq p - \in_T$. Now we prove that $\gamma_{fc} \geq p - \in_T$.

Let D be a connected dominating set with γ_{fc} nodes. Since < D > is a connected subgraph of G.

Let T_D be any spanning tree of < D >.Now we form a spanning tree T of G by adding the remaining $p - \gamma_{fc}$ nodes of G to T_D, joining each of these nodes to one node of D to which it is adjacent. T will have at least $p - \gamma_{fc}$ end nodes.
So, $\in_T \geq p - \gamma_{fc}$
$\gamma_{fc} \geq p - \in_T$, Hence $\gamma_{fc} + \in_T = p$.

Theorem: 3.12 For any connected fuzzy graph of order p, $\gamma_{fct}(G) \leq p - \Delta(G)$.

Proof: Let deg v = $\Delta(G)$. Then a spanning tree T of G can be formed in which v is adjacent to each of its neighbours. So,T has a node v of degree Δ and hence has at least Δ end nodes.

We know that $\gamma_{fc} \leq p - \in_T$.

Since $\Delta \leq \in_T$, $\gamma_{fc} \leq p - \Delta(G)$.

Theorem: 3.13 If both fuzzy graph G and its complement $\bar{G}$ are connected then, $\gamma_{fc} + \bar{\gamma}_{fc} \leq p + 1$, Where $\bar{\gamma}_{fc}$ is the fuzzy connected domination number of $\bar{G}$

Proof: We know that $\gamma_{fc} \leq p - \Delta$ and $\bar{\gamma}_{fc} \leq p - \bar{\Delta}$, where $\bar{\Delta}$ is the maximum degree in $\bar{\gamma}_{fc}$.

$$\begin{aligned}\text{Now, } \gamma_{fc} + \bar{\gamma}_{fc} &\leq (p - \Delta) + (p - \bar{\Delta}) \\ &= 2p - (\Delta + \bar{\Delta}) \\ &= 2p - (\Delta + p - 1 - \delta) \\ &= p + 1 + \delta - \Delta \\ &\leq p + 1, \text{ since } \delta - \Delta \leq 0.\end{aligned}$$

Theorem: 3.14 If G is a connected fuzzy graph with p ≥ 3 nodes, then $\bar{\gamma}_{fct} \leq \frac{2p}{3}$.

Proof: Let S be a γ_t-set of G. By minimality, each v∈S either has a private neighbor or the induced subgraph< S – {v}> contains an isolated node. Let p = {v∈ S: p_n [v, s] ≠ ϕ}. Let B be the set of isolates in <p> and A= p – B. Further, Let C be a minimum set of nodes os S–P such that each node of B is adjacent to some node of C, we note that │C│≤ │B│. Finally, let D = S – (P∪C). This definition of D implies that γ_{fct}(<D>) = │D│, and hence,<D> = k K_2, k ≥ 0. Let a_ib_i 1≤ i≤ k, be the distinct edge of <D>. The connectivity of G implies, without loss of generality, that each a_i is adjacent to some other node x_i. If $x_i \in P \cup C$, then S – {b_i} would be a smaller total dominating set then S. Hence, x_i ∈V–S. If $x_i = x_j$ for i≠j, then S – {b_i, b_j}∪{x_i} affords a similar contradiction. By the definition of D, each x_i is adjacent to atleast two nodes of S and by the definition of P, there are atleast │P│ nodes of V–S that are adjacent to exactly one node of S. Thus,

$$|P| + k \leq |V - S|$$

That is $|A| + |B| + k \leq n - \gamma_{fct}(G)$

$$\begin{aligned}\therefore \gamma_{fct}(G) &= |A| + |B| + |C| + |D| \\ &= (|A| + |B| + k) + (|C| + k).\end{aligned}$$

Since $|C| \leq |A| + |B|$, we have $\gamma_{fct}(G) \leq 2(|A| + |B| + k)$.

Hence $\gamma_{fct}(G) \leq 2(n - \gamma_{fct}(G))$

Thus, $\gamma_{fct}(G) \leq \frac{2p}{3}$.

Theorem: 3.15 For any connected fuzzy graph G of order $p \geq 2$, $\lfloor \frac{p}{q} \rfloor \leq \gamma_{fct}(G) \leq q$.

Theorem: 3.16 Let G be a fuzzy graph of order n with no isolated nodes. Then $\gamma_{ft}(G) \geq \frac{p}{\Delta(G)}$

Proof: Let S be a γ_{ft}–set of G. Then by definition, every node of G is adjacent to some node of S. That is N(S) = V(G). Since every $v \in S$ can have at most neighbors, it follows that $\Delta\gamma_{ft} \geq |V| = n$.

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