

ON $sb\hat{g}$ - CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of sets called $sb\hat{g}$ -closed sets in topological spaces. A subset A of X is said to be $sb\hat{g}$ -closed if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is $b\hat{g}$ -open in X . Also we study some of its basic properties and investigate the relationship with other existing closed sets in topological space. As an application, we introduce two new spaces namely, $T_{sb\hat{g}}$ and $T_{sb\hat{g}}^\alpha$.

Keywords: $b\hat{g}$ -open sets, semi-closure, semi-closed sets, $sb\hat{g}$ -closed sets, $T_{sb\hat{g}}$ -space and $T_{sb\hat{g}}^\alpha$ -space.

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1. INTRODUCTION

N. Levine [6] introduced semi-open sets in Topology and studied its properties in 1963. In 1970, N.Levine[7] introduced generalized closed (briefly g -closed) sets and studied their basic properties. b -open sets have been introduced and investigated by Andrijevic[2] in 1996. M.K.R.S. Veerakumar[14] defined \hat{g} -closed sets in Topological Spaces and studied their properties. Also, R.Subasree and M.MariaSingam[13] introduced $b\hat{g}$ -closed sets and studied its properties in 2013.

Now, we introduce the concept of $sb\hat{g}$ -closed sets and $sb\hat{g}$ -open sets in Topological space and study some of their properties. Applying these sets, we obtain two new spaces namely $T_{sb\hat{g}}$ -space and $T_{sb\hat{g}}^\alpha$ -space.

2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X, τ) , $Cl(A)$, $Int(A)$ and A^c denote the closure of A , interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1: A subset A of a topological space (X, τ) is called

1. a semi-open set[6] if $A \subseteq Cl(Int(A))$.
2. an α -open set[10] if $A \subseteq Int(Cl(Int(A)))$.
3. a b -open set[2] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.
4. a regular open[12] set if $A = Int(Cl(A))$.

The complement of a semi-open (resp. α -open, b -open, regular-open) set is called semi-closed (resp. α -closed, b -closed, regular-closed) set.

The intersection of all semi-closed (resp. α -closed, b -closed, regular-closed) sets of X containing A is called the semi-closure (resp. α -closure, b -closure, regular closure) of A and is denoted by $sCl(A)$ (resp. $\alpha Cl(A)$, $bCl(A)$, $rCl(A)$). The family of all semi-open (resp. α -open, b -open, regular-open) subsets of a space X is denoted by $SO(X)$ (resp. $\alpha O(X)$, $bO(X)$, $rO(X)$).

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Definition 2.2: A subset A of a topological space (X, τ) is called

- 1) a generalized closed set (briefly g -closed)[7] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 2) asg -closed set[4] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- 3) ags -closed set[3] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 4) agb -closed set[1] if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- 5) arb -closed set[9] if $rCl(A) \subseteq U$ whenever $A \subseteq U$ and U is b -open in X .
- 6) a g^*b -closed set[16] if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .
- 7) $ag\hat{g}$ -closed set[14] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- 8) $ab\hat{g}$ -closed set[13] if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in X .
- 9) a $\alpha b\hat{g}$ -closed set[11] if $\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $b\hat{g}$ -open in X .

The complement of a g -closed (resp. sg -closed, gs -closed, gb -closed, rb -closed, g^*b -closed, \hat{g} -closed, $b\hat{g}$ -closed and $\alpha b\hat{g}$ -closed) set is called g -open (resp. sg -open, gs -open, gb -open, rb -open, g^*b -open, \hat{g} -open, $b\hat{g}$ -open and $\alpha b\hat{g}$ -open) set.

Definition 2.3: $sCl(A)$ is defined as the intersection of all semi-closed sets containing A .

Definition 2.4: A space (X, τ) is called a

- (i) a T_b -space[5] if every gs -closed set in X is closed.
- (ii) a T_{gs} -space[1] if every gb -closed set in X is b -closed.
- (iii) a $T_{b\hat{g}}$ -space[13] if every $b\hat{g}$ -closed set in X is b -closed.
- (iv) a $T_{b\hat{g}}^*$ -space[13] if every $b\hat{g}$ -closed set in X is closed.
- (v) a $T_{\alpha b\hat{g}}^c$ -space[11] if every $\alpha b\hat{g}$ -closed set in X is closed.

3. $sb\hat{g}$ -CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of a topological space (X, τ) is called a $sb\hat{g}$ -closed set if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is $b\hat{g}$ -open in X . The family of all $sb\hat{g}$ -closed sets of X are denoted by $sb\hat{g}-C(X)$.

Definition 3.2: The complement of a $sb\hat{g}$ -closed set is called $sb\hat{g}$ -open set. The family of all $sb\hat{g}$ -open sets of X are denoted by $sb\hat{g}-O(X)$.

Example 3.3: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ then $\{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ are $sb\hat{g}$ -closed sets and $\{X, \phi, \{b\}, \{a\}, \{a, c\}, \{a, b\}\}$ are $sb\hat{g}$ -open sets in X .

Proposition 3.4: Every closed set is $sb\hat{g}$ -closed set

Proof: Let A be any closed set in X and U be any $b\hat{g}$ -open set in X such that $A \subseteq U$. Since A is closed, $Cl(A) = A$ for every subset A of X . Therefore, $sCl(A) \subseteq Cl(A) = A \subseteq U$. Hence, A is $sb\hat{g}$ -closed set.

The following example shows that the converse of the above proposition need not be true.

Example 3.5: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. $sb\hat{g}-C(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Here, $\{b\}, \{c\}$ are $sb\hat{g}$ -closed sets but not closed sets in X .

Proposition 3.6: A subset A of (X, τ) is semi-closed set in X iff A is $sb\hat{g}$ -closed set in X .

Proposition 3.7: Every α -closed set is $sb\hat{g}$ -closed set.

Proof: Let A be any α -closed set in X such that $A \subseteq U$ where U is $b\hat{g}$ -open. Since A is α -closed set, $sCl(A) \subseteq \alpha Cl(A) \subseteq U$. Therefore, $sCl(A) \subseteq U$. Hence, A is $sb\hat{g}$ -closed set.

The converse of the above proposition need not be true as shown in the following example.

Example 3.8: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. $\alpha-C(X) = \{X, \phi, \{a\}, \{a, b\}, \{b, c\}\}$ and $sb\hat{g}-C(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Here, $\{b\}$ is $sb\hat{g}$ -closed set but not α -closed set in X .

Proposition 3.9: Every regular closed set is $sb\hat{g}$ -closed set.

Proof: Let A be any regular closed set in X such that $A \subseteq U$ where U is $b\hat{g}$ -open. Since A is regular closed set, $sCl(A) \subseteq rCl(A) \subseteq U$. Therefore, $sCl(A) \subseteq U$. Hence, A is $sb\hat{g}$ -closed set.

The reverse implication does not hold as shown in the following example.

Example 3.10: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{b\}\}$. $r-C(X) = \{X, \phi\}$ and $sb\hat{g}-C(X) = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. Here, $\{a\}, \{c\}, \{a, c\}$ are $sb\hat{g}$ -closed sets but not regular closed sets in X .

Proposition 3.11: Every $sb\hat{g}$ -closed set is b -closed set.

Proof: Let A be any $sb\hat{g}$ -closed set in X such that $A \subseteq U$ where U is $b\hat{g}$ -open. Since A is $sb\hat{g}$ -closed set, $bCl(A) \subseteq sCl(A) \subseteq U$. Therefore, $bCl(A) \subseteq U$. Hence, A is $sb\hat{g}$ -closed set.

The converse of the above proposition need not be true as shown in the following example.

Example 3.12: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{a, b\}\}$. $bC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $sb\hat{g}-C(X) = \{X, \phi, \{c\}, \{a, b\}\}$. Here, $\{a\}, \{b\}, \{a, c\}, \{b, c\}$ are $sb\hat{g}$ -closed sets but not b -closed sets.

Proposition 3.13: Every $sb\hat{g}$ -closed set is sg -closed set

Proof: Let A be any $sb\hat{g}$ -closed set in X and U be any semi-open set in X such that $A \subseteq U$. Since “Every semi-open set is $b\hat{g}$ -open set”, we have $sCl(A) \subseteq U$ where U is semi-open. Hence, A is sg -closed.

Every sg -closed set need not be $sb\hat{g}$ -closed set as shown in the following example.

Example 3.14: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, c\}\}$. $sg-C(X) = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$ and $sb\hat{g}-C(X) = \{X, \phi, \{b\}\}$. Here, $\{a, b\}, \{b, c\}$ are sg -closed sets but not $sb\hat{g}$ -closed sets.

Proposition 3.15: Every $sb\hat{g}$ -closed set is gs -closed set

Proof: Let A be any $sb\hat{g}$ -closed set in X and U be any open set in X such that $A \subseteq U$. Since “Every open set is $b\hat{g}$ -open set”, we have $sCl(A) \subseteq U$ where U is open. Hence, A is gs -closed.

The following example shows that the converse of the above proposition need not be true.

Example 3.16: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$. $gs-C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $sb\hat{g}-C(X) = \{X, \phi, \{a\}, \{b, c\}\}$. Here, $\{b\}, \{c\}, \{a, c\}, \{a, b\}$ are gs -closed sets but not $sb\hat{g}$ -closed sets.

Proposition 3.17: Every $sb\hat{g}$ -closed set is gb -closed set.

Proof: Let A be any $sb\hat{g}$ -closed set in X . Let U be open set such that $A \subseteq U$. Since, “Every open set is $b\hat{g}$ -open”, we have $bCl(A) \subseteq sCl(A) \subseteq U$. Therefore, $bCl(A) \subseteq U$ where U is open in X . Hence, A is gb -closed set.

The converse of the above proposition need not be true as shown in the following example.

Example 3.18: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. $gb-C(X) = \{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $sb\hat{g}-C(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Here, $\{a, c\}$ is gb -closed set but not $sb\hat{g}$ -closed set.

Proposition 3.19: Every rb -closed set is $sb\hat{g}$ -closed set.

Proof: Let A be any rb -closed set in X . Let U be any b -open set in X such that $A \subseteq U$. Since “Every b -open set is $b\hat{g}$ -open set”, we have $sCl(A) \subseteq rCl(A) \subseteq U$ where U is $b\hat{g}$ -open. Therefore, $sCl(A) \subseteq U$. Hence, A is $sb\hat{g}$ -closed set.

The reverse implication does not hold as shown in the following example.

Example 3.20: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{a, c\}, \{a, b, d\}\}$. $rb-C(X) = \{X, \phi, \{b, c, d\}\}$ and $sb\hat{g}-C(X) = \{X, \phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$. Here, $\{b\}, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{b, d\}$ are $sb\hat{g}$ -closed sets but not rb -closed sets.

Proposition 3.21: Every $sb\hat{g}$ -closed set is g^*b -closed set.

Proof: Let A be any $sb\hat{g}$ -closed set in X . Let U be g -open set such that $A \subseteq U$. Since, “Every g -open set is $b\hat{g}$ -open set”, we have $bCl(A) \subseteq sCl(A) \subseteq U$. Therefore, $bCl(A) \subseteq U$ where U is g -open in X . Hence, A is g^*b -closed set.

The following example shows that the converse of the above proposition need not be true

Example 3.22: Let $X = \{a, b, c, \}$ and $\tau = \{X, \phi, \{c\}, \{a, b\}\}$. $g^*b\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $sb\hat{g}\text{-}C(X) = \{X, \phi, \{c\}, \{a, b\}\}$. Here, $\{a\}, \{b\}, \{a, c\}, \{b, c\}$ are g^*b -closed sets but not $sb\hat{g}$ -closed sets.

Proposition 3.23: Every $sb\hat{g}$ -closed set is $b\hat{g}$ -closed set.

Proof: Let A be any $sb\hat{g}$ -closed set. By proposition 3.11, A is b -closed set in X . By proposition 3.3 in [13], A is $b\hat{g}$ -closed set in X .

The converse of the above proposition need not be true as shown in the following example.

Example 3.24: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c, d\}\}$. $b\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$ and $sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$. Here $\{a, b, c\}, \{a, b, d\}$ are $b\hat{g}$ -closed sets but not $sb\hat{g}$ -closed sets.

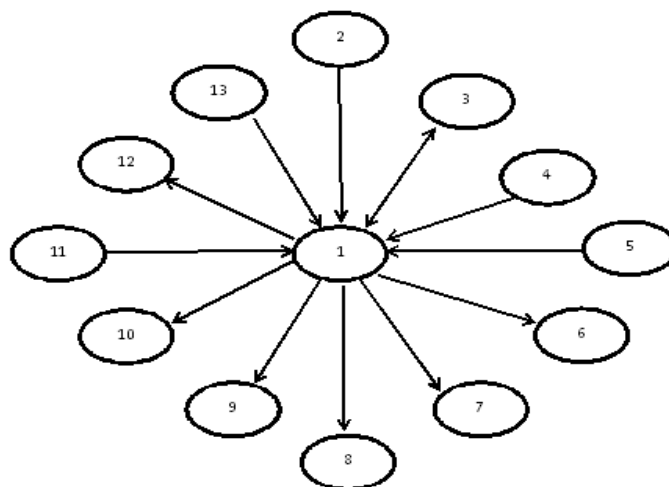
Proposition 3.25: Every $ab\hat{g}$ -closed set is $sb\hat{g}$ -closed set.

Proof: Let A be any $ab\hat{g}$ -closed set. Let U be any $b\hat{g}$ -open set in X such that $A \subseteq U$. To prove that, A is $sb\hat{g}$ -closed set. Now, $sCl(A) \subseteq \alpha Cl(A) \subseteq U$ where U is $b\hat{g}$ -open set. Therefore, A is $sb\hat{g}$ -closed set.

The converse of the above proposition need not be true as shown in the following example.

Example 3.26: Let $X = \{a, b, c, \}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. $ab\hat{g}\text{-}C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$ and $sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. Here, $\{a\}, \{b\}$ are $sb\hat{g}$ -closed sets but not $ab\hat{g}$ -closed sets.

Remark 3.27: The following diagram shows the relationship of $sb\hat{g}$ -closed sets with other known existing sets. $A \rightarrow B$ represents A implies B but not conversely.



- | | | | |
|--------------------------|--------------------|------------------|------------------------|
| 1. $sb\hat{g}$ -closed | 2. closed | 3. semi-closed | 4. α -closed |
| 5. regular-closed | 6. b -closed | 7. gs -closed | 8. sg -closed |
| 9. gb -closed | 10. g^*b -closed | 11. rb -closed | 12. $b\hat{g}$ -closed |
| 13. $ab\hat{g}$ -closed. | | | |

4. CHARACTERIZATION

Lemma 4.1: The finite union of $sb\hat{g}$ -closed sets is $sb\hat{g}$ -closed set.

Lemma 4.2: The finite intersection of $sb\hat{g}$ -closed sets is $sb\hat{g}$ -closed set.

Proposition 4.3: Let A be a $sb\hat{g}$ -closed set of X . Then $sCl(A) - A$ does not contain a non-empty $b\hat{g}$ -closed set.

Proof: Suppose A is a $sb\hat{g}$ -closed set. Let F be a $b\hat{g}$ -closed set contained in $sCl(A) - A$. Now F^c is a $b\hat{g}$ -open set of X such that $A \subseteq F^c$. Since A is $sb\hat{g}$ -closed, we have $sCl(A) \subseteq F^c$. Hence, $F \subseteq (sCl(A))^c$. Also, $F \subseteq sCl(A) - A$. Therefore, $F \subseteq sCl(A) \cap (sCl(A))^c = \phi$. Hence, F must be ϕ .

Proposition 4.5: If A is $b\hat{g}$ -open and $sb\hat{g}$ -closed set of X , then A is semi-closed.

Proof: Since A is $b\hat{g}$ -open and $sb\hat{g}$ -closed, we have $sCl(A) \subseteq A$. Hence, A is semi-closed.

Proposition 4.6: The intersection of a $sb\hat{g}$ -closed set and a semi-closed set of X is always $sb\hat{g}$ -closed set.

Proof: Let A be a $sb\hat{g}$ -closed set and B be a semi-closed set. Since A is $sb\hat{g}$ -closed, $sCl(A) \subseteq U$ whenever U is $b\hat{g}$ -open. Let B be such that $A \cap B \subseteq U$ where U is $b\hat{g}$ -open. Now, $sCl(A \cap B) \subseteq sCl(A) \cap sCl(B) \subseteq U \cap B \subseteq U$. Hence, $A \cap B$ is $sb\hat{g}$ -closed set. Therefore, intersection of any $sb\hat{g}$ -closed set and a semi-closed set of X is always $sb\hat{g}$ -closed set.

5. APPLICATIONS

As an applications of $sb\hat{g}$ -closed sets, we introduce two new spaces namely, $T_{sb\hat{g}}$ -space and $T_{sb\hat{g}}^\alpha$ -space.

Definition 5.1: A Space (X, τ) is called a $T_{sb\hat{g}}$ -space if every $sb\hat{g}$ -closed set in X is closed.

Definition 5.2: A Space (X, τ) is called a $T_{sb\hat{g}}^\alpha$ -space if every $sb\hat{g}$ -closed set in X is α -closed.

Proposition 5.3: Every T_b -space is $T_{sb\hat{g}}$ -space.

Proof: Let (X, τ) be T_b -space. Let A be $sb\hat{g}$ -closed set in (X, τ) . By proposition 3.15, A is gs -closed. Since (X, τ) is T_b -space, A is closed. Hence, (X, τ) is $T_{sb\hat{g}}$ -space.

The converse of the above proposition need not be true as shown in the following example.

Example 5.4: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{a, b\}\}$

$sb\hat{g}\text{-}C(X) = \{X, \phi, \{c\}, \{a, b\}\}$

$gs\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$C(X) = \{X, \phi, \{c\}, \{a, b\}\}$

Here, (X, τ) is $T_{sb\hat{g}}$ space but not T_b -space.

Proposition 5.5: Every $T_{sb\hat{g}}$ -space is T_{gs} -space.

Proof: Let (X, τ) be $T_{sb\hat{g}}$ -space. Let A be $s\hat{g}$ -closed set in (X, τ) . By Proposition 3.17, A is gb -closed. Since every closed set is b -closed set, A is b -closed set in X . Therefore, (X, τ) is T_{gs} -space.

The converse of the above proposition need not be true and is explained in the following example.

Example 5.6: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$

$sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$gb\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$bC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$

Here (X, τ) is T_{gs} -space but not $T_{sb\hat{g}}$ -space.

Proposition 5.7: Every $T_{sb\hat{g}}$ -space is $T_{b\hat{g}}$ -space.

Proof: Let (X, τ) be $T_{sb\hat{g}}$ -space. Let A be $s\hat{g}$ -closed set in (X, τ) . By Proposition 3.23, A is \hat{g} -closed. Since every closed set is b -closed set, A is b -closed set in X . Therefore, (X, τ) is $T_{b\hat{g}}$ -space.

The reverse implication does not hold as shown in the following example.

Example 5.8: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$

$sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$b\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$bC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

$C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$

Here, (X, τ) is $T_{b\hat{g}}$ -space but not $T_{sb\hat{g}}$ -space.

Proposition 5.9: Every $T_{b\hat{g}}^*$ -space is $T_{sb\hat{g}}$ -space.

Proof: Let (X, τ) be $T_{b\hat{g}}^*$ -space. Let A be $sb\hat{g}$ -closed set in (X, τ) . By Proposition 3.23, A is \hat{g} -closed. Since (X, τ) is $T_{b\hat{g}}^*$ -space, A is closed set in X . Therefore, (X, τ) is $T_{sb\hat{g}}$ -space.

The reverse implication need not be true as shown in the following example.

Example 5.10: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$
 $sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b, c\}\}$
 $b\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$
 $C(X) = \{X, \phi, \{a\}, \{b, c\}\}$
 Here, (X, τ) is $T_{sb\hat{g}}$ -space but not $T_{b\hat{g}}^*$ -space.

Proposition 5.11: Every $T_{b\hat{g}}^*$ -space is $T_{sb\hat{g}}^\alpha$ -space.

Proof: Let (X, τ) be $T_{b\hat{g}}^*$ -space. Let A be $sb\hat{g}$ -closed set in (X, τ) . By Proposition 3.23, A is $b\hat{g}$ -closed. Since (X, τ) is $T_{b\hat{g}}^*$ -space, A is closed set in X . Since every closed set is α -closed set, A is α -closed in X . Therefore, (X, τ) is $T_{sb\hat{g}}^\alpha$ -space.

The following example shows that the converse of the above proposition need not be true.

Example 5.12: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$
 $sb\hat{g}\text{-}C(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$
 $b\hat{g}\text{-}C(X) = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$
 $\alpha C(X) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$
 $C(X) = \{X, \phi, \{b, c\}\}$
 Here, (X, τ) is $T_{sb\hat{g}}^\alpha$ -space but not $T_{b\hat{g}}^*$ -space.

Proposition 5.13: Every $T_{sb\hat{g}}$ -space is $T_{ab\hat{g}}^c$ -space.

Proof: Let (X, τ) be $T_{sb\hat{g}}$ -space. Let A be $ab\hat{g}$ -closed set in (X, τ) . By Proposition 3.25, A is $sb\hat{g}$ -closed. Since (X, τ) is $T_{sb\hat{g}}$ -space, A is closed. Therefore, (X, τ) is $T_{ab\hat{g}}^c$ -space.

The converse of the above proposition need not be true and is explained in the following example.

Example 5.14: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$
 $sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$
 $ab\hat{g}\text{-}C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$
 $C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$
 Here, (X, τ) is $T_{ab\hat{g}}^c$ -space but not $T_{sb\hat{g}}$ -space.

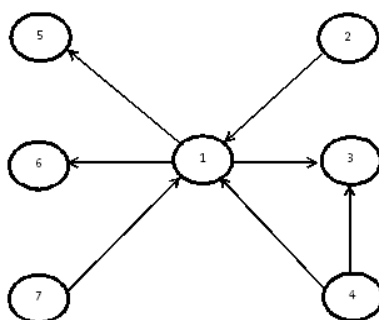
Proposition 5.15: Every $T_{sb\hat{g}}$ -space is $T_{sb\hat{g}}^\alpha$ -space.

Proof: Let (X, τ) be $T_{sb\hat{g}}$ -space. Let A be $sb\hat{g}$ -closed set in (X, τ) . Since (X, τ) is $T_{sb\hat{g}}$ -space, A is closed. Since every closed set is α -closed set, A is α -closed set in X . Therefore, (X, τ) is $T_{sb\hat{g}}^\alpha$ -space.

The following example shows that the converse of the above proposition need not be true.

Example 5.16: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{b\}\}$
 $sb\hat{g}\text{-}C(X) = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$
 $\alpha C(X) = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$
 $C(X) = \{X, \phi, \{a, c\}\}$
 Here, (X, τ) is $T_{sb\hat{g}}^\alpha$ -space but not $T_{sb\hat{g}}$ -space.

Remark 5.17: The following diagram shows the relationship about $T_{sb\hat{g}}$ -space and $T_{sb\hat{g}}^\alpha$ -space with other known existing spaces.



- | | | |
|----------------------------|-----------------------------|-----------------------------------|
| 1. $T_{sb\hat{g}}$ -space | 2. $T_{ab\hat{g}}^c$ -space | 3. $T_{sb\hat{g}}^\alpha$ - space |
| 4. $T_{b\hat{g}}^*$ -space | 5. $T_{b\hat{g}}$ -space | 6. T_{gs} -space |
| 7. T_b -space. | | |

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