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A FUZZY APPROACH FOR SOLVING MIXED INTUITIONISTIC FUZZY TRAVELLING SALESMAN PROBLEM

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ABSTRACT

In this paper we have preferred the triangular intuitionistic fuzzy numbers and ranking method for finding the initial solution, also Hungarian method is applied for finding an optimal solution for travelling salesman problem (TSP).This method requires least iteration to reach the optimality. The method is illustrated by a numerical example also.

Keywords: *Mixed travelling salesman problem, triangular intuitionistic fuzzy numbers (TrIFN), fuzzy Hungarian method, optimal solution.*

1. INTRODUCTION

The travelling salesman problem is one of the most intensively studied problems in computational mathematics. Information about real life systems is often available in the term of vague descriptions. Hence, fuzzy methods are designed to handle vague terms, and are most suited to finding optimal solutions to problems with vague parameters. The goal of the travelling salesman has to visit 'n' cities. He needs to start from a particular city visit each city once and then return to his starting point. Travelling salesman aim is to select the sequence in which the cities are visited in such a way that his total travelling time (cost) is minimized. Cost can be distance, time, money, energy etc.

The entire algorithm developed to find optimal solution of travelling salesman problem is applicable to assignment problem. However, due to it's highly degeneracy nature, a specially designed algorithm widely known as Hungarian method proposed by Kuhn [8] is used for its solution. However, in real life situations, the parameters of assignment problem are imprecise numbers instead of fixed real numbers because time/cost for doing job by a facility (machine/person) might vary due to different reasons. Zadeh [14] introduced the concept of fuzzy sets to deal with imprecision and vagueness in real life situations. In recent years, fuzzy travelling salesman problem has got great attention and the problems in fuzzy travelling salesman problem have been approached using several technique. Jain [5] introduced a new algorithm for fractional transshipment problem. Mukerjee and Basu [10] proposed a new method to solve fuzzy travelling salesman problem. Majumdar J. *et al.* [9] used genetic algorithm to solve asymmetric travelling salesman problem with fuzzy costs. Sapideh Fereidouni [12] approached the problem using multi objective linear programming. Kumar and Gupta [7] has been solved the fuzzy travelling salesman problem for LR-fuzzy parameters. Fischer and Richter [3] proposed a method for solving a multi objective travelling salesman problem by dynamic programming. Zimmermann [13] used fuzzy programming and linear programming with several objective functions. Jain and Lachhwani [6] proposed a new method for solving fuzzy bi-level linear programming problem.

In this paper, a new method is introduced to solve the mixed intuitionistic fuzzy travelling salesman problem. Thus, in this paper it is organized as follows. In section 2 some of the preliminary concepts on fuzzy number and function principal are given. In section 3 ranking of triangular intuitionistic fuzzy numbers.in setion 4 formulation of travelling salesman problem. In section 5 formulation of travelling salesman problem as an assignment problem. In section 6 defuzzyed the triangular intuitionistic fuzzy numbers (TrIFN) through ranking technique. In section 7 the proposed algorithm is discussed. In section 8 the numerical example is given and in section 8 the paper is concluded.

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2. PRELIMINARIES

Definition 1: let A be a classical set, $\mu_A(x)$ be a function from A to [0, 1]. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined by

$$A^* = \{(x, \mu_A(x)\ ); x \in A \ and \ \mu_A(x) \in [0,1]\}$$

Definition 2: Let X be denote a universe of discourse, then an intuitionistic fuzzy set A in X is given by a set of ordered triples,

$$\tilde{A}^I = \{(x, \mu_A(x), \upsilon_A(x));\ \ x \in X\}$$

Where $\mu_A, \upsilon_A : X \to [0,1]$, are functions such that $0 \le \mu_A(x) + \upsilon_A(x) \le 1, \forall x \in X$.

For each x the membership $\mu_A(x)$ and $\upsilon_A(x)$ represents the degree of membership and the degree of non-membership of the element $x \in X$ to $A \subset X$ respectively.

Definition 3: A fuzzy number A is defined to be a triangular fuzzy number if its membership function $\mu_A : R \to [0,1]$ is equal to

$$\mu_A(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} & if\ x \in [a_1, a_2] \\ \dfrac{(a_3-x)}{(a_3-a_2)} & if\ x \in [a_2, a_3] \\ 0 \quad otherwise & \end{cases}$$

Where $a_1 \le a_2 \le a_3$ .This fuzzy number is denoted by (a_1, a_2, a_3).

Definition 4: A triangular intuitionistic fuzzy number $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\upsilon_A(x)$:

$$\mu_A(x) = \begin{cases} 0 \quad for\ x \prec a_1 & \\ \dfrac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ 1 \quad for\ x = a_2 & \\ \dfrac{(a_3-x)}{(a_3-a_2)} & for\ a_2 \le x \le a_3 \\ 0 \quad for\ x \succ a_3 & \end{cases}, \upsilon_A(x) = \begin{cases} 1 \quad for\ x \prec a_1' & \\ \dfrac{(a_2-x)}{(a_2-a_1')} & for\ a_1' \le x \le a_2 \\ 0 \quad for\ x = a_2 & \\ \dfrac{(x-a_2)}{(a_3'-a_2)} & for\ a_2 \le x \le a_3' \\ 1 \quad for\ x \succ a'_3 & \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $\mu_A(x), \upsilon_A(x) \le 0.5$ for $\mu_A(x) = \upsilon_A(x), \forall x \in R$. This TrIFN is denoted by

$$\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$$

Particular cases: Let $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$ be a TrIFN. Then the following cases arise

Case-1: If $a_1' = a_1, a_3' = a_3$ then $\tilde{A}^I$ represent Triangular Fuzzy Number (TFN). It is denoted by

$$A = (a_1, a_2, a_3)$$

Case-2: If $a_1' = a_1 = a_2 = a_3 = a_3'$=m, then $\tilde{A}^I$ represent a real number m.

3. RANKING OF TRIANGULAR INTUITIONISTIC FUZZY NUMBERS

The ranking of a triangular intuitionistic fuzzy number $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$ is defined by

$$R(\tilde{A}') = \frac{1}{3}\left[\frac{(a_3' - a_1')(a_2 - 2a_3' - 2a_1') + (a_3 - a_1)(a_1 + a_2 + a_3) + 3(a_3'^2 - a_1'^2)}{a_3' - a_1' + a_3 - a_1}\right]$$

If $\upsilon_A(x) = 1 - \mu_A(x)$ then TrIFN $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$ will become the TrFN (a_1, a_2, a_3). Then $a_1' = a_1, a_3' = a_3$

$$R(\tilde{A}^I) = \frac{a_1 + a_2 + a_3}{3}$$

The rank of TrIFN $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$ is defined by $R(\tilde{A}^I) = a_2$ if $a_2 - a_1 = a_3 - a_2$ and $a_2 - a_1' = a_3' - a_2$

Let $\tilde{A}^I$ and $\tilde{B}^I$ be two TrIFN then some relational notation/operations are as follows

(i) $R(\tilde{A}^I) \succ R(\tilde{B}^I)$ *iff* $\tilde{A}^I \succ \tilde{B}^I$
(ii) $R(\tilde{A}^I) \prec R(\tilde{B}^I)$ *iff* $\tilde{A}^I \prec \tilde{B}^I$
(iii) $R(\tilde{A}^I) = R(\tilde{B}^I)$ *iff* $\tilde{A}^I \approx \tilde{B}^I$
(iv) $R(\tilde{A}^I \oplus \tilde{B}^I) = R(\tilde{A}^I) \oplus R(\tilde{B}^I)$
(v) $R(\tilde{A}^I - \tilde{B}^I) = R(\tilde{A}^I) - R(\tilde{B}^I)$
(vi) $R(\tilde{A}^I \times \tilde{B}^I) = R(\tilde{A}^I) \times R(\tilde{B}^I)$

Arithmetic Operation:

Let $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3)(b_1', b_2', b_3')$ be any two TrIFNs then arithmetic operations as follows

Addition: $\tilde{A}^I \oplus \tilde{B}^I = (a_1 \oplus b_1, a_2 \oplus b_2, a_3 \oplus b_3)(a_1' \oplus b_1', a_2 \oplus b_2, a_3' \oplus b_3')$

Subtraction: $\tilde{A}^I \oplus \tilde{B}^I = (a_1 - b_3, a_2 \oplus b_2, a_3 \oplus b_1)(a_1' \oplus b_3', a_2 \oplus b_2, a_3' \oplus b_1')$

Multiplication: If $R(\tilde{A}^I), R(\tilde{B}^I) \geq 0$ then

$$\tilde{A}^I \times \tilde{B}^I = \left[\left(\frac{a_1(b_1 + b_2 + b_3)}{3}, \frac{a_2(b_1 + b_2 + b_3)}{3}, \frac{a_3(b_1 + b_2 + b_3)}{3}\right)\left(\frac{a_1'(b_1' + b_2 + b_3')}{3}, \frac{a_2(b_1' + b_2 + b_3')}{3}, \frac{a_3'(b_1' + b_2 + b_3')}{3}\right)\right]$$

If $R(\tilde{A}^I), R(\tilde{B}^I) \leq 0$, then

$$\tilde{A}^I \times \tilde{B}^I = \left[\left(\frac{a_3(b_1 + b_2 + b_3)}{3}, \frac{a_2(b_1 + b_2 + b_3)}{3}, \frac{a_1(b_1 + b_2 + b_3)}{3}\right)\left(\frac{a_3'(b_1' + b_2 + b_3')}{3}, \frac{a_2(b_1' + b_2 + b_3')}{3}, \frac{a_1'(b_1' + b_2 + b_3')}{3}\right)\right]$$

4. FORMULATION OF TRAVELLING SALESMAN PROBLEM

The problem whose solution will yield the minimum travelling time is, let the variable x_{ij} be defined as

$$x_{ij} = \left\{\begin{matrix} 1 & \textit{from city i to j} \\ 0 & \textit{otherwise} \end{matrix}\right\}$$

Thus, the above model can be expressed as

Minimize $\tilde{Z} = \sum_i \sum_j \tilde{c}_{ij} x_{ij}$

Subject to the constraints

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1,2,.......n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1,2,.......m$$

$$x_{ij} = 0 or 1$$

5. FORMULATION TRAVELLING SALESMAN PROBLEM AS AN ASSIGNMENT PROBLEM

Let the cost of travel from i^{th} city to j^{th} city be c_{ij} and $x_{ij} = 1$ if the salesman goes directly from city i to city j and $x_{ij} = 0$ otherwise. No city is visited twice before the tour of all cities completed. In particular, he cannot go directly from city I to I itself. This possibility may be avoided in the minimization process by adopting the convention $c_{ij} = \infty$ which ensures that x_{ij} can never be unity.

The cost matrix is

$$\begin{matrix} \infty & c_{12} & c_{13} & .. & c_{1n} \\ c_{21} & \infty & c_{23} & .. & c_{2n} \\ c_{31} & c_{32} & \infty & .. & c_{3n} \\ .. & .. & .. & .. & .. \\ c_{n1} & c_{n2} & c_{n3} & .. & \infty \end{matrix}$$

Then the objective function is

Minimize $\tilde{Z} = \sum_i \sum_j \tilde{c}_{ij} x_{ij}$

Subject to the constraints

$$\sum_{i=1}^{m} x_{ij} = 1 \quad j = 1,2,.......n \quad i \neq j, i \neq m$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1,2,.......m \quad i \neq j, i \neq n$$

$$x_{mk} = 1$$

$$x_{ij} = 0 \text{ or } 1$$

6. DEFUZZIFICATION

Defuzzification is the process of finding the singleton value (crisp value) which represents the average value of the fuzzy numbers. There is no unique way to perform the defuzzification. The several existing methods for defuzzification are commonly accepted in the literature.

Graded mean integration method (4):
The graded mean integration method is used to defuzzify the triangular fuzzy number. The representation of triangular fuzzy number is $\tilde{A} = (a_1, a_2, a_3)$ and its defuzzified value is obtained by $A = \frac{a_1 + 4a_2 + a_3}{6}$.

Robust ranking method (9):
Robust ranking method is used to defuzzify the trapezoidal fuzzy number. Given a convex fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, the robust ranking index is defined by

$$R(\tilde{A}) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U) d\alpha$$

Where $\left(a_\alpha^L, a_\alpha^U\right)$ is the α level cut of the fuzzy number $\tilde{A}$ and

$$a_\alpha^L = (a_2 - a_1)\alpha + a_1$$
$$a_\alpha^U = -(a_4 - a_3)\alpha + a_4$$

In this paper we use a new ranking method to defuzzify the triangular intuitionistic fuzzy number because of its simplicity and accuracy.

7. ALGORITHM

Step-1: Construct the travelling salesman problem in which cost must be in a combination of real, fuzzy and intuitionistic fuzzy numbers.
Step-2: Calculate ranking index for each fuzzy and intuitionistic fuzzy number.
Step-3: Replace real, fuzzy and intuitionistic fuzzy numbers by their respective ranking indices.
Step-4: Solve the resulting assignment problem by using existing Hungarian method to find the optimal solution.

8. NUMERICAL EXAMPLE

The proposed method has been explained through the example given below:

$$\begin{matrix} \infty & (2,6,10)(4,6,8) & 3 & (1,3,5) \\ (3,7,11) & \infty & (3,5,7) & 9 \\ (1,8,15)(2,8,14) & (3,11,12)(4,11,13) & \infty & 12 \\ (4,12,35)(1,12,38) & (3,6,9) & (6,14,28)(3,14,31) & \infty \end{matrix}$$

Now we calculate $R(2,6,10)(4,6,8)$ by applying existing method.

$$R(\tilde{a}_{12}) = \frac{1}{3}\left[\frac{(8-4)(6-2(8)-2(4)) + (10-2)(2+6+10) + 3(8^2-4^2)}{(8-4+10-2)}\right]$$
$$R(\tilde{a}_{12}) = 6$$

Proceeding similarly, the ranking indices for the fuzzy costs $\tilde{a}_{ij}$ are calculated as:

$$R(\tilde{a}_{13}) = 3,\ R(\tilde{a}_{14}) = 3,\ R(\tilde{a}_{21}) = 7,\ R(\tilde{a}_{23}) = 5,\ R(\tilde{a}_{24}) = 9,\ R(\tilde{a}_{31}) = 8,\ R(\tilde{a}_{32}) = 11,\ R(\tilde{a}_{34}) = 12$$
$$R(\tilde{a}_{41}) = 12,\ R(\tilde{a}_{42}) = 6,\ R(\tilde{a}_{43}) = 14$$

We replace these values for their corresponding $\tilde{a}_{ij}$ in which result in a convenient assignment problem in the linear programming problem.

$$\begin{matrix} \infty & 6 & 3 & 3 \\ 7 & \infty & 5 & 9 \\ 8 & 11 & \infty & 12 \\ 12 & 6 & 14 & \infty \end{matrix}$$

By proposed method, we get

$$\begin{matrix} \infty & 6 & 3 & 3^* \\ 7 & \infty & 5^* & 9 \\ 8^* & 11 & \infty & 12 \\ 12 & 6^* & 14 & \infty \end{matrix}$$

Therefore, the total travel cost is

$$R(1,3,5)(1,3,5) + R(3,5,7)(3,5,7) + R(1,8,15)(2,8,14) + R(6,6,6)(6,6,6) = 3+5+8+6 = 22$$

CONCLUSION

In this paper, a new algorithm has been proposed to solve the fuzzy travelling salesman problems occurring in real life situation. To illustrate the algorithm a numerical example has been solved in which approximate cost is represented as different type of numbers. So, the method can be applied to solve real world travelling salesman problem where the data is not in symmetric numerical values. The proposed method is very simple and easy to understand.

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