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SEMI SYMMETRIC NON-METRIC S-CONNEXION ON AN ALMOST UNIFIED PARA-NORDEN CONTACT METRIC MANIFOLD

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ABSTRACT

In 1924, Friedmann and Schouten have introduced the idea of semi-symmetric linear connexion in a differentiable manifold. In 1970, semi-symmetric connexions were studied by K. Yano in a Riemannian manifold. In 2008, S. K. Chaubey and R. H. Ojha defined a new semi-symmetric non metric and quarter symmetric metric connexion. The purpose of the present paper is to study some properties of semi-symmetric non-metric S-Connexion on an almost unified para-norden contact metric manifold and the form of curvature tensor R of the manifold relative to this connexion has been derived. It has been shown that if almost unified para-norden contact metric manifold admits a semi-symmetric non-metric S-connexion whose curvature tensor is locally isometric to the unit sphere $S^n(1)$, then the conformal and conharmonic curvature tensors with respect to the Riemannian connexion are identical iff $n-\lambda^2(n+2)=0$. Some other useful results have been obtained, which are of great geometrical importance.

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1. INTRODUCTION:

We consider a differentiable manifold V_n of differentiability class C^∞. Let there exist in V_n a tensor F of the type (1, 1), a vector field U, a 1-form u and a Riemannian metric g satisfying

(1.1) $$\bar{\bar{X}} = \lambda^2 X - u(X)U$$

(1.2) $$\bar{U} = 0$$

(1.3) $$g(\bar{X},\bar{Y}) = \lambda^2 g(X,Y) - u(X)u(Y)$$

Where $$F(X) \stackrel{\text{def}}{=} \bar{X}$$

And λ is a complex constant.

Then the set (F, U, u, g) satisfying (1.1) to (1.3) is called an almost unified para-norden contact metric structure and V_n equipped an almost unified para-norden contact metric structure is called an almost unified para-norden contact metric manifold [4].

Remark 1.1: An almost unified para-norden contact metric manifold is an almost para contact metric manifold [2] or an almost Norden contact metric manifold [1] according as $\lambda=\pm1$ or $\lambda=\pm I$ respectively.

Agreement 1.1: All the equations which follow will hold for arbitrary vector fields X, Y, Z … etc.

It is easy to calculate that in V_n

(1.4) $$u(U) = \lambda^2$$

(1.5) $$g(X,U) \stackrel{\text{def}}{=} u(X)$$

(1.6) $$u(\bar{X}) = 0$$

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Agreement 1.2: An almost unified para-norden contact metric manifold, will always be denoted by V_n .

Agreement 1.3: V_n satisfying

(1.1) $$D_X U = F(X) \stackrel{\text{def}}{=} \bar{X}$$

will be denoted by V_n^*

In V_n^* , we can easily shown that

(1.2) $$(D_X u)(Y) = {}'F(X,Y) = (D_Y u)(X)$$

where

(1.3) $${}'F(X,Y) = g(\bar{X},Y) = g(X,\bar{Y})$$

Definition 1.1: An affine connexion B is said to be metric if

(1.4) $$B_X g = 0$$

The affine metric connexion B satisfying

(1.5) $$(B_X F)(Y) = u(Y)X - g(X,Y)U$$

is called metric S -connexion [5]

A S -connexion B is called semi-symmetric non-metric S -connexion (11] iff

(1.6) $$B_X Y = D_X Y - u(Y)X - g(X,Y)U$$

Also

$$(B_X g)(Y,Z) = 2u(Y)g(X,Z) + 2u(Z)g(X,Y)$$

which implies

(1.7) $$S(X,Y) = u(Y)X - u(X)Y$$

where S is the torsion tensor of connexion B .

The curvature tensor with respect to the semi-symmetric non-metric connexion is defined as

(1.8) $$\tilde{R}(X,Y,Z) \stackrel{\text{def}}{=} B_X B_Y Z - B_Y B_X Z - B_{[X,Y]} Z$$

Using (1.12) in (1.14), we get

(1.9) $$\tilde{R}(X,Y,Z) = K(X,Y,Z) - \alpha(X,Z)Y + \alpha(Y,Z)X - g(Y,Z)(D_X U - u(X)U) + g(X,Z)(D_Y U - u(Y)U)$$

where

(1.10) $$\alpha(X,Y) = (D_X u)(Y) + u(X)u(Y) + g(X,Y)u(U)$$

and

(1.11) $$K(X,Y,Z) \stackrel{\text{def}}{=} D_X D_Y Z - D_Y D_X Z - D_{[X,Y]} Z$$

where $\tilde{R}$ and K be the curvature tensors with respect to the connexion B and D respectively.

Using (1.7) in (1.15), we get

(1.12) $$\tilde{R}(X,Y,Z) = K(X,Y,Z) - \alpha(X,Z)Y + \alpha(Y,Z)X - g(Y,Z)(\bar{X} - u(X)U) + g(X,Z)(\bar{Y} - u(Y)U)$$

Let us consider that $\tilde{R}(X,Y,Z) = 0$ then above equation becomes

(1.13) $$K(X,Y,Z) - \alpha(X,Z)Y + \alpha(Y,Z)X - g(Y,Z)(\bar{X} - u(X)U) + g(X,Z)(\bar{Y} - u(Y)U = 0$$

Contracting above equation with respect to X , we get

(1.14) $$Ric(Y,Z) - \alpha(Y,Z) + n\alpha(Y,Z) + \lambda^2 g(Y,Z) + g(\bar{Y},Z) - u(Y)u(Z) = 0$$

Using (1.16) in (1.20), we get

(1.15) $$Ric(Y,Z) + (n-1)\{{}'F(Y,Z) + u(Y)u(Z) + \lambda^2 g(Y,Z)\} + g(\bar{Y},\bar{Z}) + g(\bar{Y},Z) = 0$$

which yields

(1.16) $$rY + n(\bar{Y} + \lambda^2 Y) + (n-2)u(Y)U = 0$$

Contracting Y in the above equation, we get

(1.17) $$R = -\lambda^2(n-1)(n+2)$$

where Ric and R are Ricci tensor and scalar curvature respectively.

The Projective curvature tensor W, conformal curvature tensor Q, Conhormonic curvature tensor L and Concircular curvature tensor C in a Riemannian manifold V_n are given by [3]

(1.18) $$W(X,Y,Z) = K(X,Y,Z) - \frac{1}{n-1}[Ric(Y,Z)X - Ric(X,Z)Y]$$

(1.19) $$Q(X,Y,Z) = K(X,Y,Z) - \frac{1}{n-2}[Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)r(X) - g(X,Z)r(Y)] + \frac{R}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]$$

(1.20) $$L(X,Y,Z) = K(X,Y,Z) - \frac{1}{n-2}[Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)r(X) - g(X,Z)r(Y)]$$

and

(1.21) $$C(X,Y,Z) = K(X,Y,Z) - \frac{R}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]$$

where

(1.22) $$'W(X,Y,Z,T) \overset{\text{def}}{=} g(W(X,Y,Z),T)$$

(1.23) $$'Q(X,Y,Z,T) \overset{\text{def}}{=} g(Q(X,Y,Z),T)$$

(1.24) $$'L(X,Y,Z,T) \overset{\text{def}}{=} g(L(X,y,Z),T)$$

(1.31) $$'C(X,Y,Z,T) \overset{\text{def}}{=} g(C(X,Y,Z),T)$$

2. CURVATURE TENSORS:

Theorem 2.1: If almost unified para-norden contact metric manifold admits a semi-symmetric non-metric S-connection whose curvature tensor is locally isometric to the unit sphere $S^n(1)$, then the conformal and con-harmonic curvature tensors with respect to the Riemannian connexion identical iff

$$n - \lambda^2(n+2) = 0$$

Proof: If the curvature tensor with respect to the semi-symmetric non metric S-connexion is locally isometric to the unit sphere $S^n(1)$, then

(2.1) $$\tilde{R}(X,Y,Z) = g(Y,Z)X - g(X,Z)Y$$

Using (2.1) in (1.18), we get

(2.2) $$g(Y,Z)X - g(X,Z) = K(X,Y,Z) - \alpha(X,Z)Y + \alpha(Y,Z)X - g(Y,Z)(\bar{X} - u(X)U) + g(X,Z)(\bar{Y} - u(Y)U)$$

Contracting above with respect to X, we get

(2.3) $$Ric(Y,Z) = (n-1)\{g(Y,Z) - 'F(Y,Z) - u(Y)u(Z) - \lambda^2 g(Y,Z)\} - g(\bar{Y},\bar{Z}) - g(\bar{Y},Z)$$

which implies

(2.4) $$rY = -n(\bar{Y} - Y) - (n-2)u(Y)U - (\lambda^2 n + 1)Y$$

Contracting Y in the above equation, we get

(2.5) $$R = (n-1)\{n - \lambda^2(n+2)\}$$

where Ric and R are Ricci tensor and scalar curvature of the manifold respectively. In view of (2.5), (1.25) and (1.26), we obtain the necessary part of the theorem. Converse part is obvious from (1.25) and (1.26).

Theorem 2.2: If almost unified para-norden contact metric manifold admits a semi-symmetric non metric S-connexion whose curvature tensor is locally isometric to the unit sphere S^n (1), then the concircular curvature tensor coincides with respect to the Riemannian connexion
$n-\lambda^2(n+2)=0$

Proof: Using (2.5) in(1.27), we get

(2.6) $$C(X,Y,Z)=K(X,Y,Z)-\frac{n-\lambda^2(n+2)}{n}[g(Y,Z)X-g(X,Z)Y]$$

which is required proves of the theorem.

Now, using (1.19) and (1.23) in (1.27), we get

(2.7) $$C(X,Y,Z)=\alpha(X,Z)Y-\alpha(Y,Z)X+g(Y,Z)\bar{X}-g(X,Z)\bar{Y}-g(Y,Z)u(X)U +g(X,Z)u(Y)U+\frac{\lambda^2(n+2)}{n}[g(Y,Z)X-g(X,Z)Y]$$

Operating g both the sides of above equation and using (1.5), (1.9) and (1.31), we get

(2.8) $$'C(X,Y,Z,T)=\alpha(X,Z)g(Y,T)-\alpha(Y,Z)g(X,T)+g(Y,Z)'F(X,T)-g(Y,Z)u(X)u(T) -g(X,Z)'F(Y,T)+g(X,Z)u(Y)u(T)+\frac{\lambda^2(n+2)}{n}[g(Y,Z)g(X,T)-g(X,Z)g(Y,T)]$$

Theorem 2.3: On a manifold V_n, we have

(2.9a) $$'C(X,Y,Z,U)=\alpha(X,Z)u(Y)-\alpha(Y,Z)u(X)-\lambda^2 g(Y,Z)u(X)+\lambda^2 g(X,Z)u(Y) +\frac{\lambda^2(n+2)}{n}[g(Y,Z)u(X)-g(X,Z)u(Y)]$$

(2.9b) $$'C(U,Y,Z,T)=\alpha(U,Z)g(Y,T)-\alpha(Y,Z)u(T)-\lambda^2 g(Y,Z)u(T)-u(Z)g(\bar{Y},T) +u(Y)u(Z)u(T)+\frac{\lambda^2(n+2)}{n}[g(Y,Z)u(T)-g(Y,T)u(Z)]$$

(2.9c) $$'C(U,Y,Z,U)=\alpha(U,Z)u(Y)-\alpha(Y,Z)\lambda^2-\lambda^4 g(Y,Z) +\lambda^2 u(Y)u(Z)+\frac{\lambda^2(n+2)}{n}[\lambda^2 g(Y,Z)-u(Y)u(Z)]$$

(2.9d) $$'C(X,Y,U,U)=\alpha(X,Z)u(Y)-\alpha(Y,U)u(X)$$

(2.9e) $$'C(\bar{X},\bar{Y},Z,U)=0$$

(2.9f) $$'C(U,Y,\bar{Z},\bar{T})=\alpha(U,\bar{Z})g(Y,\bar{T})$$

Proof: Replacing T by U in (2.8) and using (1.4), (1.5), (1.6) and (1.9), we get (2.9a).
Replacing X by U in (2.8) and using (1.2), (1.4), (1.5), (1.6) and (1.9), we get (2.9b)
Replacing T by U in (2.9b) and using (1.4) and (1.5), we get (2.9c).
Replacing Z by U in (2.9a) and using (1.5), we get (2.9d).
Replacing X by $\bar{X}$ and Y by $\bar{Y}$ in (2.9a) and using (1.4), we get (2.9e).
Replacing Z by $\bar{Z}$ and T by $\bar{T}$ in (2.9b) and using (1.4), we get (2.9f).

Now, using (1.19) and (1.21) in (1.24), we get

(2.10) $$W(X,Y,Z)=\alpha(X,Z)Y-\alpha(Y,Z)X+g(Y,Z)\bar{X}-g(Y,Z)u(X)U-g(X,Z)\bar{Y}+g(X,Z)u(Y)U +u(Y)u(Z)X-u(X)u(Z)Y+\frac{n}{n-1}[g(\bar{Y},Z)X-g(\bar{X},Z)Y] +\frac{1}{n-1}[g(\bar{Y},\bar{Z})X-g(\bar{X},\bar{Z})Y]+\lambda^2[g(Y,Z)X-g(X,Z)Y]$$

Operating g both the sides of above equation and using (1.5) and (1.28), we get

(2.11)
$$'W(X,Y,Z,T) = \alpha(X,Z)g(Y,T) - \alpha(Y,Z)g(X,T) + g(Y,Z)g(\bar{X},T)$$
$$-g(Y,Z)u(X)u(T) - g(X,Z)g(\bar{Y},T) + g(X,Z)u(Y)u(T) + u(Y)u(Z)g(X,T) - u(X)u(Z)g(Y,T)$$
$$+\frac{n}{n-1}[g(\bar{Y},Z)g(X,T) - g(\bar{X},Z)g(Y,T)] + \frac{1}{n-1}[g(\bar{Y},\bar{Z})g(X,T) - g(\bar{X},\bar{Z})g(Y,T)]$$
$$+\lambda^2[g(Y,Z)g(X,T) - g(X,Z)g(Y,T)]$$

Theorem 2.4: On a manifold V_n, we have

(2.12a)
$$'W(X,Y,Z,U) = \alpha(X,Z)u(Y) - \alpha(Y,Z)u(X) + \frac{n}{n-1}['F(Y,Z)u(X) - 'F(X,Z)u(Y)]$$
$$+\frac{1}{n-1}[g(\bar{Y},\bar{Z})u(X) - g(\bar{X},\bar{Z})u(Y)]$$

(2.12b)
$$'W(U,Y,Z,U) = \alpha(U,Z)u(Y) - \lambda^2\alpha(Y,Z) + \frac{\lambda^2 n}{n-1}g(\bar{Y},Z) + \frac{\lambda^2}{n-1}g(\bar{Y},\bar{Z})$$

(2.12c)
$$'W(\bar{X},\bar{Y},Z,U) = 0$$

(2.12d)
$$'W(X,Y,U,U) = \alpha(X,U)u(Y) - \alpha(Y,U)u(X)$$

(2.12e)
$$'W(U,Y,Z,T) = \alpha(U,Z)g(Y,T) - \alpha(Y,Z)u(T) - u(Z)g(\bar{Y},T) + 2u(Y)u(Z)u(T)$$
$$-2\lambda^2 g(Y,T)u(Z) + \frac{n}{n-1}g(\bar{Y},Z)u(T) + \frac{1}{n-1}g(\bar{Y},\bar{Z})u(T)$$

(2.12f)
$$'W(U,Y,Z,U) = \alpha(U,Z)u(Y) - \lambda^2\alpha(Y,Z) + \frac{\lambda^2}{n-1}[ng(\bar{Y},Z) - g(\bar{Y},\bar{Z})]$$

Proof: Replacing T by U in (2.11) and using (1.4), (1.6) and (1.9) we get (2.12a).
Replacing *X* by *U* in (2.12a) and using (1.2), (1.4), (1.6) and (1.9) we get (2.12b).
Replacing *X* by $\bar{X}$ and Y by $\bar{Y}$ in (2.12a) and using (1.6), we get (2.12c).
Replacing Z by U in (2.12a) and using (1.2) and (1.9), we get (2.12d).
Replacing *X* by *U* in (2.11) and using (1.2), (1.4), (1.5), we get (2.12e).
Replacing T by U in (2.12e) and using (1.4), (1.5) and (1.6), we get (2.12f).

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