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SOME OPERATORS DEFINED OVER INTUITIONISTIC FUZZY SETS AND INTERVAL VALUED INTUITIONISTIC FUZZY SETS

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ABSTRACT

In this paper, two operators defined over IFS, that will be an analogous as of Operations "extraction" as well as of operation "Multiplication of an IFS A^n with $\frac{1}{n}$ and multiplication of an IFS with $A^{\frac{1}{n}}$ with the natural number n are proved, also we extended the same operators over IVIFS are proved.

AMS Classification: *03E72.*

Keywords: *Intuitionistic fuzzy sets, Interval valued intuitionistic fuzzy sets, Operations over intuitionistic fuzzy sets, Operations over Interval valued intuitionistic fuzzy sets.*

1. INTRODUCTION

The notion of intuitionistic fuzzy sets was introduced by Atanassov. K (1986) [1] as a generalization of the concept of fuzzy sets was introduced by Zadeh. L. A (1965) [23]. Intuitionistic fuzzy sets are characterized by two functions expressing the degree of membership and the degree of non-membership respectively. The concept of intuitionistic fuzzy sets has been successfully applied in numerous fields, such as pattern recognition, machine learning, image processing and decision making, and etc. A lot of operations are introduced and proved over the intuitionistic fuzzy sets. Atanassov. K (1994) [3] proposed new operations defined over the intuitionistic fuzzy sets. Supriya Kumar De, Ranjit Biswas and Akhil Ranjan Roy(2000) [17,18] proposed Some operations on intuitionistic fuzzy sets and also Supriya Kumar De, Ranjit Biswas and Akhil Ranjan Roy (2001) proposed an application of intuitionistic fuzzy sets in medical diagnosis. Riecan, B., Atanassov K. (2006) [13] proposed n-extraction operation over intuitionistic fuzzy sets. Riecan, B., Atanassov. K. (2010) [14] proposed Operation division by n over intuitionistic fuzzy sets. Atanassov. K (2010)[4]proposed remarks on equalities between intuitionistic fuzzy sets. Liu. Q, Ma. C and Zhou. X (2008) [11] proposed On properties of some IFS operators and operations. Vasilev. T (2008) [12] proposed Four equalities connected with intuitionistic fuzzy sets. Verma, R. K and Sharma, B. D (2011) [20] proposed Intuitionistic fuzzy sets: Some new results. Atanassov. Kand Gargov. G (1989) [5] introduced the notion of interval-valued intuitionistic fuzzy sets which is a generalization of both intuitionistic fuzzy sets and interval-valued fuzzy sets. After the introduction of IVIFS, many researchers have shown interest in the IVIFS theory and its application. Atanassov.K (1994) [6]proposed Operators over interval-valued intuitionistic fuzzy sets. Xu. Z.S (2007) [22] proposed methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. Rangasamy Parvathi, Beloslav Riecan and Krassimir Atanassov (2012) [15] proposed Properties of some operations defined over intuitionistic fuzzy sets. MonoranjanBhowmik and Madhumangal Pal (2012) [12] proposed Some Results on Generalized Interval-Valued Intuitionistic Fuzzy Sets. Said Brunei and Florentin Smarandache (2014) [16] proposed new Operations over Interval Valued Intuitionistic Hesitant Fuzzy Set. Ezhilmaran. D and Sudharsan. S (2014) [7, 8] proposed An Interval-valued Intuitionistic Fuzzy Weighted Entropy (IVIFWE) method for selection of vendor and also application of generalized interval valued intuitionistic fuzzy relation with fuzzy max- min composition technique in medical diagnosis. In 2014,

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Ezhilmaran. D and Sudharsan. S [9, 10] are proposed two new operators defined over intuitionistic fuzzy sets and also two new operators defined over an interval valued intuitionistic fuzzy sets. This paper proceeds as follows: In section 2 some basic definitions related to intuitionistic fuzzy sets (IFSs), interval valued intuitionistic fuzzy sets (IVIFSs) and different relations and operation are introduced over the IFSs and (IVIFSs) are presented. In section 3 introduced two operators $\frac{1}{n}A^n$ and $n\,A^{\frac{1}{n}}$ over IFSs and IVIFSs and proved.

2. PRELIMINARIES

Definition 2.1: Intuitionistic fuzzy set. Let a set X be fixed. An Intuitionistic fuzzy set A in X is an object having the form $A = \{[x, \mu_A(x), \gamma_A(x)] \mid x \epsilon X\}$, where the functions $\mu_A : X \to [0\,,1]$ and $\gamma_A : X \to [0,1]$ define the degree of membership and the degree of non- membership of the element $x \in X$ to the set A which is a subset of X, respectively, and for every $x \in X$: $0 \le \mu_A(x) + \gamma_A(x) \le 1$. Let for every $\in X$: $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$. Therefore, Function determines the degree of uncertainty. Let us define the empty IFS, the totally uncertain IFS, and the unit IFS by: $O^* = \{\langle x, 0{,}1\rangle \mid x \in X\}, U^* = \{\langle x, 0{,}0\rangle \mid x \in X\}\; and\; E^* = \{\langle x, 1{,}0\rangle \mid x \in X\}$.

Definition 2.2: Interval valued Intuitionistic Fuzzy Set. An Interval valued intuitionistic fuzzy set A in the finite universe X is defined as $A = \{\, x, [\mu_A(x), \gamma_A(x)] \mid x \in X\,\}$, where $\mu_A : X \to [0{,}1]$ and $\gamma_A : X \to [0{,}1]$ with the condition $0 \le \sup(\mu_A(x)) + \sup(\gamma_A(x)) \le 1$, for any $x \in X$. The intervals $\mu_A(x)$ and $\gamma_A(x)$ denote the degree of membership function and the degree of non-membership of the element x to the set A. For every $x \in X$, $\mu_A(x)$ and $\gamma_A(x)$ are closed intervals and their Left and Right end points are denoted by $\mu_A^L(x), \mu_A^R(x)$, $\gamma_A^L(x)$, and $\gamma_A^R(x)$. Let us denote $A = \{[x, (\mu_A^L(x), \mu_A^R(x)), (\gamma_A^L(x), \gamma_A^R(x))] \mid x \in X\}$ where $0 \le \mu_A^R(x) + \gamma_A^R(x) \le 1$, $\mu_A^L(x) \ge 0, \gamma_A^L(x) \ge 0$. We call the interval $[1 - \mu_A^R(x) - \gamma_A^R(x)\,,\; 1 - \mu_A^L(x) - \gamma_A^L(x)]$, abbreviated by $[\pi_A^L(x),\, \pi_A^R(x)]$ and denoted by $\pi_A(x)$, the interval-valued intuitionistic index of x in A, which is a hesitancy degree of x to A. Especially if $\mu_A(x) = \mu_A^L(x) = \mu_A^R(x)$ and $\gamma_A(x) = \gamma_A^L(x) = \gamma_A^R(x)$ then the given IVIFS A is reduced to an ordinary IFS. Let us define the empty IVIFS, the totally uncertain IVIFS, and the unit IVIFS by:
$O^* = \{\langle x, (0{,}0), (1{,}1)\rangle \mid x \in X\}$, $U^* = \{\langle x, (0{,}0), (1{,}1)\rangle \mid x \in X\}\; and\; E^* = \{\langle x, (1{,}1), (0{,}0)\rangle \mid x \in X\}$.

Definition 2.3: Set operations on IFS. Let A and B be two IFSs on the universe X, where
$A = \{[x, \mu_A(x), \gamma_A(x)] \mid x \epsilon X\}$, $B = \{[x, \mu_B(x), \gamma_B(x)] \mid x \epsilon X\}$

Here, we define some set operations for IFSs:

$A^c = \{[x, \gamma_A(x), \mu_A(x)] \mid x \epsilon X\}$

$A \cap B = \{[x, min(\mu_A(x), \mu_B(x)), max(\gamma_A(x), \gamma_B(x))] \mid x \epsilon X\}$

$A \cup B = \{[x, max(\mu_A(x), \mu_B(x)), min(\gamma_A(x), \gamma_B(x))] \mid x \epsilon X\}$

$A + B = \{[x,\; \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x)\;,\; \gamma_A(x)\gamma_B(x)] \mid x \epsilon X\}$

$A \cdot B = \{[x,\; \mu_A(x)\mu_B(x)\;,\; \gamma_A(x) + \gamma_B(x) - \gamma_A(x)\gamma_B(x)] \mid x \epsilon X\}$

$\Box A = \{[x, \mu_A(x), 1 - \mu_A(x)] | x \in X\}$

$\Diamond A = \{[x, 1 - \gamma_A(x), \gamma_A(x)] | x \in X\}$

$nA = \{[x, 1 - (1 - \mu_A(x))^n, (\gamma_A(x))^n] | x \in X\}$

$A^n = \{[x, (\mu_A(x))^n, 1 - (1 - \gamma_A(x))^n] | x \in X\}$

$\sqrt[n]{A} = \left\{\left[x\,, \sqrt[n]{(\mu_A(x))}, 1 - \sqrt[n]{(1 - \gamma_A(x))}\right] | x \in X\right\}$

$\frac{1}{n}A = \left\{\left[x, 1 - \sqrt[n]{1 - (\mu_A(x))}, \sqrt[n]{(\gamma_A(x))}\right] | x \in X\right\}$, Where $n \ge 1$ is natural number.

Definition 2.4: Set operations on IVIFS. Let A and B be two IVIFSs on the universe X, where
$A = \{[\, x, (\mu_A^L(x), \mu_A^R(x)), (\gamma_A^L(x), \gamma_A^R(x))] \mid x \in X\}$
$B = \{[x, (\mu_B^L(x), \mu_B^R(x)), (\gamma_B^L(x), \gamma_B^R(x))] \mid x \in X\}$

Here, we define some set operations for IVIFSs:

$$A \cup B = \left\{\begin{bmatrix} x, \left(Max(\mu_A^L(x), \mu_B^L(x))\,, Max(\mu_A^R(x), \mu_B^R(x))\right), \\ \left(Min(\gamma_A^L(x), \gamma_B^L(x))\,, Min(\gamma_A^R(x), \gamma_B^R(x))\right)\end{bmatrix} \mid x \in X\right\}$$

$$A \cap B = \left\{\begin{bmatrix} x, \left(Min(\mu_A^L(x), \mu_B^L(x))\,, Min(\mu_A^R(x), \mu_B^R(x))\right), \\ \left(Max(\gamma_A^L(x), \gamma_B^L(x))\,, Max(\gamma_A^R(x), \gamma_B^R(x))\right)\end{bmatrix} \mid x \in X\right\}$$

$$A + B = \left\{\begin{bmatrix} x, \left(\mu_A^L(x) + \mu_B^L(x) - \mu_A^L(x)\mu_B^L(x), \mu_A^R(x)\mu_B^R(x) - \mu_A^R(x)\mu_B^R(x)\right), \\ \left(\gamma_A^L(x), \gamma_B^L(x)\,, \gamma_A^R(x)\gamma_B^R(x)\right)\end{bmatrix} \mid x \in X\right\}$$

$$A \cdot B = \left\{ \begin{bmatrix} x, \left(\mu_A^L(x)\mu_B^L(x), \mu_A^R(x), \mu_B^R(x)\right), \\ \left(\gamma_A^L(x) + \gamma_B^L(x) - \gamma_A^L(x)\gamma_B^L(x), \gamma_A^R(x) + \gamma_B^R(x) - \gamma_A^R(x)\gamma_B^R(x)\right) \end{bmatrix} \mid x \in X \right\}$$

$$A^c = \left\{\left[x, \left(\gamma_A^L(x), \gamma_A^R(x)\right), \left(\mu_A^L(x), \mu_A^R(x)\right)\right] \mid x \in X\right\}$$

$$\Box A = \left\{\left[x, \left(\mu_A^L(x), \mu_A^R(x)\right), \left(1 - \mu_A^L(x), 1 - \mu_A^R(x)\right)\right] | x \in X \right\}$$

$$\Diamond A = \left\{\left[x, \left(1 - \gamma_A^L(x), 1 - \gamma_A^R(x)\right), \left(\gamma_A^L(x), \gamma_A^R(x)\right)\right] | x \in X \right\}$$

$$nA = \left\{\left[x, \left(1 - \left(1 - \mu_A^L(x)\right)^n, 1 - \left(1 - \mu_A^R(x)\right)^n\right), \left(\left(\gamma_A^L(x)\right)^n, \left(\gamma_A^R(x)\right)^n\right)\right] | x \in X \right\}$$

$$A^n = \left\{\left[x, \left(\left(\mu_A^L(x)\right)^n, \left(\mu_A^R(x)\right)^n\right), \left(1 - \left(1 - \gamma_A^L(x)\right)^n, 1 - \left(1 - \gamma_A^R(x)\right)^n\right)\right] | x \in X \right\}$$

$$\sqrt[n]{A} = \left\{\left[x, \left(\sqrt[n]{\left(\mu_A^L(x)\right)}, \sqrt[n]{\left(\mu_A^R(x)\right)}\right), \left(1 - \sqrt[n]{\left(1 - \gamma_A^L(x)\right)}, 1 - \sqrt[n]{\left(1 - \gamma_A^R(x)\right)}\right)\right] | x \in X \right\}$$

$\frac{1}{n}A = \left\{\left[x, \left(1 - \sqrt[n]{\left(1 - \mu_A^L(x)\right)}, 1 - \sqrt[n]{\left(1 - \mu_A^R(x)\right)}\right), \left(\sqrt[n]{\left(\gamma_A^L(x)\right)}, \sqrt[n]{\left(\gamma_A^R(x)\right)}\right)\right] | x \in X \right\}$, Where $n \geq 1$ is natural number.

3. SOME NEW OPERATORS DEFINED OVER INTUITIONISTIC FUZZY SETS AND INTERVAL VALUED INTUITIONISTIC FUZZY SETS

We will introduce some new operator defined over IFS, that will be an analogous as of operations "extraction" as well as of operation "Multiplication of an IFSA^n with$\frac{1}{n}$ andMultiplication of an IFS $A^{\frac{1}{n}}$with$n$". It has the form for every IFS and for every natural number $n \geq 1$:

$$(a).\ \frac{1}{n}A^n = \left\{\left[x, 1 - \sqrt[n]{1 - \left(\mu_A(x)\right)^n}, \sqrt[n]{1 - \left(1 - \gamma_A(x)\right)^n}\right] \mid x \in X\right\}$$

$$(b).\ n\, A^{\frac{1}{n}} = \left\{\left[x, 1 - \left(1 - \sqrt[n]{\left(\mu_A(x)\right)}\right)^n, \left(1 - \left(\sqrt[n]{1 - \gamma_A(x)}\right)\right)^n\right] \mid x \in X\right\}$$

Also, we extended the same operators over IVIFS are proved.

$$(c).\ \frac{1}{n}A^n = \left\{ \begin{bmatrix} x, \left(1 - \sqrt[n]{1 - \left(\mu_A^L(x)\right)^n}, 1 - \sqrt[n]{1 - \left(\mu_A^R(x)\right)^n}\right), \\ \left(\sqrt[n]{1 - \left(1 - \gamma_A^L(x)\right)^n}, \sqrt[n]{1 - \left(1 - \gamma_A^R(x)\right)^n}\right) \end{bmatrix} |x \in X \right\}$$

$$(d).\ n\, A^{\frac{1}{n}} = \left\{ \begin{bmatrix} x, \left(1 - \left(1 - \sqrt[n]{\mu_A^L(x)}\right)^n, 1 - \left(1 - \sqrt[n]{\mu_A^R(x)}\right)^n\right), \\ \left(\left(1 - \sqrt[n]{\left(1 - \gamma_A^L(x)\right)}\right)^n, \left(1 - \sqrt[n]{\left(1 - \gamma_B^R(x)\right)}\right)^n\right) \end{bmatrix} |x \in X \right\}$$

First for (a), we must check that in a result of the operation we obtain an IFS. Really, for given IFS A^n, for each $x \epsilon X$, and for each$n \geq 1$:$1 - \sqrt[n]{1 - \left(\mu_A(x)\right)^n}n \geq 1$:

$$1 - \sqrt[n]{1 - \left(\mu_A(x)\right)^n} + \sqrt[n]{1 - \left(1 - \gamma_A(x)\right)^n} \leq 1,$$

Because from $\mu_A(x) \leq 1 - \gamma_A(x), 1 - \left(\mu_A(x)\right)^n \leq 1 - \left(1 - \gamma_A(x)\right)^n$

it follows that $\sqrt[n]{1 - \left(\mu_A(x)\right)^n} \leq \sqrt[n]{1 - \left(1 - \gamma_A(x)\right)^n}$.

Obviously, for every natural number $n \geq 1$: $\frac{1}{n}(O^*)^n = O^*$, $\frac{1}{n}(U^*)^n = U^*$, $\frac{1}{n}(E^*)^n = E^*$. By similar to the above way for (b) also.For (c), we must check that in a result of the operation we obtain an IVIFS. Really, for given IVIFS A^n, for each $x \epsilon X$, and for each $n \geq 1$:$1 - \sqrt[n]{1 - \left(\mu_A^R(x)\right)^n} + \sqrt[n]{1 - \left(1 - \gamma_B^R(x)\right)^n} \leq 1$,

$1 - \sqrt[n]{1 - \left(\mu_A^L(x)\right)^n} \geq 0, \sqrt[n]{1 - \left(1 - \gamma_A^L(x)\right)^n} \geq 0$,

Because from $\mu_A^R(x) \leq 1 - \gamma_B^R(x), 1 - \left(\mu_A^R(x)\right)^n \leq 1 - \left(1 - \gamma_A(x)\right)^n$.

It follows that $\sqrt[n]{1-\left(\mu_A^R(x)\right)^n} \leq \sqrt[n]{1-\left(1-\gamma_B^R(x)\right)^n}$. Obviously, for every natural number $n \geq 1$: $\frac{1}{n}(O^*)^n = O^*$, $\frac{1}{n}(U^*)^n = U^*$, $\frac{1}{n}(E^*)^n = E^*$. By similar to the above way for (d) also. Using the four operators, we can prove the following theorems.

Theorem 3.1: For any two IFSs A and B and for every natural number $n \geq 1$:
$(a).\ \frac{1}{n}(A^n \cap B^n) = \frac{1}{n}A^n \cap \frac{1}{n}B^n,\qquad (b).\ \frac{1}{n}(A^n \cup B^n) = \frac{1}{n}A^n \cup \frac{1}{n}B^n$
$(c).\ n\left(A^{\frac{1}{n}} \cap B^{\frac{1}{n}}\right) = nA^{\frac{1}{n}} \cap nB^{\frac{1}{n}},\qquad (d).\ n\left(A^{\frac{1}{n}} \cup B^{\frac{1}{n}}\right) = nA^{\frac{1}{n}} \cup nB^{\frac{1}{n}}$

Proof: (a).

$$\frac{1}{n}(A^n \cap B^n) = \frac{1}{n}\left(\left\{\left[x, \left(\mu_A(x)\right)^n, 1-\left(1-\gamma_A(x)\right)^n\right] \cap \left[x, \left(\mu_B(x)\right)^n, 1-\left(1-\gamma_B(x)\right)^n\right] \mid x \in X\right\}\right)$$
$$= \frac{1}{n}\left(\left\{\left[x, Min\left(\left(\mu_A(x)\right)^n, \left(\mu_B(x)\right)^n\right), Max\left(1-\left(1-\gamma_A(x)\right)^n, 1-\left(1-\gamma_B(x)\right)^n\right)\right] \mid x \in X\right\}\right)$$
$$= \left\{\left[\begin{array}{c} x, 1-\sqrt[n]{1-Min\left(\left(\mu_A(x)\right)^n, \left(\mu_B(x)\right)^n\right)}, \\ \sqrt[n]{Max\left(1-\left(1-\gamma_A(x)\right)^n, 1-\left(1-\gamma_B(x)\right)^n\right)} \end{array}\right] \mid x \in X\right\}$$
$$= \left\{\left[\begin{array}{c} x, 1-\sqrt[n]{Max\left(1-\left(\mu_A(x)\right)^n, 1-\left(\mu_B(x)\right)^n\right)}, \\ Max\left(\sqrt[n]{1-\left(1-\gamma_A(x)\right)^n}, \sqrt[n]{1-\left(1-\gamma_B(x)\right)^n}\right) \end{array}\right] \mid x \in X\right\}$$
$$= \left\{\left[\begin{array}{c} x, 1-Max\left(\sqrt[n]{1-\left(\mu_A(x)\right)^n}, \sqrt[n]{1-\left(\mu_B(x)\right)^n}\right), \\ Max\left(\sqrt[n]{1-\left(1-\gamma_A(x)\right)^n}, \sqrt[n]{1-\left(1-\gamma_B(x)\right)^n}\right) \end{array}\right] \mid x \in X\right\}$$
$$= \left\{\left[\begin{array}{c} x, Min\left(1-\sqrt[n]{1-\left(\mu_A(x)\right)^n}, 1-\sqrt[n]{1-\left(\mu_B(x)\right)^n}\right), \\ Max\left(\sqrt[n]{1-\left(1-\gamma_A(x)\right)^n}, \sqrt[n]{1-\left(1-\gamma_B(x)\right)^n}\right) \end{array}\right] \mid x \in X\right\}$$
$$= \left(\begin{array}{c} \left\{\left[x, 1-\sqrt[n]{1-\left(\mu_A(x)\right)^n}, \sqrt[n]{1-\left(1-\gamma_A(x)\right)^n}\right] \mid x \in X\right\} \\ \cap \\ \left\{\left[x, 1-\sqrt[n]{1-\left(\mu_B(x)\right)^n}, \sqrt[n]{1-\left(1-\gamma_B(x)\right)^n}\right] \mid x \in X\right\} \end{array}\right)$$
$$= \frac{1}{n}A^n \cap \frac{1}{n}B^n$$

Hence $(a)$is proved and similarly (b), (c) & (d) is proved by analogy.

Theorem 3.2: For every IFS A and for every natural number $n \geq 1$:
$(a).\ \Box\frac{1}{n}A^n = \frac{1}{n}\Box A^n (b).\ \Diamond\frac{1}{n}A^n = \frac{1}{n}\Diamond A^n$
$(c).\ \Box n\, A^{\frac{1}{n}} = n\Box A^{\frac{1}{n}} (d).\ \Diamond nA^{\frac{1}{n}} = n\Diamond A^{\frac{1}{n}}$

Proof: (a).

$$\Box\frac{1}{n}A^n = \Box\frac{1}{n}\left\{\left[x, \left(\mu_A(x)\right)^n, 1-\left(1-\gamma_A(x)\right)^n\right] \mid x \in X\right\}$$
$$= \Box\left\{\left[x, 1-\sqrt[n]{\left(1-\left(\mu_A(x)\right)^n\right)}, \sqrt[n]{\left(1-\left(1-\gamma_A(x)\right)^n\right)}\right] \mid x \in X\right\}$$
$$= \left\{\left[x, 1-\sqrt[n]{\left(1-\left(\mu_A(x)\right)^n\right)}, 1-\left(1-\sqrt[n]{\left(1-\left(\mu_A(x)\right)^n\right)}\right)\right] \mid x \in X\right\} \quad (3.1)$$

$$\frac{1}{n}\Box A^n = \frac{1}{n}\Box\{[x,(\mu_A(x))^n, 1-(1-\gamma_A(x))^n]|x \in X\}$$
$$= \frac{1}{n}\{[x,(\mu_A(x))^n, 1-(\mu_A(x))^n]|x \in X\}$$
$$= \frac{1}{n}\left\{\left[x, 1-\sqrt[n]{(1-(\mu_A(x))^n)}, \sqrt[n]{(1-(\mu_A(x))^n)}\right]|x \in X\right\} \tag{3.2}$$

From (3.1) and (3.2), we get

$$\Box\frac{1}{n}A^n = \frac{1}{n}\Box A^n$$

Hence $(a)$is proved and similarly $(b), (c)\&(d)$ is proved by analogy.

Theorem 3.3: For any two IFSs A and B and for every natural number $n \geq 1$:

$(a).\ \frac{1}{n}(A^n + B^n) = \frac{1}{n}A^n + \frac{1}{n}B^n,$ $\quad (b).\ \frac{1}{n}(A^n \cdot B^n) = \frac{1}{n}A^n \cdot \frac{1}{n}B^n$

$(c).\ n\left(A^{\frac{1}{n}} + B^{\frac{1}{n}}\right) = nA^{\frac{1}{n}} + nB^{\frac{1}{n}},$ $\quad (d).\ n\left(A^{\frac{1}{n}} \cdot B^{\frac{1}{n}}\right) = nA^{\frac{1}{n}} \cdot nB^{\frac{1}{n}}$

Proof: (a)

$$\frac{1}{n}(A^n + B^n) = \frac{1}{n}(\{[x,(\mu_A(x))^n, 1-(1-\gamma_A(x))^n] + [x,(\mu_B(x))^n, 1-(1-\gamma_B(x))^n]|x \in X\})$$
$$= \frac{1}{n}\left\{\begin{bmatrix} x,(\mu_A(x))^n + (\mu_B(x))^n - (\mu_A(x))^n(\mu_B(x))^n, \\ (1-(1-\gamma_A(x))^n)(1-(1-\gamma_B(x))^n) \end{bmatrix} \\ |x \in X\right\}$$
$$= \left\{\left[x, \begin{pmatrix} 1-\sqrt[n]{(1-(\mu_A(x))^n)} + 1-\sqrt[n]{(1-(\mu_B(x))^n)} \\ -\left(1-\sqrt[n]{(1-(\mu_A(x))^n)}\right)\left(1-\sqrt[n]{(1-(\mu_B(x))^n)}\right) \end{pmatrix}, \atop \sqrt[n]{(1-(1-\gamma_A(x))^n)}\sqrt[n]{(1-(1-\gamma_B(x))^n)}\right]|x \in X\right\}$$
$$= \left\{\left[x, 1-\sqrt[n]{(1-(\mu_A(x))^n)}, \sqrt[n]{(1-(1-\gamma_A(x))^n)}\right]|x \in X\right\}$$
$$+\left\{\left[x, 1-\sqrt[n]{(1-(\mu_B(x))^n)}, \sqrt[n]{(1-(1-\gamma_B(x))^n)}\right]|x \in X\right\}$$
$$= \frac{1}{n}\{[x,(\mu_A(x))^n, 1-(1-\gamma_A(x))^n]|x \in X\} + \frac{1}{n}\{[x,(\mu_B(x))^n, 1-(1-\gamma_B(x))^n]|x \in X\}$$
$$= \frac{1}{n}A^n + \frac{1}{n}B^n$$

Hence $(a)$is proved and similarly $(b), (c)\&(d)$ is proved by analogy.

Theorem 3.4: For every IFS A and for every natural number $n \geq 1$:

$(a).\ n\left(\frac{1}{n}A^n\right) = A^n$ $\qquad (b).\ \frac{1}{n}(n\,A^n) = A^n$

Proof: (a).

$$n\left(\frac{1}{n}A^n\right) = n\left(\frac{1}{n}\{[x,(\mu_A(x))^n, 1-(1-\gamma_A(x))^n]|x \in X\}\right)$$
$$= n\left\{\left[x, 1-\sqrt[n]{1-(\mu_A(x))^n}, \sqrt[n]{1-(1-\gamma_A(x))^n}\right] |\ x \in X\right\}$$
$$= \left\{\left[x, \left(1-\left(1-\left(1-\sqrt[n]{1-(\mu_A(x))^n}\right)\right)^n\right), \left(\sqrt[n]{1-(1-\gamma_A(x))^n}\right)^n\right] |\ x \in X\right\}$$
$$= \left\{\left[x, \left(1-\left(\sqrt[n]{1-(\mu_A(x))^n}\right)^n\right), 1-(1-\gamma_A(x))^n\right] |\ x \in X\right\}$$
$$= \{[x, 1-(1-(\mu_A(x))^n), 1-(1-\gamma_A(x))^n]|\ x \in X\}$$
$$= \{[x,(\mu_A(x))^n, 1-(1-\gamma_A(x))^n]|\ x \in X\}$$
$$= A^n$$

Hence $(a)$is proved and similarly $(b)$is proved by analogy.

Theorem 3.5: For any two IVIFSs A and B and for every natural number $n \geq 1$:

$(a).\frac{1}{n}(A^n \cap B^n) = \frac{1}{n}A^n \cap \frac{1}{n}B^n$, $\quad (b).\ \frac{1}{n}(A^n \cup B^n) = \frac{1}{n}A^n \cup \frac{1}{n}B^n$

$(c).\ n\left(A^{\frac{1}{n}} \cap B^{\frac{1}{n}}\right) = nA^{\frac{1}{n}} \cap nB^{\frac{1}{n}}$, $\quad (d).\ n\left(A^{\frac{1}{n}} \cup B^{\frac{1}{n}}\right) = nA^{\frac{1}{n}} \cup nB^{\frac{1}{n}}$

Proof: (a).

$$\frac{1}{n}(A^n \cap B^n) = \frac{1}{n}\left(\left\{\begin{matrix} [x, ((\mu_A^L(x))^n, (\mu_A^R(x))^n), (1-(1-\gamma_A^L(x))^n, 1-(1-\gamma_A^R(x))^n)] \\ \cap \\ [x, ((\mu_B^L(x))^n, (\mu_B^R(x))^n), (1-(1-\gamma_B^L(x))^n, 1-(1-\gamma_B^R(x))^n)] \mid x \in X \end{matrix}\right\}\right)$$

$$= \frac{1}{n}\left(\left\{\begin{bmatrix} x, \left(Min\left((\mu_A^L(x))^n, (\mu_B^L(x))^n\right), Min\left((\mu_A^R(x))^n, (\mu_B^R(x))^n\right)\right), \\ \begin{pmatrix} Max\left(1-(1-\gamma_A^L(x))^n, 1-(1-\gamma_B^L(x))^n\right), \\ Max\left(1-(1-\gamma_A^R(x))^n, 1-(1-\gamma_B^R(x))^n\right) \end{pmatrix} \end{bmatrix} \mid x \in X\right\}\right)$$

$$= \left\{\begin{bmatrix} x, \left(1-\sqrt[n]{1-Min\left((\mu_A^L(x))^n, (\mu_B^L(x))^n\right)}, 1-\sqrt[n]{1-Min\left((\mu_A^R(x))^n, (\mu_B^R(x))^n\right)}\right), \\ \begin{pmatrix} \sqrt[n]{Max\left(1-(1-\gamma_A^L(x))^n, 1-(1-\gamma_B^L(x))^n\right)}, \\ \sqrt[n]{Max\left(1-(1-\gamma_A^R(x))^n, 1-(1-\gamma_B^R(x))^n\right)} \end{pmatrix} \end{bmatrix} \mid x \in X\right\}$$

$$= \left\{\begin{bmatrix} x, \begin{pmatrix} 1-\sqrt[n]{Max\left(1-(\mu_A^L(x))^n, 1-(\mu_B^L(x))^n\right)}, \\ 1-\sqrt[n]{Max\left(1-(\mu_A^R(x))^n, 1-(\mu_B^R(x))^n\right)} \end{pmatrix}, \\ \begin{pmatrix} Max\left(\sqrt[n]{1-(1-\gamma_A^L(x))^n}, \sqrt[n]{1-(1-\gamma_B^L(x))^n}\right) \\ , Max\left(\sqrt[n]{1-(1-\gamma_A^R(x))^n}, \sqrt[n]{1-(1-\gamma_B^R(x))^n}\right) \end{pmatrix} \end{bmatrix} \mid x \in X\right\}$$

$$= \left\{\begin{bmatrix} x, \begin{pmatrix} 1-Max\left(\sqrt[n]{1-(\mu_A^L(x))^n}, \sqrt[n]{1-(\mu_B^L(x))^n}\right), \\ 1-Max\left(\sqrt[n]{1-(\mu_A^R(x))^n}, \sqrt[n]{1-(\mu_B^R(x))^n}\right) \end{pmatrix}, \\ \begin{pmatrix} Max\left(\sqrt[n]{1-(1-\gamma_A^L(x))^n}, \sqrt[n]{1-(1-\gamma_B^L(x))^n}\right), \\ Max\left(\sqrt[n]{1-(1-\gamma_A^R(x))^n}, \sqrt[n]{1-(1-\gamma_B^R(x))^n}\right) \end{pmatrix} \end{bmatrix} \mid x \in X\right\}$$

$$= \left\{\begin{bmatrix} x, \begin{pmatrix} Min\left(1-\sqrt[n]{1-(\mu_A^L(x))^n}, 1-\sqrt[n]{1-(\mu_B^L(x))^n}\right), \\ Min\left(1-\sqrt[n]{1-(\mu_A^R(x))^n}, 1-\sqrt[n]{1-(\mu_B^R(x))^n}\right) \end{pmatrix}, \\ \begin{pmatrix} Max\left(\sqrt[n]{1-(1-\gamma_A^L(x))^n}, \sqrt[n]{1-(1-\gamma_B^L(x))^n}\right), \\ Max\left(\sqrt[n]{1-(1-\gamma_A^R(x))^n}, \sqrt[n]{1-(1-\gamma_B^R(x))^n}\right) \end{pmatrix} \end{bmatrix} \mid x \in X\right\}$$

$$= \left(\begin{array}{c} \left\{ \begin{bmatrix} x, \left(1-\sqrt[n]{1-\left(\mu_A^L(x)\right)^n}, 1-\sqrt[n]{1-\left(\mu_A^R(x)\right)^n}\right), \\ \left(\sqrt[n]{1-\left(1-\gamma_A^L(x)\right)^n}, \sqrt[n]{1-\left(1-\gamma_A^R(x)\right)^n}\right) \end{bmatrix} | x \in X \right\} \\ \cap \\ \left\{ \begin{bmatrix} x, \left(1-\sqrt[n]{1-\left(\mu_B^L(x)\right)^n}, 1-\sqrt[n]{1-\left(\mu_B^R(x)\right)^n}\right), \\ \left(\sqrt[n]{1-\left(1-\gamma_B^L(x)\right)^n}, \sqrt[n]{1-\left(1-\gamma_B^R(x)\right)^n}\right) \end{bmatrix} | x \in X \right\} \end{array} \right)$$

$$= \frac{1}{n}A^n \cap \frac{1}{n}B^n$$

Hence $(a)$is proved and similarly (b), (c)&(d) is proved by analogy.

Theorem3.6: For every IFS A and for every natural number $n \geq 1$:

(a). $\Box \frac{1}{n}A^n = \frac{1}{n}\Box A^n$ (b). $\Diamond \frac{1}{n}A^n = \frac{1}{n}\Diamond A^n$

(c). $\Box n\, A^{\frac{1}{n}} = n\Box A^{\frac{1}{n}}$ (d). $\Diamond\, nA^{\frac{1}{n}} = n \Diamond A^{\frac{1}{n}}$

Proof: (a).

$$\Box \frac{1}{n}A^n = \Box \frac{1}{n}\left\{\left[x, \left(\left(\mu_A^L(x)\right)^n, \left(\mu_A^R(x)\right)^n\right), \left(1-\left(1-\gamma_A^L(x)\right)^n, 1-\left(1-\gamma_A^R(x)\right)^n\right)\right] | x \in X\right\}$$

$$= \Box \left\{ \begin{bmatrix} x, \left(1-\sqrt[n]{\left(1-\left(\mu_A^L(x)\right)^n\right)}, 1-\sqrt[n]{\left(1-\left(\mu_A^R(x)\right)^n\right)}\right), \\ \left(\sqrt[n]{\left(1-\left(1-\gamma_A^L(x)\right)^n\right)}, \sqrt[n]{\left(1-\left(1-\gamma_A^R(x)\right)^n\right)}\right) \end{bmatrix} \\ | x \in X \right\}$$

$$= \left\{ \begin{bmatrix} x, \left(1-\sqrt[n]{\left(1-\left(\mu_A^L(x)\right)^n\right)}, 1-\sqrt[n]{\left(1-\left(\mu_A^R(x)\right)^n\right)}\right), \\ \left(1-\left(1-\sqrt[n]{\left(1-\left(\mu_A(x)\right)^n\right)}\right), 1-\left(1-\sqrt[n]{\left(1-\left(\mu_A(x)\right)^n\right)}\right)\right) \end{bmatrix} | x \in X \right\} \tag{3.3}$$

$$\frac{1}{n}\Box A^n = \frac{1}{n}\Box\left\{\left[x, \left(\left(\mu_A^L(x)\right)^n, \left(\mu_A^R(x)\right)^n\right), \left(1-\left(1-\gamma_A^L(x)\right)^n, 1-\left(1-\gamma_A^R(x)\right)^n\right)\right] | x \in X\right\}$$

$$= \frac{1}{n}\left\{\left[x, \left(\left(\mu_A^L(x)\right)^n, \left(\mu_A^R(x)\right)^n\right), \left(1-\left(\mu_A^L(x)\right)^n, 1-\left(\mu_A^R(x)\right)^n\right)\right] | x \in X\right\}$$

$$= \frac{1}{n}\left\{ \begin{bmatrix} x, \left(1-\sqrt[n]{\left(1-\left(\mu_A^L(x)\right)^n\right)}, 1-\sqrt[n]{\left(1-\left(\mu_A^R(x)\right)^n\right)}\right), \\ \left(\sqrt[n]{\left(1-\left(\mu_A^L(x)\right)^n\right)}, \sqrt[n]{\left(1-\left(\mu_A^R(x)\right)^n\right)}\right) \end{bmatrix} \\ | x \in X \right\} \tag{3.4}$$

From (3.3) and (3.4), we get

$$\Box \frac{1}{n}A^n = \frac{1}{n}\Box A^n$$

Hence $(a)$is proved and similarly (b), (c)&$(d)$is proved by analogy.

Theorem 3.7: For any two IVIFSs A and B and for every natural number $n \geq 1$:

(a). $\frac{1}{n}(A^n + B^n) = \frac{1}{n}A^n + \frac{1}{n}B^n$, (b). $\frac{1}{n}(A^n \cdot B^n) = \frac{1}{n}A^n \cdot \frac{1}{n}B^n$

(c). $n\left(A^{\frac{1}{n}} + B^{\frac{1}{n}}\right) = nA^{\frac{1}{n}} + nB^{\frac{1}{n}}$, (d). $n\left(A^{\frac{1}{n}} \cdot B^{\frac{1}{n}}\right) = nA^{\frac{1}{n}} \cdot nB^{\frac{1}{n}}$

Proof: (a)

$$\frac{1}{n}(A^n + B^n) = \frac{1}{n}\left(\left\{ \begin{array}{c} \left[x, \left(\left(\mu_A^L(x)\right)^n, \left(\mu_A^R(x)\right)^n\right), \left(1-\left(1-\gamma_A^L(x)\right)^n, 1-\left(1-\gamma_A^R(x)\right)^n\right)\right] \\ +\left[x, \left(\left(\mu_B^L(x)\right)^n, \left(\mu_B^R(x)\right)^n\right), \left(1-\left(1-\gamma_B^L(x)\right)^n, 1-\left(1-\gamma_B^R(x)\right)^n\right)\right] \\ | x \in X \end{array} \right\} \right)$$

$$= \frac{1}{n}\left\{\left[x, \begin{pmatrix} \left(\mu_A^L(x)\right)^n + \left(\mu_B^L(x)\right)^n - \left(\mu_A^L(x)\right)^n\left(\mu_B^L(x)\right)^n, \\ \left(\mu_A^R(x)\right)^n + \left(\mu_B^R(x)\right)^n - \left(\mu_A^R(x)\right)^n\left(\mu_B^R(x)\right)^n \end{pmatrix}, \begin{pmatrix} \left(1-\left(1-\gamma_A^L(x)\right)^n\right)\left(1-\left(1-\gamma_B^L(x)\right)^n\right) \\ ,\left(1-\left(1-\gamma_A^R(x)\right)^n\right)\left(1-\left(1-\gamma_B^R(x)\right)^n\right) \end{pmatrix}\right] \middle| x \in X\right\}$$

$$= \left\{\left[x, \left(\begin{pmatrix} 1-\sqrt[n]{\left(1-\left(\mu_A^L(x)\right)^n\right)} + 1-\sqrt[n]{\left(1-\left(\mu_B^L(x)\right)^n\right)} \\ -\left(1-\sqrt[n]{\left(1-\left(\mu_A^L(x)\right)^n\right)}\right)\left(1-\sqrt[n]{\left(1-\left(\mu_B^L(x)\right)^n\right)}\right) \end{pmatrix}, \begin{pmatrix} 1-\sqrt[n]{\left(1-\left(\mu_A^R(x)\right)^n\right)} + 1-\sqrt[n]{\left(1-\left(\mu_B^R(x)\right)^n\right)} \\ -\left(1-\sqrt[n]{\left(1-\left(\mu_A^R(x)\right)^n\right)}\right)\left(1-\sqrt[n]{\left(1-\left(\mu_B^R(x)\right)^n\right)}\right) \end{pmatrix}\right), \begin{pmatrix} \sqrt[n]{\left(1-\left(1-\gamma_A^L(x)\right)^n\right)}\sqrt[n]{\left(1-\left(1-\gamma_B^L(x)\right)^n\right)}, \\ \sqrt[n]{\left(1-\left(1-\gamma_A^R(x)\right)^n\right)}\sqrt[n]{\left(1-\left(1-\gamma_B^R(x)\right)^n\right)} \end{pmatrix}\right] \middle| x \in X\right\}$$

$$= \left\{\left[x, \begin{pmatrix} \left(1-\sqrt[n]{\left(1-\left(\mu_A^L(x)\right)^n\right)}, 1-\sqrt[n]{\left(1-\left(\mu_A^R(x)\right)^n\right)}\right), \\ \left(\sqrt[n]{\left(1-\left(1-\gamma_A^L(x)\right)^n\right)}, \sqrt[n]{\left(1-\left(1-\gamma_A^R(x)\right)^n\right)}\right) \end{pmatrix}\right] \middle| x \in X\right\}$$

$$+ \left\{\left[x \begin{pmatrix} \left(1-\sqrt[n]{\left(1-\left(\mu_B^L(x)\right)^n\right)}, 1-\sqrt[n]{\left(1-\left(\mu_B^R(x)\right)^n\right)}\right), \\ \left(\sqrt[n]{\left(1-\left(1-\gamma_B^L(x)\right)^n\right)}, \sqrt[n]{\left(1-\left(1-\gamma_B^R(x)\right)^n\right)}\right) \end{pmatrix}\right] \middle| x \in X\right\}$$

$$= \frac{1}{n}\left\{\left[x, \left(\left(\mu_A^L(x)\right)^n, \left(\mu_A^R(x)\right)^n\right), \left(1-\left(1-\gamma_A^L(x)\right)^n, 1-\left(1-\gamma_A^R(x)\right)^n\right)\right] | x \in X\right\}$$
$$+\frac{1}{n}\left\{\left[x, \left(\left(\mu_B^L(x)\right)^n, \left(\mu_B^R(x)\right)^n\right), \left(1-\left(1-\gamma_B^L(x)\right)^n, 1-\left(1-\gamma_B^R(x)\right)^n\right)\right] | x \in X\right\}$$
$$= \frac{1}{n}A^n + \frac{1}{n}B^n$$

Hence $(a)$is proved and similarly $(b), (c)\&(d)$ is proved by analogy.

Theorem 3.8: For every IVIFS A and for every natural number $n \geq 1$:
$(a).\ n\left(\frac{1}{n}A^n\right) = A^n (b).\ \frac{1}{n}(n\, A^n) = A^n$

Proof: (a).

$$n\left(\frac{1}{n}A^n\right) = n\left(\frac{1}{n}\left\{\left[x, \left(\left(\mu_A^L(x)\right)^n, \left(\mu_A^R(x)\right)^n\right), \left(1-\left(1-\gamma_A^L(x)\right)^n, 1-\left(1-\gamma_A^R(x)\right)^n\right)\right] | x \in X\right\}\right)$$

$$= n\left\{\left[\begin{matrix} x, \left(1-\sqrt[n]{1-\left(\mu_A^L(x)\right)^n}, 1-\sqrt[n]{1-\left(\mu_A^R(x)\right)^n}\right), \\ \left(\sqrt[n]{1-\left(1-\gamma_A^L(x)\right)^n}, \sqrt[n]{1-\left(1-\gamma_A^R(x)\right)^n}\right) \end{matrix}\right] \middle| x \in X\right\}$$

$$= \left\{\left[\begin{matrix} x, \left(\left(1-\left(1-\left(1-\sqrt[n]{1-\left(\mu_A^L(x)\right)^n}\right)\right)^n\right), \left(1-\left(1-\left(1-\sqrt[n]{1-\left(\mu_A^R(x)\right)^n}\right)\right)^n\right)\right), \\ \left(\left(\sqrt[n]{1-\left(1-\gamma_A^L(x)\right)^n}\right)^n, \left(\sqrt[n]{1-\left(1-\gamma_A^R(x)\right)^n}\right)^n\right) \end{matrix}\right] \middle| x \in X\right\}$$

$$= \left\{ \begin{bmatrix} x, \left(\left(1 - \left(\sqrt[n]{1 - \left(\mu_A^L(x)\right)^n}\right)^n\right), \left(1 - \left(\sqrt[n]{1 - \left(\mu_A^R(x)\right)^n}\right)^n\right)\right), \\ \left(1 - \left(1 - \gamma_A^L(x)\right)^n, 1 - \left(1 - \gamma_A^R(x)\right)^n\right) \end{bmatrix} \\ \mid x \in X \right\}$$

$$= \left\{ \begin{bmatrix} x, \left(1 - \left(1 - \left(\mu_A^L(x)\right)^n\right), 1 - \left(1 - \left(\mu_A^R(x)\right)^n\right)\right), \\ \left(1 - \left(1 - \gamma_A^L(x)\right)^n, 1 - \left(1 - \gamma_A^R(x)\right)^n\right) \end{bmatrix} \\ \mid x \in X \right\}$$

$$= \left\{\left[x, \left(\left(\mu_A^L(x)\right)^n, \left(\mu_A^R(x)\right)^n\right), \left(1 - \left(1 - \gamma_A^L(x)\right)^n, 1 - \left(1 - \gamma_A^R(x)\right)^n\right)\right] \mid x \in X\right\}$$

$$= A^n$$

Hence $(a)$is proved and similarly $(b)$is proved by analogy.

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