

HOMOMORPHISM AND ANTIHOMOMORPHISM
IN INTERVAL VALUED Q-FUZZY SUBHEMIRINGS OF A HEMIRING

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ABSTRACT

In this paper, we study some of the properties of interval valued Q-fuzzy subhemiring of a hemiring under homomorphism, antihomomorphism and prove some results on these.

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Keywords: Interval valued fuzzy subset, interval valued Q-fuzzy subhemiring, and interval valued Q-fuzzy normal subhemiring.

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring $(R, +, \cdot)$. Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (called also half rings) are algebras $(R, +, \cdot)$ share the same properties as a ring except that $(R, +)$ is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra $(R, +, \cdot)$ is said to be a semi ring $(R, +)$ and (R, \cdot) are semi groups satisfying $a \cdot (b+c) = a \cdot b + a \cdot c$ and $(b+c) \cdot a = b \cdot a + c \cdot a$ for all a, b and c in R . A semi ring R is said to be additively commutative if $a+b=b+a$ for all a, b and c in R . A semi ring R may have an identity 1 , defined by $1 \cdot a = a \cdot 1$ and a zero 0 , defined by $0+a=a+0$ and $a \cdot 0=0 \cdot a$ for all a in R . A semi ring R is said to be a hemiring if it is an additively commutative with zero. Interval valued fuzzy sets were introduced independently by Zadeh [10], Grattan-Guinness [4], Jahn [6], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function $\mu_{\tilde{A}}: X \rightarrow D[0,1]$. Y.B and Kin.K [7] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju. A and Nagarajan.R [9] defined the characterization of interval valued Anti fuzzy Left h-ideals over hemirings. Azriel Rosenfeld [2] defined fuzzy groups. Osman Kazanci, Sultan yamark and serife yilmaz in [11] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear rings) in a near-ring and investigated some related properties. Solairaju. A and Nagarajan.R [14], have given a new structure in the construction of Q-fuzzy groups and subgroups [15]. We introduced the concept of interval valued Q-fuzzy subhemiring of a hemiring under homomorphism, anti homomorphism and established some results.

1. PRELIMINARIES

1.1 Definition: Let X be any nonempty set. A mapping $[M]: X \rightarrow D[0,1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X , where $D[0,1]$ denoted the family of all closed subintervals of $[0,1]$ and $[M](x) = [M^-(x), M^+(x)]$, for all x in X , where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X . Thus $[M](x)$ is an interval (a closed subset of $[0,1]$) and not number from the interval $[0,1]$ as in the case of fuzzy subset. Note that $[0] = [0, 0]$ and $[1] = [1, 1]$.

1.2 Remark: Let D^X be the set of all interval valued fuzzy subsets of X , where D means $D[0,1]$.

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1.3 Definition: Let $[M] = \{ \langle x, [M^-(x), M^+(x)] \rangle / x \in X \}$, $[N] = \{ \langle x, [N^-(x), N^+(x)] \rangle / x \in X \}$ be any two interval valued fuzzy subsets of x . We define the following relations and operations:

- (i) $[M] \subseteq [N]$ if and only if $M^-(x) \leq N^-(x)$ and $M^+(x) \leq N^+(x)$, for all x in X .
- (ii) $[M] = [N]$ if and only if $M^-(x) = N^-(x)$ and $M^+(x) = N^+(x)$, for all x in X .
- (iii) $[M] \cap [N] = \{ \langle x, [\min\{M^-(x), N^-(x)\}, \min\{M^+(x), N^+(x)\}] \rangle / x \in X \}$.
- (iv) $[M] \cup [N] = \{ \langle x, [\max\{M^-(x), N^-(x)\}, \max\{M^+(x), N^+(x)\}] \rangle / x \in X \}$.
- (v) $[M]^c = [1] - [M] = \{ \langle x, [1 - M^+(x), 1 - M^-(x)] \rangle / x \in X \}$

1.4 Definition: Let X be a non-empty set and Q be a non-empty set. A (Q, L) -fuzzy subset A of X is function $A: X \times Q \rightarrow [0,1]$.

1.5 Definition: Let $(R, +, \cdot)$ be a hemiring. A interval valued Q -fuzzy subset $[M]$ of R is said to be an interval valued Q -fuzzy subhemiring (IVFSHR) of R if the following conditions are satisfied:

- (i) $[M](x + y, q) \geq \min([M](x, q), [M](y, q))$
- (ii) $[M](xy, q) \geq \min([M](x, q), [M](y, q))$, for all x and y in R , and q in Q .

1.6 Definition: Let $(R, +, \cdot)$ be a hemiring. A interval valued Q -fuzzy subhemiring $[A]$ of R is said to be an interval valued Q -fuzzy normal subhemiring (IVFNSHR) of R if $[A](xy, q) = [A](yx, q)$, for all x and y in R and q in Q .

1.7 Definition: Let X and X' be any two sets. Let $f: X \rightarrow X'$ be any function and $[A]$ be an interval valued Q -fuzzy subset in X , $[V]$ be an interval valued Q -fuzzy subset in $f(X) = X'$, defined by $[V](y, q) = \sup_{x \in f^{-1}(y)} [A](x, q)$ for all x in X and y in X' and q in Q . Then $[A]$ is called a pre-image of $[V]$ under f and is denoted by $f^{-1}([V])$.

2. PROPERTIES OF INTERVAL VALUED Q-FUZZY SUBHEMIRINGS

2.1 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic image of an interval valued Q -fuzzy subhemiring of R is an interval valued Q -fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Then, $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x)f(y)$, for all x and y in R . Let $[V] = f([A])$, where $[A]$ is an interval valued Q -fuzzy subhemiring of R . We have to prove that $[V]$ is an interval valued Q -fuzzy subhemiring of R' . Now, for $f(x), f(y)$ in R' & q in Q .

$[V](f(x), q) + (f(y), q) = [V](f(x + y), q) \geq [A](x + y, q) \geq \min\{[A](x, q), [A](y, q)\}$ Which implies that $[V](f(x), q) + (f(y), q) \geq \min([V](f(x), q), [V](f(y), q))$.

Again,

$[V](f(x), q)(f(y), q) = [V](f(xy), q) \geq [A](xy, q) \geq \min\{[A](x, q), [A](y, q)\}$ Which implies that $[V](f(x), q)(f(y), q) \geq \min([V](f(x), q), [V](f(y), q))$. Hence $[V]$ is an interval valued Q -fuzzy subhemiring of R' .

2.2 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic preimage of an interval valued Q -fuzzy subhemiring of R' is interval valued Q -fuzzy subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Then, $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x)f(y)$, for all x and y in R . Let $[V] = f([A])$, where $[V]$ is an interval valued Q -fuzzy subhemiring of R' . We have to prove that $[A]$ is an interval valued Q -fuzzy subhemiring of R . Let x and y in R and q in Q . Then

$[A](x + y, q) = [V](f(x + y), q) \geq [V](f(x, q) + f(y, q)) \geq \min\{[V](f(x), q), [V](f(y), q)\} = \min\{[A](x, q), [A](y, q)\}$, which implies that $[A](x + y, q) = \min\{[A](x, q), [A](y, q)\}$

Again,

$[A](xy, q) = [V](f(xy), q) \geq [V](f(x, q)f(y, q)) \geq \min\{[V](f(x), q), [V](f(y), q)\} = \min\{[A](x, q), [A](y, q)\}$ which implies that $[A](xy, q) = \min\{[A](x, q), [A](y, q)\}$.

Hence $[A]$ is an interval valued Q -fuzzy subhemiring of R .

2.3 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The anti-homomorphic image of an interval valued Q -fuzzy subhemiring of R is an interval valued Q -fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a anti-homomorphism. Then, $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x)f(y)$, for all x and y in R . Let $[V] = f([A])$, where $[A]$ is an interval valued Q -fuzzy subhemiring of R . We have to prove that $[V]$ is an interval valued Q -fuzzy subhemiring of R' . Now, for $f(x), f(y)$ in R' & q in Q $[V](f(x), q) + (f(y), q) = [V](f(y), q) + (f(x), q) \geq [A](y + x, q) \geq \min\{[A](y, q), [A](x, q)\}$ which implies that $[V](f(x), q) + (f(y), q) \geq \min([V](f(x), q), [V](f(y), q))$.

Again,

$[V]((f(x), q)(f(y), q)) = [V](f(yx), q) \geq [A](yx, q) \geq \min\{[A](y, q), [A](x, q)\} = \min\{[A](x, q), [A](y, q)\}$ which implies that $[V]((f(x), q)(f(y), q)) \geq \min\{[V](f(x), q), [V](f(y), q)\}$.

Hence $[V]$ is an interval valued Q-fuzzy subhemiring of R' .

2.4 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The anti-homomorphic preimage of an interval valued Q-fuzzy subhemiring of R' is an interval valued Q-fuzzy subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a anti-homomorphism. Then $f(x+y)=f(y)+f(x)$ and $f(xy)=f(y)f(x)$, for all x and y in R .

Let $[V] = f([A])$ where V is an anti (Q, L) – fuzzysubhemiring of R' . Let x and y in R & q in Q . where $[V]$ is an interval valued Q-fuzzy subhemiring of R' . We have to prove that $[A]$ is an interval valued Q – fuzzy subhemiring of R . Let x and y in R and q in Q . Then

$[A](x + y, q) = [V](f(x + y), q) = [V](f(y + x), q) \geq \min\{[V](f(y), q), [V](f(x), q)\} \geq \min\{[V](f(x), q), [V](f(y), q)\} = \min\{[A](x, q), [A](y, q)\}$, which implies that $[A](x + y, q) = \min\{[A](x, q), [A](y, q)\}$. Again, $[A](xy, q) = [V](f(xy), q) \geq [V](f(x), q)(f(y), q) \geq \min\{[V](f(x), q), [V](f(y), q)\} \geq \min\{[V](f(x), q), [V](f(y), q)\} = \min\{[A](x, q), [A](y, q)\}$ which implies that $[A](xy, q) = \min\{[A](x, q), [A](y, q)\}$. Hence $[A]$ is an interval valued Q-fuzzy subhemiring of R .

2.5 Theorem: Let $[A]$ be an interval valued Q-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q-fuzzy subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q-fuzzy subhemiring of a hemiring H . Then we have, $([A] \circ f)(x + y, q) = [A](f(x + y), q) = [A](f(x), q) + (f(y), q) \geq \min\{[A](f(x), q), [A](f(y), q)\} = \min\{([A] \circ f)(x, q), ([A] \circ f)(y, q)\}$. which implies that $([A] \circ f)(x + y, q) \geq \min\{([A] \circ f)(x, q), ([A] \circ f)(y, q)\}$. And, $([A] \circ f)(xy, q) = [A](f(xy), q) = [A](f(x), q)(f(y), q) \geq \min\{[A](f(x), q), [A](f(y), q)\} = \min\{([A] \circ f)(x, q), ([A] \circ f)(y, q)\}$. which implies that $([A] \circ f)(xy, q) \geq \min\{([A] \circ f)(x, q), ([A] \circ f)(y, q)\}$. Therefore $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of R .

2.6 Theorem: Let $[A]$ be an interval valued Q-fuzzy subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q-fuzzy subhemiring of R .

Proof: Let x and y in R . Then we have, $([A] \circ f)(x + y, q) = [A](f(x + y), q) = [A](f(y), q) + f(x, q) \geq \min\{[A](f(x), q), [A](f(y), q)\} = \min\{([A] \circ f)(x, q), ([A] \circ f)(y, q)\}$. which implies that $([A] \circ f)(x + y, q) \geq \min\{([A] \circ f)(x, q), ([A] \circ f)(y, q)\}$. and, $([A] \circ f)(xy, q) = [A](f(yx), q) = [A](f(y), q)f(x, q) \geq \min\{[A](f(x), q), [A](f(y), q)\} = \min\{([A] \circ f)(x, q), [A] \circ f(y, q)\}$. which implies that $[A] \circ f(x, y, q) \geq \min\{[A] \circ f(x, q), [A] \circ f(y, q)\}$. Therefore $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of the hemiring R .

2.7 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic image of an interval valued Q-fuzzy normal subhemiring of R is an interval valued Q-fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Let $[A]$ is an interval valued Q-fuzzy normal subhemiring of R . We have to prove that $[V]$ is an interval valued Q-fuzzy normal subhemiring of $f(R) = R'$. Now, for $f(x), f(y)$ in R' & q in Q . clearly, $[V]$ is an interval valued Q-fuzzy subhemiring of $f(R) = R'$. since $[A]$ is an interval valued Q-fuzzy subhemiring of R .

Now, $[V]((f(x), q)(f(y), q)) = [V](f(xy), q) \geq [A](xy, q) = [A](yx, q) \leq [V](f(yx), q) = [V]((f(y), q)(f(x), q))$ which implies that $[V]((f(x), q)(f(y), q)) = [V]((f(y), q)(f(x), q))$. Hence $[V]$ is an interval valued Q-fuzzy normal subhemiring of the hemiring R' .

2.8 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic preimage of an interval valued Q-fuzzy normal subhemiring of R' is interval valued Q-fuzzy normal subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Let $[V]$ is an interval valued Q-fuzzy normal subhemiring of $f(R) = R'$. we have to prove that $[A]$ is an interval valued Q – fuzzy normal subhemiring of R . Let x and y in R and q in Q . Then, clearly, $[A]$ is an interval valued Q-fuzzy subhemiring of the hemiring R .

Now, $[A](xy, q) = [V](f(xy), q) = [V]\{(f(x), q)(f(y), q)\} = [V]\{(f(y), q)(f(x), q)\} = [V](f(yx), q) = [A](yx, q)$ which implies that $[A](xy, q) = [A](yx, q)$, for all x and y in R and q in Q . Hence $[A]$ is an interval valued Q -fuzzy normal subhemiring of the hemiring R .

2.9 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The antihomomorphic image of an interval valued Q -fuzzy normal subhemiring of R is an interval valued Q -fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a anti homomorphism. Let $[A]$ is an interval valued Q -fuzzy normal subhemiring of R . We have to prove that $[V]$ is an interval valued Q -fuzzy normal subhemiring of $f(R) = R'$. Now, for $f(x), f(y)$ in R' & q in Q . clearly, $[V]$ is an interval valued Q -fuzzy subhemiring of R^A . since $[A]$ is an interval valued Q -fuzzy subhemiring of R . Now, $[V]\{(f(x), q)(f(y), q)\} = [V](f(yx), q) \geq [A](yx, q) = [A](xy, q) \leq [V]\{(f(xy), q)\} = [V]\{(f(y), q)(f(x), q)\}$ which implies that $[V]\{(f(x), q)(f(y), q)\} = [V]\{(f(y), q)(f(x), q)\}$. Hence $[V]$ is an interval valued Q -fuzzy normal subhemiring of the hemiring R' .

2.10 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The anti homomorphic preimage of an interval valued Q -fuzzy normal subhemiring of R' is interval valued Q -fuzzy normal subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be anti homomorphism. Let $[V]$ is an interval valued Q -fuzzy normal subhemiring of $f(R) = R'$. We have to prove that $[A]$ is an interval valued Q -fuzzy normal subhemiring of R . Let x and y in R and q in Q . Then clearly, $[A]$ is an interval valued Q -fuzzy subhemiring of the hemiring R . since $[V]$ is an interval valued Q -fuzzy normal subhemiring of the hemiring R' .

Now, $[A](xy, q) = [V](f(xy), q) = [V]\{(f(y), q)(f(x), q)\} = [V]\{(f(x), q)(f(y), q)\} = [V](f(yx), q) = [A](yx, q)$ which implies that $[A](xy, q) = [A](yx, q)$, for all x and y in R and q in Q . Hence $[A]$ is an interval valued Q -fuzzy normal subhemiring of the hemiring R .

2.11 Theorem: Let $[A]$ be an interval valued Q -fuzzy normal subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q -fuzzy normal subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q -fuzzy normal subhemiring of a hemiring H . Then clearly, $([A] \circ f)$ is an interval valued Q -fuzzy subhemiring of the hemiring R . Then we have $([A] \circ f)(xy, q) = [A](f(xy), q) = [A]\{(f(x), q)(f(y), q)\} = [A]\{(f(y), q)(f(x), q)\} = [A](f(yx), q) = ([A] \circ f)(yx, q)$ which implies that $([A] \circ f)(xy, q) = ([A] \circ f)(yx, q)$ therefore $([A] \circ f)$ is an interval valued Q -fuzzy normal subhemiring of R .

2.12 Theorem: Let $[A]$ be an interval valued Q -fuzzy normal subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q -fuzzy normal subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q -fuzzy normal subhemiring of a hemiring H . Then clearly, $([A] \circ f)$ is an interval valued Q -fuzzy subhemiring of the hemiring R . Then we have $([A] \circ f)(xy, q) = [A](f(xy), q) = [A]\{(f(y), q)(f(x), q)\} = [A]\{(f(x), q)(f(y), q)\} = [A](f(yx), q) = ([A] \circ f)(yx, q)$ which implies that $([A] \circ f)(xy, q) = ([A] \circ f)(yx, q)$, for all x and y in R and q in Q . Hence, $([A] \circ f)$ is an interval valued Q -fuzzy normal subhemiring of hemiring R .

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