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EFFECT OF MHD FLOWS DUE TO ROTATING POROUS DISK AND A THIRD GRADE FLUID AT INFINITY WITH PARTIAL SLIP

R. LAKSHMI[1], S. SELVANAYAKI*[2]

[1]Assistant Professor, Department of Mathematics,
PSGR Krishnammal College for women, Coimbatore - 641004, (T.N.), India.

[2]Research Scholar, Department of Mathematics,
PSGR Krishnammal College for women, Coimbatore - 641004, (T.N.), India.

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ABSTRACT

The aim of the present paper is to study the effect of MHD flows due to rotating porous disk and a third grade fluid at infinity with partial slip. The arising non-linear problem is solved numerically using modified crank-Nicolson implicit scheme through MATLAB. The variation of the velocity profile with distance from the disk for various values of slip parameter λ is discussed graphically and the results are reported for conclusion.

Keywords: *Non-Newtonian fluid, Third grade fluid, MHD flows, Finite difference method, Partial slip parameter.*

INTRODUCTION

The equations which govern the flows of viscous fluids are the Navier-stokes equations. In nature, there are many fluids which do not obey the Newtonian law of viscosity and the Navier-stokes equations are inadequate for such fluids. These fluids are termed as the non-Newtonian fluids. Non – Newtonian fluid flow play an important roles in several industrial manufacturing processes. That are used in various branches of science, engineering technology: particularly in material processing, chemical industry, geophysics and bio-engineering. Moreover, the non-Newtonian fluids such as mercury amalgams, liquid metals, biological fluids, plastic extrusions, paper coating lubrication oils and greases have applications in many areas with or without magnetic field. Due to complexity of fluids in nature, non-Newtonian fluids are classified on the basis of their behaviour in shear. Among the many fluid models which have been used to describe the viscoelastic behaviour exhibited by these fluids, the fluid of second grade and third grade have received a special attention. The normal stress differences are described in second grade of non-Newtonian fluid, but they cannot predict shear thinning or thickening properties due to their constant apparent viscosity. The third grade fluid model attempt includes such characteristics of visco-elastic fluids. Despite various complexities in the constitutive equations, several researchers have investigated the flows of third – grade fluids taking into account various aspects. Several researchers discussed the slip effect on fluid flow. For instance, S. Asghar, M. Mudassar Gulzar, M. Ayub (2006) discussed an analytical study of on the rotating flow of third grade fluid past a porous plate with the partial slip effects. Sajid and Hayat (2007) discussed the two – dimensional boundary layer flow of a third grade fluid over stretching sheet. Sajit *et al.* (2007) considered heat transfer characteristics in an electrically conducting third grade fluid. Miccal and James (2008) discussed the effect of replacing the standard no slip boundary condition of fluid mechanics applying for the so called Falkner-Skan solutions, with a boundary condition that allows some degree of tangential fluid slip. Ellahi (2009) discuss the slip condition of an Oldroyd 8 – constant fluid. Sahoo (2010) computed the numerical solutions for heat transfer in Heimenz flow of third –grade fluid. Hayat *et al.* (2010) examined the simultaneous effects of heat and mass transfer on an unsteady flow of third-grade fluid. Rashidi and Pour (2010) examined the three dimensional problem of a condensation film inclined on a rotating disk by a differential transform method. Recently Hayat and Nawaz (2011) discussed unsteady stagnation point flow over a rotating disk. However, the effect of MHD flows due to rotating porous disk and a third grade fluid at infinity with partial slip is not established in the literature. The present work deals with the variation of the velocity profiles for the different values of the partial slip parameter.

Corresponding Author: S. Selvanayaki[2]*
[2]Research scholar, Department of Mathematics,
PSGR Krishnammal college for women, Coimbatore - 641004, (T.N.), India.

MATHEMATICAL FORMULATION

Consider the Cartesian coordinate system with z-axis normal to the porous disk which lies in the plane $z = 0$. The common angular velocity of the disk and the fluid at infinity is Ω. The fluid is electrically conducting in the presence of an applied constant magnetic field B_0.

The velocity field is of the form

$$u = -\Omega y + f(z,t), \quad v = \Omega x + g(z,t), \quad w = -w_0 \tag{1}$$

where u, v, w are the components of the velocity vector V, in the directions x, y, z respectively. Obviously $w_0 > 0$ is the suction velocity and $w_0 < 0$ is the blowing velocity. The velocity field satisfies $\nabla . V = 0$

For magnetohydrodynamic fluid, the governing equation of motion is,

$$\rho \frac{d\boldsymbol{V}}{dt} = div\, \boldsymbol{T} + \boldsymbol{J} \times \boldsymbol{B} \tag{2}$$

where $\boldsymbol{J}$ is the current density and $\boldsymbol{B}$ is the total magnetic field.

The constitutive equation of the third – grade fluid is

$$T = -p_1 I + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr A_1^2) A_1 \tag{3}$$

Where T is the stress tensor, I is the identity, A_1, A_2 and A_3 are the Rivlin - Ericsen tensors of the first, second and third orders respectively, p_1 is the static fluid pressure $(p = p(x,y,z))$, μ is the dynamic viscosity, coefficients $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ are the material constants. Rivlin - Ericksen tensors of order first and nth order are given by

$$A_1 = (gradV) + (gradV)^T, \quad A_n = \frac{dA_{n-1}}{dt} + A_{n-1}(gradV) + (gradV)^T A_{n-1}, \; n \geq 1, \tag{4}$$

The equation (4) to be compatible with thermodynamics and the free energy to be minimum when the fluid is at rest, the material constant should satisfy the relation

$$\mu \geq 0, \; \alpha_1 \geq 0, \; |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}, \; \beta_1 = 0, \; \beta_2 = 0, \; \beta_3 \geq 0 \tag{5}$$

and specific Helmholtz free energy φ has the form

$$\varphi = \varphi(\theta, L) = \varphi(\theta, 0) + \frac{\alpha_1}{4\rho} \tag{6}$$

where $L = gradV$

Here, the fluid is thermodynamically compatible; hence the stress constitutive relation (2) reduces to

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 (tr A_1^2) A_1 \tag{7}$$

The flow is governed through the following equation

$$\frac{\alpha_1}{\rho}\left[\frac{\partial^3 F^*}{\partial t \partial z^2} - w_0 \frac{\partial^3 F^*}{\partial z^3}\right] + \left(v - i\frac{\alpha_1 \Omega}{\rho}\right)\frac{\partial^2 F^*}{\partial z^2} + w_0 \frac{\partial F^*}{\partial z} - \frac{\partial F^*}{\partial t} - \Omega(i + N_1)F^* + \frac{2\beta_3}{\rho}\frac{\partial}{\partial z}\left[\left\{\left(\frac{\partial F^*}{\partial z}\right)^2 \frac{\partial \overline{F^*}}{\partial z}\right\}\right] = 0 \tag{8}$$

$$F^*(0,t) = -\cos k^* t, \; F^*(\infty, t) = 0, \; F^*(z, 0) = 0 \tag{9}$$

where $F^* = \frac{f + ig}{\Omega l} - \cos k^* t$, $\overline{F^*} = \frac{f - ig}{\Omega l} - \cos k^* t$,

On introducing non-dimensional parameter

$$\xi = \sqrt{\frac{\Omega}{2v}} z, \; \tau = \Omega t, \; F^{**}(\xi, \tau) = \frac{F^*(z,t)}{\Omega l}, \; \overline{F^{**}}(\xi, \tau) = \frac{\overline{F^*}(z,t)}{\Omega l} \quad c = \frac{k}{\Omega}$$

$$\beta = \frac{\Omega^3 l^2 \beta_3}{\rho v^2}, \alpha = \frac{\Omega \alpha_1}{\rho v}, \; n = \frac{N_1}{\Omega}, \; \epsilon = \frac{w_0}{\sqrt{2v\Omega}}, \; N_1 = \frac{\sigma}{\rho \Omega} B_0^2, \tag{10}$$

Using equation (10) and equation (8) becomes

$$\alpha \frac{\partial^3 F^{**}}{\partial \tau \partial \xi^2} - \alpha\epsilon \frac{\partial^3 F^{**}}{\partial \xi^3} + (1 - i\alpha)\frac{\partial^2 F^{**}}{\partial \xi^2} + 2\epsilon \frac{\partial F^{**}}{\partial \xi} - 2\frac{\partial F^{**}}{\partial \tau} - 2(i + n)F^{**} + \beta \frac{\partial}{\partial \xi}\left[\left\{\left(\frac{\partial F^{**}}{\partial \xi}\right)^2 \left(\frac{\partial \overline{F^{**}}}{\partial \xi}\right)\right\}\right] = 0 \tag{11}$$

$$F^{**}(0, \tau) = -\cos c\tau, \; F^{**}(\infty, \tau) = 0, \; F^{**}(\xi, 0) = 0. \tag{12}$$

It is noted that equation (11) is a third order and non - linear partial differential equation.

For case of partial slip the boundary condition at $z = 0$ is of the following form

$$u - \lambda_2 \tau_{xz} = \Omega_y - \cos k^* t, v - \lambda_2 \tau_{yz} = \Omega_x \quad at\; z = 0 \; for\; t > 0 \;, \tag{13}$$

where $\lambda = \lambda_2 \sqrt{\frac{\Omega}{v}}$ is slip parameter

The governing problem consist of

$$\alpha \frac{\partial^3 F^{**}}{\partial\tau\partial\xi^2} - \alpha\epsilon\frac{\partial^3 F^{**}}{\partial\xi^3} + (1-i\alpha)\frac{\partial^2 F^{**}}{\partial\xi^2} + 2\epsilon\frac{\partial F^{**}}{\partial\xi} - 2\frac{\partial F^{**}}{\partial\tau} - 2(i+n)F^{**} + \beta\frac{\partial}{\partial\xi}\left[\left\{\left(\frac{\partial F^{**}}{\partial\xi}\right)^2\left(\frac{\partial\overline{F^{**}}}{\partial\xi}\right)\right\}\right] = 0 \quad (14)$$

$$F^{**}(0,\tau) = -\cos c\tau + \lambda\left[\frac{\partial F^{**}}{\partial\xi} + \alpha\left\{\frac{\partial^2 F^{**}}{\partial\tau\partial\xi} - \epsilon\frac{\partial^2 F^{**}}{\partial\xi^2} - i\frac{\partial F^{**}}{\partial\xi}\right\} + \beta\left\{\left(\frac{\partial F^{**}}{\partial\xi}\right)^2\left(\frac{\partial\overline{F^{**}}}{\partial\xi}\right)\right\}\right] \quad (15)$$

NUMERICAL SOLUTION OF THE PROBLEM

The governing equation (14) is highly non-linear partial differential equation together with the initial and boundary condition (15). Also the boundary condition at z=0 is non-linear. Using finite difference method the following algebraic equation is obtained from equation (14)

$$\frac{\alpha}{k^*h^2}\begin{bmatrix}\left(F^{**}_{i+1,j+1} - 2F^{**}_{i,j+1} + F^{**}_{i-1,j+1}\right)\\ -\left(F^{**}_{i+2,j} - 2F^{**}_{i+1,j} + F^{**}_{i,j}\right)\end{bmatrix} - \frac{\alpha\epsilon}{4h^3}\left[\left(F^{**}_{i+2,j+1} - 2F^{**}_{i+1,j+1} + 2F^{**}_{i-1,j+1} + F^{**}_{i-2,j+1}\right) + \left(F^{**}_{i+2,j} - 2F^{**}_{i+1,j} + 2F^{**}_{i-1,j} - F^{**}_{i-2,j}\right)\right] + \frac{1}{2h^2}(1-i\alpha)\left[\left(F^{**}_{i+1,j+1} - 2F^{**}_{i,j+1} + F^{**}_{i-1,j+1}\right) + \left(F^{**}_{i+1,j} - 2F^{**}_{i,j} + F^{**}_{i-1,j}\right)\right] + \frac{2\epsilon}{4h}\left[\left(F^{**}_{i+1,j+1} - F^{**}_{i-1,j+1}\right) + \left(F^{**}_{i+1,j} - F^{**}_{i-1,j}\right)\right] - \frac{2}{k^*}\left[F^{**}_{i,j+1} - F^{**}_{i,j}\right] - 1(i+n)\left[F^{**}_{i,j+1} + F^{**}_{i,j}\right] +$$

$$\frac{\beta}{4h^4}\begin{bmatrix}2\left(F^{**}_{i+1,j} - F^{**}_{i-1,j}\right)\left(F^{**}_{i+2,j} - 2F^{**}_{i+1,j} + F^{**}_{i,j}\right)\begin{pmatrix}\overline{F^{**}}_{i+1,j}\\ -\overline{F^{**}}_{i-1,j}\end{pmatrix}\\ +\left(F^{**}_{i+1,j} - F^{**}_{i-1,j}\right)^2\left(\overline{F^{**}}_{i+2,j} - 2\overline{F^{**}}_{i+1,j} + \overline{F^{**}}_{i,j}\right)\end{bmatrix} = 0. \quad (16)$$

The problem consisting of the above equation along with initial and boundary condition becomes

$$a_iF^{**}_{i-2,j+1} + b_iF^{**}_{i-1,j+1} + c_iF^{**}_{i,j+1} + d_iF^{**}_{i+1,j+1} + e_iF^{**}_{i+2,j+1} = h_i, \quad (17)$$

$$F^{**}_{0,j} = -\cos cjk^*, F^{**}_{N,j} = 0, \quad F^{**}_{i,0} = 0 i = 0,1,2\ldots,N, \quad (18)$$

In which

$$a_i = \frac{k^*\alpha\epsilon}{4h^3}, \quad (19)$$

$$b_i = -\frac{k^*\alpha\epsilon}{2h^3} + \frac{2\alpha}{h^2} + \frac{k^*(1-i\alpha)}{2h^2} - \frac{k^*\epsilon}{4h}, \quad (20)$$

$$c_i = -\frac{2\alpha}{h^2} - \frac{k^*(1-i\alpha)}{h^2} - 2 - (i+n)k^*, \quad (21)$$

$$d_i = \frac{k^*\alpha\epsilon}{2h^3} + \frac{\alpha}{h^2} + \frac{k^*(1-i\alpha)}{2h^2} + \frac{k^*\epsilon}{2h}, \quad (22)$$

$$e_i = -\frac{k^*\alpha\epsilon}{4h^3}, \quad (23)$$

$$h_i = \frac{\alpha}{h^2}\left[F^{**}_{i+2,j} - 2F^{**}_{i+1,j} + F^{**}_{i,j}\right] + \frac{\alpha\epsilon}{4h^3}\left[F^{**}_{i+2,j} - 2F^{**}_{i+1,j} + 2F^{**}_{i-1,j} - F^{**}_{i-2,j}\right] - \frac{k^*(1-i\alpha)}{2h^2}\left[F^{**}_{i+2,j} - 2F^{**}_{i+1,j} + F^{**}_{i,j}\right] - \frac{k^*\beta}{4h^4}\left[2\left(F^{**}_{i+1,j} - F^{**}_{i-1,j}\right)\left(F^{**}_{i+2,j} - 2F^{**}_{i+1,j} + F^{**}_{i,j}\right)\left(\overline{F^{**}}_{i+1,j} - \overline{F^{**}}_{i-1,j}\right) + \left(F^{**}_{i+1,j} - F^{**}_{i-1,j}\right)^2\left(\overline{F^{**}}_{i+2,j} - 2\overline{F^{**}}_{i+1,j} + F^{**}_{i,j}\right.\right. . \quad (24)$$

when $i = 1$ then equation (17) becomes

$$a_1F^{**}_{-1,j+1} + b_1F^{**}_{0,j+1} + c_1F^{**}_{1,j+1} + d_1F^{**}_{2,j+1} + e_1F^{**}_{3,j+1} = h_1. \quad (25)$$

The value of F^{**} at the fictitious point ξ_{-1} is approximated by means of the Lagrange polynomial of third degree

$$F^{**}_{-1,j+1} = l_0F^{**}_{0,j+1} + l_1F^{**}_{1,j+1} + l_2F^{**}_{2,j+1} + l_3F^{**}_{3,j+1}, \quad (26)$$

in which

$$l_q = \Pi\frac{(\xi_{-1} - \xi_p)}{(\xi_q - \xi_p)}, p = 0,1,2,3 \; and \; p \neq q.$$

From equations (25) and (26)

$$(a_1l_0 + b_1)F^{**}_{0,j+1} + (a_1l_1 + c_1)F^{**}_{1,j+1} + (a_1l_2 + d_1)F^{**}_{2,j+1} + (a_1l_3 + e_1)F^{**}_{3,j+1} = h_1. \quad (27)$$

Since $F^{**}_{0,j+1}$ is known, so equation (27) must be written as

$$c_1'F^{**}_{1,j+1} + d_1'F^{**}_{2,j+1} + e_1'F^{**}_{3,j+1} = h_1', \quad (28)$$

where

$c_1' = a_1l_1 + c_1, \quad d_1' = a_1l_2 + d_1,$

$e_1' = a_1l_3 + e_1, \quad h_1' = h_1 - (a_1l_0 + b_1)F^{**}_{0,j+1}.$

For $i = 2$ we have

$$a_2 F^{**}_{0,j+1} + b_2 F^{**}_{1,j+1} + c_2 F^{**}_{2,j+1} + d_2 F^{**}_{3,j+1} + e_2 F^{**}_{4,j+1} = h_2. \quad (29)$$

(or)

$$b_2 F^{**}_{1,j+1} + c_2 F^{**}_{2,j+1} + d_2 F^{**}_{3,j+1} + e_2 F^{**}_{4,j+1} = h_2',$$

in which

$$h_2' = h_2 - a_2 F^{**}_{0,j+1}. \quad (30)$$

When $3 \le i \le N-3$, the equations are given by

$$a_i F^{**}_{i-2,j+1} + b_i F^{**}_{i-1,j+1} + c_i F^{**}_{i,j+1} + d_i F^{**}_{i+1,j+1} + e_i F^{**}_{i+2,j+1} = h_i.$$

For $i = N-2$ we have

$$a_{N-2} F^{**}_{N-4,j+1} + b_{N-2} F^{**}_{N-3,j+1} + c_{N-2} F^{**}_{N-2,j+1} + d_{N-2} F^{**}_{N-1,j+1} + e_{N-2} F^{**}_{N,j+1} = h_{N-2.} \quad (31)$$

For known $F^{**}_{N,j+1}$ equation (31) is

$$a_{N-2} F^{**}_{N-4,j+1} + b_{N-2} F^{**}_{N-3,j+1} + c_{N-2} F^{**}_{N-2,j+1} + d_{N-2} F^{**}_{N-1,j+1} = h_{N-2}', \quad (32)$$

where

$$h_{N-2}' = h_{N-2} - e_{N-2} F^{**}_{N,j+1}.$$

For $i = N-1$, we have

$$a_{N-1} F^{**}_{N-3,j+1} + b_{N-1} F^{**}_{N-2,j+1} + c_{N-1} F^{**}_{N-1,j+1} + d_{N-1} F^{**}_{N,j+1} + e_{N-1} F^{**}_{N+1,j+1} = h_{N-1} \quad (33)$$

To find the value of h_1 at the time level j, we must have the value of $F^{**}_{N+1,j}$. We use the augmentation process and write

$$\frac{\partial F^{**}(\infty,\tau)}{\partial \xi} = 0. \quad (34)$$

This boundary condition is discretized to give

$$\frac{F^{**}_{N+1,j} - F^{**}_{N,j}}{h} = 0 \quad i.e\ F^{**}_{N+1,j} = F^{**}_{N,j}\ . \quad (35)$$

Thus for $i = N-1$, equation (33) is

$$a_{N-1} F^{**}_{N-3,j+1} + b_{N-1} F^{**}_{N-2,j+1} + c_{N-1} F^{**}_{N-1,j+1} = h_{N-1}',$$

where

$$h_{N-1}' = h_{N-1} - (d_{N-1} + e_{N-1}) F^{**}_{N,j+1} \quad (36)$$

It is noted that there are $N-1$ equations. In matrix form, it can be written as

$$\begin{bmatrix} c_1' & d_1' & e_1' & 0 & 0 & 0 & 0 & 0 & . & . & 0 \\ b_2 & c_2 & d_2 & e_2 & 0 & 0 & 0 & 0 & . & . & 0 \\ a_3 & b_3 & c_3 & d_3 & e_3 & 0 & 0 & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & . & 0 & a_i & b_i & c_i & d_i & e_i & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & a_{N-3} & b_{N-3} & c_{N-3} & d_{N-3} & e_{N-3} \\ 0 & 0 & 0 & 0 & . & . & 0 & a_{N-2} & b_{N-2} & c_{N-2} & d_{N-2} \\ 0 & 0 & 0 & 0 & 0 & . & . & 0 & a_{N-1} & b_{N-1} & c_{N-1} \end{bmatrix} X \begin{bmatrix} F_1^{**} \\ F_2^{**} \\ F_3^{**} \\ . \\ F_i^{**} \\ . \\ F_{N-3}^{**} \\ F_{N-2}^{**} \\ F_{N-1}^{**} \end{bmatrix} = \begin{bmatrix} h_1' \\ h_2' \\ h_3 \\ . \\ h_i \\ . \\ h_{N-3} \\ h_{N-2}' \\ h_{N-1}' \end{bmatrix} \quad (37)$$

Obviously the matrix involved in equation (37) is pentadiagonal.

It is observed that the partial slip boundary condition (15) is highly non-linear. Thus the following same procedure adopted for discritization of equation (14), equation (15) may be discritized as

$$F^{**}_{0,j} = r_1 F^{**}_{1,j} + r_2 F^{**}_{2,j} + r_3 \left[F^{**}_{0,j-1} - F^{**}_{1,j-1}\right] + r_4 \left[BT_j - \cos cjk^*\right] \quad (38)$$

In the above equation

$$r_0 = h^2 k^* + \lambda(1 - i\alpha) h k^* + \alpha\lambda h + \alpha\epsilon\lambda k^* \quad (39)$$

$$r_1 = [\lambda(1 - i\alpha) h k^* + \alpha\lambda h + 2\alpha\epsilon\lambda k^*]/r_0 \quad (40)$$

$$r_2 = -[\alpha\epsilon\lambda k^*]/r_0 \quad (41)$$

$$r_3 = [\alpha\lambda h]/r_0 \quad (42)$$

$$r_4 = [h^2 k^*]/r_0 \quad (43)$$

$$BT_j = \frac{\beta\lambda}{h^3}\left[(F^{**}_{1,j} - F^{**}_{0,j})^2 \left(\overline{F^{**}}_{1,j} - \overline{F^{**}}_{0,j}\right)\right] \quad (44)$$

To evaluate $F^{**}_{0,j+1}$, first take $BT_{j+1} = BT_j$ in the system of algebraic equation and the solution of the system is sought which results in known values of $F^{**}_{i,j+1},\ i = 1,2,3 \ldots\ldots N-1$. Second, we update $F^{**}_{0,j+1}$ by using iterative method as follows

$$F^{**}{}^{k^*+1}_{0,j+1} = r_1 F^{**}_{1,j+1} + r_2 F^{**}_{2,j+1} + r_3\left[F^{**}{}^{k^*}_{0,j} - F^{**}_{1,j}\right] + r_4 \frac{\beta\lambda}{h^3}\left[(F^{**}_{1,j+1} - F^{**}{}^{k^*}_{0,j+1})^2\left(\overline{F^{**}}_{i,j+1} - \overline{F^{**k^*}}_{0,j+1}\right)\right] + r_4[-\cos c(j+1)k^*] \tag{45}$$

where, $F^{**}{}^{0}_{0,j+1} = F^{**}_{0,j+1}$, and this iterative procedure is continued until $F^{**}{}^{k^*+1}_{0,j+1} \approx F^{**}{}^{k^*}_{0,j+1}$. Futhermore, $F^{**}_{0,0}$ is evaluated by letting $F^{**}_{0,-1} = F^{**}_{1,-1} = 0$, and by using iterative method as described above with $F^{**}{}^{0}_{0,0} = 0$ as initial guess.

For $i = 1,\ i = 2, 3 \le i \le N-3,\ i = N-2$ *and* $i = N-1$, We respectively have

$$c_1' F^{**}{}_{1,j+1} + d_1' F^{**}{}_{2,j+1} + e_1' F^{**}{}_{3,j+1} = h_1', \tag{46}$$
$$b_2' F^{**}{}_{1,j+1} + c_2' F^{**}{}_{2,j+1} + d_2 F^{**}{}_{3,j+1} + e_2 F^{**}{}_{4,j+1} = h_2', \tag{47}$$
$$a_i F^{**}{}_{i-2,j+1} + b_i F^{**}{}_{i-1,j+1} + c_i F^{**}{}_{i,j+1} + d_i F^{**}{}_{i+1,j+1} + e_i F^{**}{}_{i+2,j+1} = h_i\ , \tag{48}$$
$$a_{N-2} F^{**}_{N-4,j+1} + b_{N-2} F^{**}{}_{N-3,j+1} + c_{N-2} F^{**}{}_{N-2,j+1} + d_{N-2} F^{**}{}_{M-1,j+1} = h_{N-2}', \tag{49}$$
$$a_{N-1} F^{**}{}_{N-3,j+1} + b_{N-1} F^{**}{}_{N-2,j+1} + c_{N-1} F^{**}{}_{N-1j+1} = h_{N-1}' \tag{50}$$

where a_i, b_i, c_i, d_i, e_i and h_i are given through equations (19) to (24)

In the above equations

$$c_1' = a_1 l_1 + c_1 + r_1(b_1 + l_0 a_1), \tag{51}$$
$$d_1' = a_1 l_2 + d_1 + r_2(b_1 + l_0 a_1) \tag{52}$$
$$e_1' = a_1 l_3 + e_1, \tag{53}$$
$$h_1' = h_1 - (a_1 l_0 + b_1)\left[r_3\left(F^{**}_{0,j} - F^{**}_{1,j}\right) + r_4(BT_{j+1} - \cos c(j+1)k^*\right] \tag{54}$$
$$b_2' = b_2 + r_1 a_2 \tag{55}$$
$$c_2' = c_2 + r_2 a_2 \tag{56}$$
$$h_2' = h_2 - a_2\left[r_3\left(F^{**}_{0,j} - F^{**}_{1,j}\right) + r_4\left(BT_{j+1} - \cos c\,(j+1)k^*\right)\right], \tag{57}$$
$$h_{N-2}' = h_{N-2} - e_{N-2} F^{**}_{N,j+1}, \tag{58}$$
$$h_{N-1}' = h_{N-1} - (d_{N-1} + e_{N-1}) F^{**}_{N,j+1} \tag{59}$$

The matrix form of the above set of *N-1* equation is

$$\begin{bmatrix} c_1' & d_1' & e_1' & 0 & 0 & 0 & 0 & 0 & . & . & 0 \\ b_2' & c_2' & d_2 & e_2 & 0 & 0 & 0 & 0 & . & . & 0 \\ a_3 & b_3 & c_3 & d_3 & e_3 & 0 & 0 & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & . & 0 & a_i & b_i & c_i & d_i & e_i & 0 & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & a_{N-3} & b_{N-3} & c_{N-3} & d_{N-3} & e_{N-3} \\ 0 & 0 & 0 & 0 & . & . & 0 & a_{N-2} & b_{N-2} & c_{N-2} & d_{N-2} \\ 0 & 0 & 0 & 0 & 0 & . & . & 0 & a_{N-1} & b_{N-1} & c_{N-1} \end{bmatrix} X \begin{bmatrix} F_1^{**} \\ F_2^{**} \\ F_3^{**} \\ . \\ F_i^{**} \\ . \\ F_{N-3}^{**} \\ F_{N-2}^{**} \\ F_{N-1}^{**} \end{bmatrix} = \begin{bmatrix} h_1' \\ h_2' \\ h_3 \\ . \\ h_i \\ . \\ h_{N-3} \\ h_{N-2}' \\ h_{N-1}' \end{bmatrix} \tag{60}$$

RESULTS AND DISCUSSION

In this paper, the effect of MHD flows due to rotating porous disk and a third grade fluid at infinity with partial slip is studied. The variations of the velocity profile with distance from the disk for various values of partial slip parameter λ are discussed graphically. Figures 1 to 6 are prepared for both real and imaginary parts of the velocity profile. Figures 1 to 2 illustrate the effect of partial slip parameter $\lambda(= 0,1,2)$ on the flow of viscous fluid when $\alpha = 0.08,\ \beta = 1,$ $c = 1.5,\ n = 0,\ \tau = 1$. Here the real and imaginary part increases with increase of slip parameter λ. Illustration of Figures 3 to 4 show the effect of partial slip parameter $\lambda(= 0,1,2)$ on the flow of viscous fluid when $\alpha = 0.08,$ $\beta = 1,\ c = 1,\ n = 0,\ \tau = 1$. It seen that the real and imaginary part increases with increase of slip parameter λ.Illustration of Figures 5 and 6 shows the effect of partial slip parameter $\lambda(= 0,1,2)$ on the flow of viscous fluid when $\alpha = 0.08,\ \beta = 1,\ c = 0.5,\ n = 0,\ \tau = 1$.

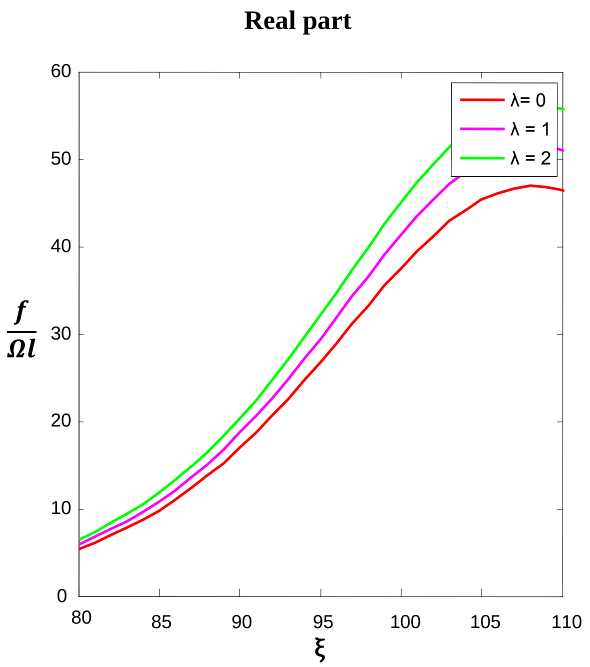


Fig. 1

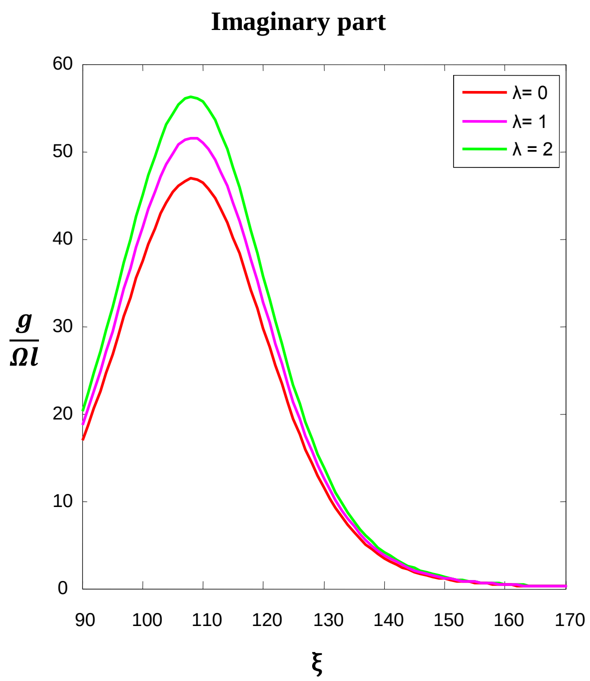
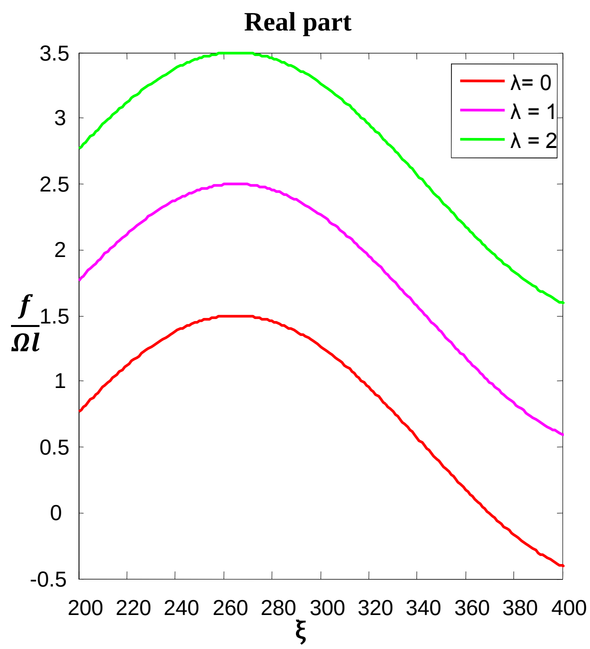
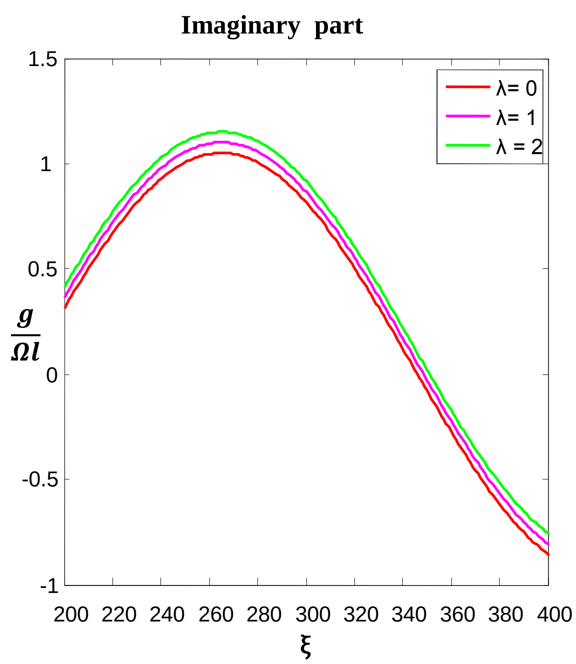


Fig. 2

The variation of the velocity profile with distance from the disk for various values of partial slip parameter λ when $\alpha = 0.08,\ \beta = 1,\ c = 1.5,\ n = 0,\ \tau = 1.$



Fig, 3



Fig, 4

The variation of the velocity profile with distance from the disk for various values of partial slip parameter λ when $\alpha = 0.08,\ \beta = 1,\ c = 1,\ n = 0,\ \tau = 1.$

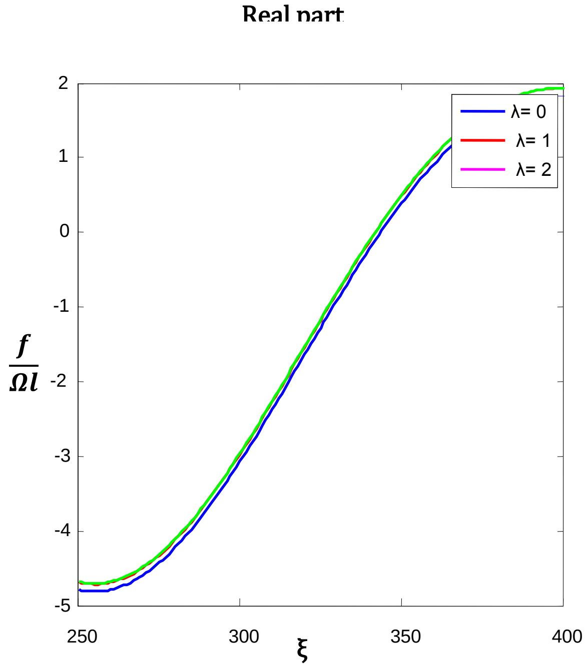


Fig. 5

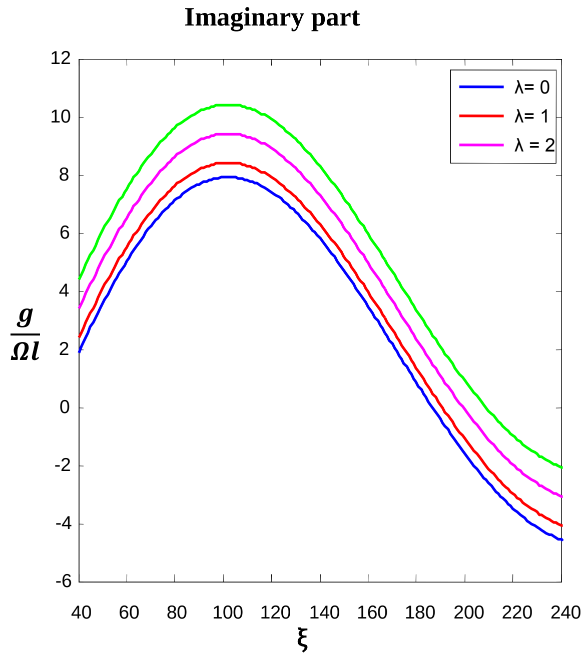


Fig. 6

The variation of the velocity profile with distance from the disk for various values of partial slip parameter λ when $\alpha = 0.08,\ \beta = 1,\ c = 0.5,\ n = 0,\ \tau = 1.$

CONCULSION

In this paper, effect of MHD flows due to rotating porous disk and a third grade fluid at infinity with partial slip is studied. The nonlinear partial differential equations are solved numerically by finite difference method through MATLAB. The results are shown graphically and it is found that the real and imaginary part of the velocity profile increases with increase of partial slip parameter. It is further found that the partial slip causes the reduction in the boundary layer thickness.

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