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ON2-2 NIM EXTENSIVE FORM GAME

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ABSTRACT

In the present paper a particular extensive form game, 2-2 Nim game is reduced in normal form and bimatrix form. A new technique is used to deal with the problem to obtain the value of game for best strategies of the players.

Keywords: *Nims game, optimal strategy, best inequality.*

INTRODUCTION

In the recent years extensive games are an intuitive formalism for modelling interaction between agents in a sequential decision making. The best strategies can be found using linear programming in [4].The iterative Brown and Robinson method is one of the common techniques to find solution of extensive form games. Brown conjectured and Robinson proved the convergence of this process for bimatrix games [6].

2-2. NIM EXTENSIVE FORM GAME

Four matches are set out in two piles of two matches each. Two players take alternate turns. At each turn player selects a pile that has at least one match and removes at least one match from this pile. The player may take several matches, but only from one pile. When both piles have no matches there are no more turns and game is ended. The player who removed last match loses

GAME TREE

The letter in each box is player to move. Numbers in large type are the sizes of the piles, and each state is numbered in small type for later reference.

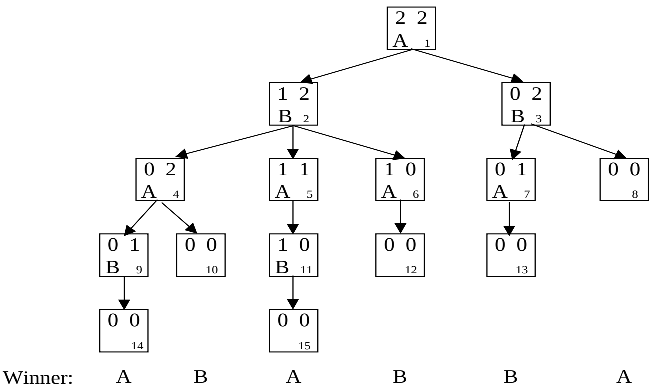

Diagram: 1

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Pure strategies are used for, 2—2 Nim; in this table the expression x □ y means "If I find myself at box x then move to y". If A chooses A_3 then the game can never reach state 4 so there is no entry 4□x .The normal form of a game can only be constructed for a 2 person game. It is a matrix in which (i, j) th cell shows the outcome if the first player chooses i^{th} pure strategy and the 2^{nd} player chooses j^{th} strategy. Usually the entry in each cell shows the final state and some indication of the payoffs to the players. In the normal form the rows of the matrix correspond to the 1^{st} player's strategies, and the columns correspond to the 2^{nd} player's strategies.

Table-1: Pure strategies

Pure Strategies for A		Pure Strategies for B	
Name	Moves	Name	Moves
A1	1□2 , 4□9	B1	2□4 , 3□7
		B2	2□5 , 3□7
A2	1□2 , 4□10	B3	2□6 , 3□7
		B4	2□4 , 3□8
A3	1□3	B5	2□5 , 3□8
		B6	2□6 , 3□8

Table-2: Normal form

	B1	B2	B3	B4	B5	B6
A1	A(14)	A(15)	B(12)	A(14)	A(15)	B(12)
A2	B(10)	A(15)	B(13)	B(12)	A(15)	B(12)
A3	B(13)	B(13)	B(13)	A(8)	A(8)	A(8)

Table-3: Matrix form

	B1	B2	B3	B4	B5	B6
A1	1	1	-1	1	1	-1
A2	-1	1	-1	-1	1	-1
A3	-1	-1	-1	1	1	1

Table-4: Pay off

State	14	10	15	12	13	8
Payoff to A	1	-4	1	-4	-4	1
Payoff to B	-5	1	-5	1	1	-3

Table-5: Bimatrix form

	(Q_1, Q'_1)	(Q_2, Q'_2)	(Q_3, Q'_3)	(Q_4, Q'_4)	(Q_5, Q'_5)	(Q_6, Q'_6)
(P_1, P'_1)	(1,-5)	(1,-5)	(-4,1)	(1,-5)	(1,-5)	(-4,1)
(P_2, P'_2)	(-4,1)	(1,-5)	(-4,1)	(-4,1)	(1,-5)	(-4,1)
(P_3, P'_3)	(-4,1)	(-4,1)	(-4,1)	(1,-3)	(1,-3)	(1,-3)

NEW TECHNIQUE TO FIND VALUE OF THE GAME

In the approximation method two players play a long sequence of plays in given game. Each of them plays in such a way as to maximize his expectation under the assumption that the future will resemble the past. At each point of the sequence one can calculate upper and lower bounds for the value of the game as well as an approximation to an optimal strategy for each player.

Table-6: Expectations of player 1

Play	Player1 chooses	Player2 chooses	Expectation of player 1		
			(P_1, P'_1)	(P_2, P'_2)	(P_3, P'_3)
1.	(P_1, P'_1)	(Q_1, Q'_1)	(1, -5)	(-4, 1)	(-4, 1)
2.	(P_1, P'_2)	(Q_1, Q'_1)	(2, -10)	(-8, 2)	(-8, 2)
3.	(P_1, P'_2)	(Q_1, Q'_2)	(3, -15)	(-12, -3)	(-12, 3)
4.	(P_1, P'_3)	(Q_3, Q'_2)	(-1, -20)	(-16, -8)	(-16, 4)
5.	(P_1, P'_3)	(Q_3, Q'_5)	(-5, -25)	(-20, -13)	(-20, 1)
6.	(P_1, P'_3)	(Q_6, Q'_5)	(-9, -30)	(-25, -18)	(-19, -2)
7.	(P_1, P'_3)	(Q_6, Q'_5)	(-13, -35)	(-29, -23)	(-18, -5)
8.	(P_1, P'_3)	(Q_6, Q'_5)	(-17, -40)	(-33, -28)	(-17, -8)
10.	(P_3, P'_3)	(Q_3, Q'_5)	(-25, -50)	(-41, -38)	(-20, -14)

Table-7: Expectation of player 2

Play	Player1 Chooses	Player2 Chooses	Expectation of player2					
			(Q_1, Q'_1)	(Q_2, Q'_2)	(Q_3, Q'_3)	(Q_4, Q'_4)	(Q_5, Q'_5)	(Q_6, Q'_6)
1.	(P_1, P'_1)	(Q_1, Q'_1)	(1, 5)	(1, 5)	(-4, -1)	(1, 5)	(1, 5)	(-4, -1)
2.	(P_1, P'_2)	(Q_1, Q'_1)	(0, 4)	(0, 10)	(0, -2)	(0, 4)	(0, 10)	(0, -2)
3.	(P_1, P'_2)	(Q_1, Q'_2)	(-1, 3)	(-1, 15)	(4, -3)	(-1, 3)	(-1, 15)	(4, -3)
4.	(P_1, P'_3)	(Q_3, Q'_2)	(-2, 2)	(-2, 14)	(8, -4)	(-2, 0)	(-2, 18)	(8, 0)
5.	(P_1, P'_3)	(Q_3, Q'_5)	(-3, 1)	(-3, 13)	(12, -5)	(-3, 3)	(-3, 21)	(12, 3)
6.	(P_1, P'_3)	(Q_6, Q'_5)	(-4, 0)	(-4, 12)	(16, -6)	(-4, 6)	(-4, 24)	(16, 6)
7.	(P_1, P'_3)	(Q_6, Q'_5)	(-5, -1)	(-5, 11)	(20, -7)	(-5, 9)	(-5, 27)	(20, 9)
8.	(P_1, P'_3)	(Q_6, Q'_5)	(-6, -2)	(-6, 10)	(24, -8)	(-6, -12)	(-6, 30)	(24, 12)
9.	(P_3, P'_3)	(Q_6, Q'_5)	(-2, -3)	(-2, 9)	(28, -9)	(-7, 15)	(-7, 33)	(23, 15)
10.	(P_3, P'_3)	(Q_3, Q'_5)	(2, -4)	(2, 8)	(32, -10)	(-8, 18)	(-8, 36)	(22, 18)

$(\overline{v_i}, \overline{v_i'})$ →Maximum of numbers in the row under the heading expectation of player1 and expectation of player2 respectively.

$(\underline{v_i}, \underline{v'_i})$→The negative of maximum of numbers in the row of the table under the heading expectation of player1 and expectation of player2 respectively.

Interval of game value is obtained using the inequality $([\frac{\underline{v_i}}{i}, \frac{\overline{v_i}}{i}], [\frac{\underline{v'_i}}{i}, \frac{\overline{v'_i}}{i}])$

Table-8: Value intervals

Play	$(\underline{v_i}, \underline{v'_i})$	$(\overline{v_i}, \overline{v'_i})$	$([\frac{\underline{v_i}}{i}, \frac{\overline{v_i}}{i}], [\frac{\underline{v'_i}}{i}, \frac{\overline{v'_i}}{i}])$
1.	(-1, -5)	(1, 1)	([-1, 1], [-5, 1])
2.	(0, -10)	(2, 2)	([0, 1], [-5, 1])
3.	(-4, -15)	(3, 3)	([-1.3333, 1], [-5, 1])
4.	(-8, -18)	(-1, 4)	([-2, -0.25], [-4.5, 1])
5.	(-12, -21)	(-5, 1)	([-2.4, -1], [-4.2, 0.2])
6.	(-16, -24)	(-9, -2)	([-2.6667, -1.5], [-4, -0.3333])
7.	(-20, -27)	(-13, -5)	([-2.8571, -1.8571], [-3.8571, -0.7143])
8.	(-24, -30)	(-17, -8)	([-3, -2.125], [-3.75, -1])
9.	(-28, -33)	(-16, -11)	([-3.1111, -1.7778], [-3.6667, -1.2222])
10.	(-32, -36)	(-20, -14)	([-3.2, -2], [-3.6, -1.4])

Table-9: Strategies of player 1

Play	Strategies for player1 $(X^{(i)} = [x_1^{(i)}, x_2^{(i)}, x_3^{(i)}] X'^{(i)} = [x'^{(i)}_1, x'^{(i)}_2, x'^{(i)}_3])$
1.	[1, 0, 0] [1, 0, 0]
2.	[1, 0, 0] [0.5, 0.5, 0]
3.	[1, 0, 0] [0.3333, 0.6667, 0]
4.	[1, 0, 0] [0.25, 0.5, 0.25]
5.	[1, 0, 0] [0.2, 0.4, 0.4]
6.	[1, 0, 0] [0.1667, 0.3333, 0.5]
7.	[1, 0, 0] [0.1428, 0.2857, 0.5714]
8.	[1, 0, 0] [0.125, 0.25, 0.625]
9.	[0.8889, 0, 0.1111] [0.1111, 0.2222, 0.6667]
10.	[0.8, 0, 0.2] [0.1, 0.2, 0.7]

Table-10: Strategies of player 2

Play	Strategies for player2 $Y^{(i)} = [y_1^{(i)}, y_2^{(i)}, y_3^{(i)}, y_4^{(i)}, y_5^{(i)}, y_6^{(i)}]$, $Y'^{(i)} = [y'^{(i)}_1, y'^{(i)}_2, y'^{(i)}_3, y'^{(i)}_4, y'^{(i)}_5, y'^{(i)}_6])$
1.	[1, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]
2.	[1, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0]
3.	[1, 0, 0, 0, 0, 0], [0.6667, 0.3333, 0, 0, 0, 0]
4.	[0.75, 0, 0.25, 0, 0, 0], [0.5, 0.5, 0, 0, 0, 0]
5.	[0.6, 0, 0.4, 0, 0, 0], [0.4, 0.4, 0, 0, 0.2, 0]
6.	[0.5, 0, 0.3333, 0, 0, 0.1667], [0.3333, 0.3333, 0, 0, 0.3333, 0]
7.	[0.4286, 0, 0.2857, 0, 0, 0.2857], [0.2857, 0.2857, 0, 0, 0.4286, 0]

8.	[0.375, 0, 0.25, 0, 0, 0.375], [0.25, 0.25, 0, 0, 0.5, 0]
9.	[0.3333, 0, 0.2222, 0, 0, 0.4444], [0.2222, 0.2222, 0, 0, 0.5556, 0]
10.	[0.3, 0, 0.3, 0, 0, 0.4], [0.2, 0.2, 0, 0, 0.6, 0]

4. CONCLUSION

This technique is new to solve 2-2 Nim game; best strategies are determined by tables (9) and (10). Also for solution of game we get the best inequality by this method as,

$$-2.125 \leq v \leq 0$$
$$-3.6 \leq v' \leq -1.4$$

REFERENCES

1. Jones A. J. (2000), Mathematical models of conflict. Game Theory, Ellis Harwood series.
2. Robinson J. (1951), an iterative method for solving a game. Annals of mathematics, Vol. 54, pp. 296-301,
3. Sergienko V. Lebedeva O. O. and Roshchin V. A. (1980). Approximate Methods to solve Discrete Optimization Problems [in Russian], Naukova Dumka, Kyiv,
4. Koller D. Megiddo N. and Vonstengel B. (1994). Fast algorithms for finding randomized strategies in game trees. In annual ACM symposium on theory of computing, STOC'94, pages 750-759
5. Zinkevich M. Johanson M. Bowling M. and Piccione C. (2007). Regret minimization in Games with incomplete information. Technical Report TR 07-14, Department of computing science, University of Alberta.
6. Robinson J. (1961) Iterative method for solving Games in matrix Games. Russian translation, Fizmatgiz, Moscow.
7. Denskin J. M. (1963) Iterative method for solving continuous games in infinite zero sum games. Russian translation, Fizmatgiz, Moscow.
8. Lemets O. O. and Yuyan N. (2007). Solving games problems on permutations. Naukovi Visti NTUU KPI, No. 3, 47-52.
9. OLkhovskaja E. V. and Yemets O. A. (2011) Iterative method for solving combinatorial optimization problems of the game type on arrangements. J. Automat. Information science, 43, Issue 5, 52-63.

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