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## ON2-2 NIM EXTENSIVE FORM GAME

S. C. SHARMA ${ }^{1}$, GANESH KUMAR*2<br>${ }^{1}$ Associate Professor, ${ }^{2}$ Assistant Professor, Department of Mathematics, Rajasthan University Jaipur - (R.J.), India.

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#### Abstract

In the present paper a particular extensive form game, 2-2 Nim game is reduced in normal form and bimatrix form. A new technique is used to deal with the problem to obtain the value of game for best strategies of the players.


Keywords: Nims game, optimal strategy, best inequality.

## INTRODUCTION

In the recent years extensive games are an intuitive formalism for modelling interaction between agents in a sequential decision making. The best strategies can be found using linear programming in [4].The iterative Brown and Robinson method is one of the common techniques to find solution of extensive form games. Brown conjectured and Robinson proved the convergence of this process for bimatrix games [6].

## 2-2. NIM EXTENSIVE FORM GAME

Four matches are set out in two piles of two matches each. Two players take alternate turns. At each turn player selects a pile that has at least one match and removes at least one match from this pile. The player may take several matches, but only from one pile. When both piles have no matches there are no more turns and game is ended. The player who removed last match loses

## GAME TREE

The letter in each box is player to move. Numbers in large type are the sizes of the piles, and each state is numbered in small type for later reference.


Diagram: 1

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Pure strategies are used for, $2-2 \mathrm{Nim}$; in this table the expression $\mathrm{x} \square \mathrm{y}$ means "If I find myself at box x then move to $y$ ". If $A$ chooses $A_{3}$ then the game can never reach state 4 so there is no entry $4 \square \mathrm{x}$. The normal form of a game can only be constructed for a 2 person game. It is a matrix in which ( $\mathrm{i}, \mathrm{j}$ ) th cell shows the outcome if the first player chooses $\mathrm{i}^{\text {th }}$ pure strategy and the $2^{\text {nd }}$ player chooses $\mathrm{j}^{\text {th }}$ strategy. Usually the entry in each cell shows the final state and some indication of the payoffs to the players. In the normal form the rows of the matrix correspond to the $1^{\text {st }}$ player's strategies, and the columns correspond to the $2^{\text {nd }}$ player's strategies.

Table-1: Pure strategies

| Pure Strategies for A |  |  |  |
| :--- | :--- | :--- | :--- |
| Name | Moves | Pure Strategies for B |  |
| Name | Moves |  |  |
| A1 | $1 \square 2,4 \square 9$ | B1 | $2 \square 4,3 \square 7$ |
|  |  | B2 | $2 \square 5,3 \square 7$ |
| A2 | $1 \square 2,4 \square 10$ | B3 | $2 \square 6,3 \square 7$ |
|  |  | B4 | $2 \square 4,3 \square 8$ |
| A3 | $1 \square 3$ | B5 | $2 \square 5,3 \square 8$ |
|  |  | B6 | $2 \square 6,3 \square 8$ |

Table-2: Normal form

|  | B 1 | B 2 | B 3 | B 4 | B 5 | B 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A 1 | $\mathrm{~A}(14)$ | $\mathrm{A}(15)$ | $\mathrm{B}(12)$ | $\mathrm{A}(14)$ | $\mathrm{A}(15)$ | $\mathrm{B}(12)$ |
| A 2 | $\mathrm{~B}(10)$ | $\mathrm{A}(15)$ | $\mathrm{B}(13)$ | $\mathrm{B}(12)$ | $\mathrm{A}(15)$ | $\mathrm{B}(12)$ |
| A 3 | $\mathrm{~B}(13)$ | $\mathrm{B}(13)$ | $\mathrm{B}(13)$ | $\mathrm{A}(8)$ | $\mathrm{A}(8)$ | $\mathrm{A}(8)$ |

Table-3: Matrix form

|  | B1 | B2 | B3 | B4 | B5 | B6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A1 | 1 | 1 | -1 | 1 | 1 | -1 |
| A2 | -1 | 1 | -1 | -1 | 1 | -1 |
| A3 | -1 | -1 | -1 | 1 | 1 | 1 |

Table-4: Pay off

| State | 14 | 10 | 15 | 12 | 13 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Payoff to A | 1 | -4 | 1 | -4 | -4 | 1 |
| Payoff to B | -5 | 1 | -5 | 1 | 1 | -3 |

Table-5: Bimatrix form

|  | $\left(\mathrm{Q}_{1}, \mathrm{Q}_{1}^{\prime}\right)$ | $\left(\mathrm{Q}_{2}, \mathrm{Q}^{\prime}{ }_{2}\right)$ | $\left(\mathrm{Q}_{3}, \mathrm{Q}^{\prime}{ }_{3}\right)$ | $\left(\mathrm{Q}_{4}, \mathrm{Q}_{4}^{\prime}\right)$ | $\left(\mathrm{Q}_{5}, \mathrm{Q}^{\prime}{ }_{5}\right)$ | $\left(\mathrm{Q}_{6}, \mathrm{Q}^{\prime}{ }_{6}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{1}\right)$ | $(1,-5)$ | $(1,-5)$ | $(-4,1)$ | $(1,-5)$ | $(1,-5)$ | $(-4,1)$ |
| $\left(\mathrm{P}_{2}, \mathrm{P}^{\prime}{ }_{2}\right)$ | $(-4,1)$ | $(1,-5)$ | $(-4,1)$ | $(-4,1)$ | $(1,-5)$ | $(-4,1)$ |
| $\left(\mathrm{P}_{3}, \mathrm{P}_{3}{ }_{3}\right)$ | $(-4,1)$ | $(-4,1)$ | $(-4,1)$ | $(1,-3)$ | $(1,-3)$ | $(1,-3)$ |

## NEW TECHNIQUE TO FIND VALUE OF THE GAME

In the approximation method two players play a long sequence of plays in given game. Each of them plays in such a way as to maximize his expectation under the assumption that the future will resemble the past. At each point of the sequence one can calculate upper and lower bounds for the value of the game as well as an approximation to an optimal strategy for each player.

Table-6: Expectations of player 1

| Play | Player1 chooses | Player2 chooses | Expectation of player 1 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{1}\right)$ | $\left(\mathrm{P}_{2}, \mathrm{P}^{\prime}{ }_{2}\right)$ | $\left(\mathrm{P}_{3}, \mathrm{P}^{\prime}{ }_{3}\right)$ |
| 1. | $\left(\mathrm{P}_{1}, \mathrm{P}_{1}{ }_{1}\right)$ | $\left(\mathrm{Q}_{1}, \mathrm{Q}^{\prime}{ }_{1}\right)$ | $(1,-5)$ | $(-4,1)$ | $(-4,1)$ |
| 2. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{2}\right)$ | $\left(\mathrm{Q}_{1}, \mathrm{Q}^{\prime}{ }_{1}\right)$ | $(2,-10)$ | $(-8,2)$ | $(-8,2)$ |
| 3. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{2}\right)$ | $\left(\mathrm{Q}_{1}, \mathrm{Q}^{\prime}{ }_{2}\right)$ | $(3,-15)$ | $(-12,-3)$ | $(-12,3)$ |
| 4. | $\left(\mathrm{P}_{1}, \mathrm{P}_{3}\right)$ | $\left(\mathrm{Q}_{3}, \mathrm{Q}^{\prime}{ }_{2}\right)$ | $(-1,-20)$ | $(-16,-8)$ | $(-16,4)$ |
| 5. | $\left(\mathrm{P}_{1}, \mathrm{P}_{3}{ }_{3}\right)$ | $\left(\mathrm{Q}_{3}, \mathrm{Q}^{\prime}{ }_{5}\right)$ | $(-5,-25)$ | $(-20,-13)$ | $(-20,1)$ |
| 6. | $\left(\mathrm{P}_{1}, \mathrm{P}_{3}{ }_{3}\right)$ | $\left(\mathrm{Q}_{6}, \mathrm{Q}_{5}\right)$ | $(-9,-30)$ | $(-25,-18)$ | $(-19,-2)$ |
| 7. | $\left(\mathrm{P}_{1}, \mathrm{P}_{3}{ }_{3}\right)$ | $\left(\mathrm{Q}_{6}, \mathrm{Q}_{5}{ }_{5}\right)$ | $(-13,-35)$ | $(-29,-23)$ | $(-18,-5)$ |
| 8. | $\left(\mathrm{P}_{1}, \mathrm{P}_{3}{ }_{3}\right)$ | $\left(\mathrm{Q}_{6}, \mathrm{Q}^{\prime}{ }_{5}\right)$ | $(-17,-40)$ | $(-33,-28)$ | $(-17,-8)$ |
| 10. | $\left(\mathrm{P}_{3}, \mathrm{P}_{3}\right)$ | $\left(\mathrm{Q}_{3}, \mathrm{Q}^{\prime}{ }_{5}\right)$ | $(-25,-50)$ | $(-41,-38)$ | $(-20,-14)$ |

Table-7: Expectation of player 2

| Play | Player1 Chooses | Player2 Chooses | Expectation of player2 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | ( $\mathrm{Q}_{1}, \mathrm{Q}^{\prime}{ }_{1}$ ) | ( $\mathrm{Q}_{2}, \mathrm{Q}^{\prime}{ }_{2}$ ) | ( $\mathrm{Q}_{3}, \mathrm{Q}^{\prime}{ }_{3}$ ) | ( $\mathrm{Q}_{4}, \mathrm{Q}^{\prime}{ }_{4}$ ) | ( $\mathrm{Q}_{5}, \mathrm{Q}^{\prime}{ }_{5}$ ) | ( $\mathrm{Q}_{6}, \mathrm{Q}^{\prime}{ }_{6}$ ) |
| 1. | ( $\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{1}$ ) | $\left(\mathrm{Q}_{1}, \mathrm{Q}^{\prime}{ }_{1}\right)$ | $(1,5)$ | $(1,5)$ | (-4, -1) | $(1,5)$ | $(1,5)$ | (-4, -1) |
| 2. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{2}\right)$ | $\left(\mathrm{Q}_{1}, \mathrm{Q}^{\prime}{ }_{1}\right)$ | $(0,4)$ | $(0,10)$ | $(0,-2)$ | $(0,4)$ | $(0,10)$ | (0, -2) |
| 3. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{2}\right)$ | $\left(Q_{1}, Q^{\prime}{ }_{2}\right)$ | $(-1,3)$ | $(-1,15)$ | $(4,-3)$ | $(-1,3)$ | $(-1,15)$ | $(4,-3)$ |
| 4. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{3}\right)$ | $\left(Q_{3}, Q^{\prime}{ }_{2}\right)$ | $(-2,2)$ | $(-2,14)$ | $(8,-4)$ | $(-2,0)$ | $(-2,18)$ | $(8,0)$ |
| 5. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{3}\right)$ | ( $\mathrm{Q}_{3}, \mathrm{Q}^{\prime}{ }_{5}$ ) | $(-3,1)$ | $(-3,13)$ | $(12,-5)$ | $(-3,3)$ | $(-3,21)$ | $(12,3)$ |
| 6. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{3}\right)$ | $\left(Q_{6}, Q^{\prime}{ }_{5}\right)$ | $(-4,0)$ | $(-4,12)$ | $(16,-6)$ | $(-4,6)$ | $(-4,24)$ | $(16,6)$ |
| 7. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{3}\right)$ | $\left(Q_{6}, Q^{\prime}{ }_{5}\right)$ | $(-5,-1)$ | $(-5,11)$ | $(20,-7)$ | $(-5,9)$ | $(-5,27)$ | $(20,9)$ |
| 8. | $\left(\mathrm{P}_{1}, \mathrm{P}^{\prime}{ }_{3}\right)$ | $\left(\mathrm{Q}_{6}, \mathrm{Q}^{\prime}{ }_{5}\right)$ | $(-6,-2)$ | $(-6,10)$ | $(24,-8)$ | $(-6,-12)$ | $(-6,30)$ | $(24,12)$ |
| 9. | $\left(\mathrm{P}_{3}, \mathrm{P}^{\prime}{ }_{3}\right)$ | $\left(Q_{6}, Q^{\prime}{ }_{5}\right)$ | $(-2,-3)$ | $(-2,9)$ | $(28,-9)$ | $(-7,15)$ | $(-7,33)$ | $(23,15)$ |
| 10. | $\left(\mathrm{P}_{3}, \mathrm{P}^{\prime}{ }_{3}\right)$ | ( $\left.\mathrm{Q}_{3}, \mathrm{Q}^{\prime}{ }_{5}\right)$ | (2, -4) | $(2,8)$ | (32, -10) | $(-8,18)$ | $(-8,36)$ | $(22,18)$ |

$\left(\overline{v_{i}}, \overline{v_{i}}\right) \rightarrow$ Maximum of numbers in the row under the heading expectation of player1 and expectation of player2 respectively.
$\left(\underline{v_{i}}, \underline{v_{i}^{\prime}}\right) \rightarrow$ The negative of maximum of numbers in the row of the table under the heading expectation of playerl and expectation of player2 respectively.

Interval of game value is obtained using the inequality $\left(\left[\begin{array}{l}v_{i} \\ \overline{\underline{v_{i}}} \\ \frac{\bar{i}}{i}\end{array}\right],\left[\frac{v_{i}^{\prime}}{i}, \frac{\overline{v_{i}^{i}}}{i}\right]\right)$
Table-8: Value intervals

| Play | $\left(\underline{v_{i}}, \underline{v_{i}^{\prime}}\right)$ | $\left(\overline{v_{i}} \overline{v_{i}^{\prime}}\right)$ | $\left(\left[\underline{i} \overline{\overline{v_{i}}}, \frac{\overline{v_{i}}}{i}\right],\left[\stackrel{v_{i}^{\prime}}{i}, \frac{\overline{v_{i}^{\prime}}}{i}\right]\right)$ |
| :---: | :---: | :---: | :---: |
| 1. | $(-1,-5)$ | $(1,1)$ | $([-1,1],[-5,1])$ |
| 2. | $(0,-10)$ | $(2,2)$ | $([0,1],[-5,1])$ |
| 3. | $(-4,-15)$ | $(3,3)$ | $([-1.3333,1],[-5,1])$ |
| 4. | $(-8,-18)$ | $(-1,4)$ | $([-2,-0.25],[-4.5,1])$ |
| 5. | $(-12,-21)$ | $(-5,1)$ | $([-2.4,-1],[-4.2,0.2])$ |
| 6. | $(-16,-24)$ | $(-9,-2)$ | $([-2.6667,-1.5],[-4,-0.3333])$ |
| 7. | $(-20,-27)$ | $(-13,-5)$ | $([-2.8571,-1.8571],[-3.8571,-0.7143])$ |
| 8. | $(-24,-30)$ | $(-17,-8)$ | $([-3,-2.125],[-3.75,-1])$ |
| 9. | $(-28,-33)$ | $(-16,-11)$ | $([-3.1111,-1.7778],[-3.6667,-1.2222])$ |
| 10. | $(-32,-36)$ | $(-20,-14)$ | $([-3.2,-2],[-3.6,-1.4])$ |

Table-9: Strategies of player 1

| Play | Strategies for player1 <br> $\left(X^{(i)}=\left[x_{1}^{(i)}, x_{2}^{(i)}, x_{3}^{(i)}\right] X^{\prime(i)}=\left[x_{1}^{\prime(i)}, x_{2}^{\prime(i)}, x_{3}^{\prime(i)}\right]\right)$ |  |
| :--- | :--- | :---: |
| 1. | $[1,0,0]$ | $[1,0,0]$ |
| 2. | $[1,0,0]$ | $[0.5,0.5,0]$ |
| 3. | $[1,0,0][0.3333,0.6667,0]$ | $[0.25,0.5,0.25]$ |
| 4. | $[1,0,0]$ | $[0.2,0.4,0.4]$ |
| 5. | $[1,0,0]$ | $[0.1667,0.3333,0.5]$ |
| 6. | $[1,0,0]$ | $[0.1428,0.2857,0.5714]$ |
| 7. | $[1,0,0]$ | $[0.125,0.25,0.625]$ |
| 8. | $[1,0,0]$ | $[0.1111,0.2222,0.6667]$ |
| 9. | $[0.8889,0,0.1111]$ | $[0.1,0.2,0.7]$ |
| 10. | $[0.8,0,0.2]$ |  |

Table-10: Strategies of player 2
\(\left.\begin{array}{|l|l|}\hline Play \& \left.\begin{array}{l}Strategies for player2 <br>

Y^{(i)}=\left[y_{1}^{(i)}, y_{2}^{(i)}, y_{3}^{(i)}, y_{4}^{(i)}, y_{5}^{(i)}, y_{6}^{(i)}\right],\end{array} \quad Y^{\prime(i)}=\left[y_{1}^{\prime(i)}, y_{2}^{(i)}, y_{3}^{(i)}, y_{4}^{\prime(i)}, y_{5}^{\prime(i)}, y_{6}^{\prime(i)}\right]\right)\end{array}\right]\)| 1. | $[1,0,0,0,0,0],[1,0,0,0,0,0]$ |
| :--- | :--- |
| 2. | $[1,0,0,0,0,0],[1,0,0,0,0,0]$ |
| 3. | $[1,0,0,0,0,0],[0.6667,0.3333,0,0,0,0]$ |
| 4. | $[0.75,0,0.25,0,0,0],[0.5,0.5,0,0,0,0]$ |
| 5. | $[0.6,0,0.4,0,0,0],[0.4,0.4,0,0,0.2,0]$ |
| 6. | $[0.5,0,0.3333,0,0,0.1667],[0.3333,0.3333,0,0,0.3333,0]$ |
| 7. | $[0.4286,0,0.2857,0,0,0.2857],[0.2857,0.2857,0,0,0.4286,0]$ |

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| 8. | $[0.375,0,0.25,0,0,0.375],[0.25,0.25,0,0,0.5,0]$ |
| :--- | :--- |
| 9. | $[0.3333,0,0.2222,0,0,0.4444],[0.2222,0.2222,0,0,0.5556,0]$ |
| 10. | $[0.3,0,0.3,0,0,0.4],[0.2,0.2,0,0,0.6,0]$ |

## 4. CONCLUSION

This technique is new to solve 2-2 Nim game; best strategies are determined by tables (9) and (10). Also for solution of game we get the best inequality by this method as,

$$
\begin{aligned}
& -2.125 \leq \mathrm{v} \leq 0 \\
& -3.6 \leq \mathrm{v}^{\prime} \leq-1.4
\end{aligned}
$$

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