International Journal of Mathematical Archive-6(9), 2015, 182-186
Available online through www.ijma.info ISSN 2229 – 5046

CONNECTED ROMAN BLOCK DOMINATION IN GRAPHS

[1]M. H. MUDDEBIHAL, [2]VEDULA PADMAVATHI*

[1]Department of Mathematics,
Gulbarga University, Gulbarga – 585106, Karnataka, India.

[2]Department of Mathematics,
Gulbarga University, Gulbarga – 585106, Karnataka, India.

(Received On: 08-09-15; Revised & Accepted On: 30-09-15)

ABSTRACT

A Roman dominating function (RDF) on a block graph B(G) = (H, X) is defined as a function f: H→{0, 1, 2} satisfying the condition that every vertex u for which f(u) = 0 is adjacent to at least one vertex v for which f(v) = 2. The weight of a Roman dominating function of B(G) is defined as $f(H) = \sum_{v\in H} f(v)$. The Roman domination number of a block graph B(G) is denoted by $\gamma_{RB}(G)$, equals the minimum weight of a RDF of B(G). A Roman dominating function of B(G) is connected Roman dominating function of B(G) if either $\langle V_1 \cup V_2\rangle$ or $\langle V_2\rangle$ is connected. The connected Roman block domination number $\gamma_{cRB}(G)$ is the minimum weight of a connected Roman block dominating function of B(G). In this paper we establish some results on $\gamma_{cRB}(G)$ in terms of elements of G. Further we develop its relationship with other different dominating parameters.

Key words: *Block graph, Roman domination, Roman block domination, Connected Roman Block domination.*

***Subject classification numbers**: AMS – 05C69, 05C70.*

INTRODUCTION

Let $G = (V, E)$ be a simple, undirected (p, q) graph with $p = |V|$, $q = |E|$. Any undefined terms or notations can be found in [2]. We denote open neighbourhood of a vertex v of G by $N(v)$. The degree of a vertex v denotes the number of neighbours of v in G and $\Delta(G)$ is the maximum degree, $\delta(G)$ is the minimum degree of G. For any connected graph G, a vertex $v \in V(G)$ is called a cut vertex of G if $G - v$ is no longer connected. A maximal induced subgraph without cutvertex is called a block of G. For any (p, q) graph G, a block graph is the graph whose vertices correspond to the blocks of G and two vertices in $B(G)$ are adjacent whenever the corresponding blocks contain a common cutvertex in G.

A set D of a graph G is a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set. A dominating set D is called connected dominating set if induced subgraph $\langle D\rangle$ is connected. The connected domination number $\gamma_c(G)$ is the minimum cardinality of a connected dominating set of G. A dominating set D of a graph G is a strong split dominating set if the induced subgraph $\langle V - D\rangle$ is totally disconnected with at least two vertices. The strong split domination number $\gamma_{ss}(G)$ of G is the minimum cardinality of a strong split dominating set of G. From [1], a Roman dominating function (RDF) on a graph $G = (V,E)$ is a function $f : V \to \{0,1,2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight of a Roman dominating function is the value $f(V) = \sum_{v\in V} f(v)$. The minimum weight of a Roman dominating function on a graph G is called the Roman domintion number and is denoted by $\gamma_R(G)$.

A Roman dominating function $f = (V_0^{\mathrm{I}}, V_1^{\mathrm{I}}, V_2^{\mathrm{I}})$ on a graph G is a connected Roman dominating function (CRDF) on G if $\langle V_1^{\mathrm{I}} \cup V_2^{\mathrm{I}}\rangle$ or $\langle V_2^{\mathrm{I}}\rangle$ is connected. The minimum weight of a CRDF is called a connected Roman domination number of G and is denotd by $\gamma_{RC}(G)$. This concept is introduced by M. H. Muddebihal and Sumangaladevi [5].

Corresponding Author: [2]Vedula Padmavathi, [2]Department of Mathematics, Gulbarga University, Gulbarga – 585106, Karnataka, India.*

A Roman dominating function on a block graph $B(G) = (H, X)$ is a function $f: H \rightarrow \{0,1,2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight of a Roman dominating function on $B(G)$ is the value $f(H) = \sum_{v \in H} f(v)$. The minimum weight of a Roman dominating function on a block graph $B(G)$ is called the Roman block domintion number and is denoted by $\gamma_{RB}(G)$, see [6] .

A Roman block dominating function $f = (V_0, V_1, V_2)$ of a graph G is a connected Roman block dominating function if either $\langle V_1 \cup V_2 \rangle$ or $\langle V_2 \rangle$ is connected in $B(G)$. The minimum weight of a connected Roman block dominating function is denoted by $\gamma_{cRB}(G)$ and is called connected Roman block domination number of G.

PREREQUISITES

Theorem A[3] : If any tree T has p vertices, then its block graph $B(T)$ has $(p - 1)$ vertices.

Theorem B [4]: For any connected graph G on n vertices, $\gamma_R(G) \leq n - \left\lceil \frac{1 + diam\ G}{3} \right\rceil$.

Theorem C [7]: For any connected graph G, $\gamma(G) \leq \gamma_c(G)$.

We used above Theorems in our further results.

RESULTS

Theorem 1:
(i) For any non - separable graph G, $\gamma_{cRB}(G) = 1$.
(ii) For any path P_p, $\gamma_{cRB}(P_p) = p - 2 \ if \ p = 4$
$= p - 1 \ if \ p \neq 4$.
(iii) For any graph G with exactly one cut vertex, $\gamma_{cRB}(G) = 2$.
(iv) For any graph G whose $B(G)$ is a star, $\gamma_{cRB}(G) = 2$.

In the following Theorems we establish relations between$\gamma_{cRB}(G)$, $\gamma_{RC}(G)$ and $\gamma_{RB}(G)$.

Theorem 2: For any connected graph G, $\gamma_{cRB}(G) \leq \gamma_{RC}(G)$.

Proof: Let G be any connected graph with a CRDF $f = (V_0^{\mathrm{I}}, V_1^{\mathrm{I}}, V_2^{\mathrm{I}})$ and let $f = (V_0, V_1, V_2)$ be the CRDF in $B(G)$. We consider the following cases.

Case-1: Suppose G be a non – trivial tree T. Let $V_{en} = \{v_1, v_2, v_3, \dots, v_{en}\}$ be the set of all end vertices, $V_c = \{v_1, v_2, v_3, \dots, v_c\}$ be the set of all cutvertices in T such that $V(T) = V_c \cup V_{en}$ and $V_c^{\mathrm{I}} \subseteq V_c$ be the set of all cutvertices adjacent to endvertices in T. Then $\forall\ v_i \in V_c^{\mathrm{I}}$, $w(v_i) = 2$ and $\forall\ v_j \in V_c \setminus V_c^{\mathrm{I}}$, $w(v_j) = 1$ such that $w\left(N(v_i) \cap N(v_j)\right) = 1 \ or \ 2$. Then $\langle v_i v_j \rangle$ is connected. Hence V_c forms γ_{RC}–set in T and $|V_c| = |V_1^{\mathrm{I}}| + |V_2^{\mathrm{I}}| = \gamma_{RC}(T)$.

Let $A = \{B_1, B_2, \dots, B_n\}$ be the set of all blocks of G, $M^{\mathrm{I}} = \{B_1, B_2, \dots B_k\}, k < n$ be the set of all non - end blocks and $E^{\mathrm{I}} = \{B_1, B_2, \dots B_m\}, m < n$ be set of all endblocks of G.

Let $H = \{b_1, b_2, \dots, b_n\}$ be the corresponding block vertex set of A, $M = \{b_1, b_2, \dots b_k\}, k < n$ be the set of all cutvertices and $M_{nc} = \{b_1, b_2, \dots b_m\}, m < n$ be set of all non - cutvertices corresponding to $M^{\mathrm{I}}, E^{\mathrm{I}}$ respectively in $B(G)$. Since $G \cong T$, $\forall\ b_i \in M$ in $B(T)$ are connected and $w(b_i) = 1 \ or \ 2$ and $\forall\ b_j \in M_{nc}$ in $B(T)$,$w(b_j) = 0$. Hence M forms γ_{cRB} – set in $B(T)$. By Theorem A, T has one vertex more than that of $B(T)$. Hence $|V_1| + |V_2| \leq |V_1^{\mathrm{I}}| + |V_2^{\mathrm{I}}|$ give $\gamma_{cRB}(T) \leq \gamma_{RC}(T)$.

Case-2: Suppose G is not a tree. Then assume G is a tree and any two non – adjacent vertices u, v with $d(u, v) = 2$ are joined by an edge. Then the graph G contains a smallest block B_i with $p = 3$ vertices. If $B_i \in E^{\mathrm{I}}$ in G, then the two non – adjacent vertices u, v of $B_i \in V_0^{\mathrm{I}}$ and the corresponding block vertex b_i of B_i belongs to V_0 in $B(G)$. If $B_i \in M^{\mathrm{I}}$ in G, then $u, v \in V_c$ where V_c is γ_{RC} – set and the corresponding block vertex $b_i \in \gamma_{cRB}$ – set in $B(G)$. Hence one can easily verify that $\gamma_{cRB}(G) \leq \gamma_{RC}(G)$.

Theorem 3: For any connected graph G, $\gamma_{RB}(G) \leq \gamma_{cRB}(G)$.

Proof: Let $H = \{b_1, b_2, \dots, b_n\}$ be the set of all vertices of $B(G)$. Let $H_1 = \{b_1, b_2, \dots b_i\}, 1 \leq i < n$ such that $H_1 \subset H$ and Then $H_2 = H \setminus H_1$ and $\forall\ b_j \in H_2$, $w(b_j) = 0 \ or \ 1$. Suppose each b_i has weight 2. Then $\langle H_1 \rangle = \langle V_2 \rangle$ and is connected which gives $|V_2| = \gamma_{cRB}(G)$. Otherwise $\exists\ b_j \in H_2,\ w(b_j) = 1$ and $N(b_i) = b_i \in H_1$. Then $\langle V_1 \cup V_2 \rangle$ is connected. Hence $|V_1 \cup V_2| = \gamma_{cRB}(G)$. Suppose $H_1 - \{b_k\}$ or $H_1 - \{b_k \cup \{b_n\}\}$, $1 < k \leq i, 1 \leq n \leq j$ such that $|H_1 - \{b_k\}|$ *or* $|H_1 - \{\{b_k\} \cup \{b_n\}\}|$ gives γ_{RB}- set. Hence $|H_1 - \{b_k\}|$ *or* $|H_1 - \{\{b_k\} \cup \{b_n\}\}| \leq |V_1 \cup V_2|$ which gives $\gamma_{RB}(G) \leq \gamma_{cRB}(G)$.

In the following Theorem we establish the relation between $\gamma(T)$ and $\gamma_{cRB}(T)$.

Theorem 4: For any connected tree T, $\gamma(T) \leq \gamma_{cRB}(T)$.

Proof: Let $V = \{v_1, v_2, \ldots\ldots, v_p\}$ be the set of all vertices of T and suppose $D = \{v_1, v_2, \ldots\ldots, v_l\}, l < p$ be the minimal dominating set of T such that $|D| = \gamma(T)$. Let $A = \{B_1, B_2, \ldots\ldots, B_{p-1}\}$ be the set of all blocks of T and $H = \{b_1, b_2, \ldots\ldots, b_{p-1}\}$ be the corresponding block vertices in $B(T)$. $\forall\ B_i$ adjacent to endblocks containing $v_i \in D$ in T, there exist a corresponding block vertex set $\{b_i\}$ in $B(T)$ such that $\{b_i\} \in V_2 \cup V_1$ and $\forall\ B_j$ not adjacent to endblocks in T there exist a corresponding block vertex set $\{b_j\}$ in $B(T)$ such that $\{b_j\} \in V_1$. Hence $\langle b_i \cup b_j \rangle$ is connected and it forms γ_{cRB} – set such that $|V_1| + |V_2| = |D_{cRB}| = \gamma_{cRB}(T)$. Clearly $|D| \leq |D_{cRB}|$ gives $\gamma(T) \leq \gamma_{cRB}(T)$.

The following Theorem relates $\gamma(G)$, $\gamma_{cRB}(G)$ *and* $\gamma_c(G)$.

Theorem 5: For any connected (p, q) graph G, $\gamma_{cRB}(G) \leq \gamma(G) + \gamma_C(G)$.

Proof: Let $D = \{v_1, v_2, \ldots\ldots, v_l\}, l < p$ be the set of vertices of G such that every $v_j \in V - D$ is adjacent to at least one vertex of D. Then D is a γ – set. Suppose for some $v_i \in D$ and $\forall\ v_k \in D - \{v_i\}$ is not adjacent to at least one $v_j^{\mathrm{I}} \in V - D \cup \{v_k\}$. Then D is a minimal dominating set.

For $v_i \in D, v_j \in D$ if $N(v_i) \cap N(v_j) \neq \emptyset$, then $\langle D \rangle$ is connected otherwise $\exists$ at least one vertex $x \in V - D$ such that $N(v_i) \cap N(v_j) = x$. Then $D \cup \{x\}$ forms a minimal connected dominating set and $D \cup \{x\} = D_c$. Let $A = \{B_1, B_2, \ldots\ldots, B_n\}$ be the set of all blocks of G, $M^{\mathrm{I}} = \{B_1, B_2, \ldots\ldots B_k\}, k < n$ be the set of all non - end blocks and $E^{\mathrm{I}} = \{B_1, B_2, \ldots\ldots B_m\}, m < n$ be set of all endblocks of G. Let $H = \{b_1, b_2, \ldots\ldots, b_n\}$ be the corresponding block vertex set of A, $M = \{b_1, b_2, \ldots\ldots b_k\}, k < n$ be the set of all cutvertices and $M_{nc} = \{b_1, b_2, \ldots\ldots b_m\}, m < n$ be set of all non - cutvertices corresponding to $M^{\mathrm{I}}, E^{\mathrm{I}}$ respectively in $B(G)$. Let $H_1 = \{b_1, b_2, \ldots\ldots b_i\}, i \leq k$ such that $H_1 \subseteq M$ and $\forall\ b_i \in H_1, w(b_i) = 2$. Then $H_2 = H \setminus H_1$ and $\forall\ b_j \in H_2, w(b_j) = 0$ *or* 1.

Suppose $H_1 = M$ and each b_i has weight 2. Then $\langle H_1 \rangle = \langle V_2 \rangle$ and is connected gives $|V_2| = \gamma_{cRB}(G)$ otherwise $\exists\ b_j \in H_2$ with $w(b_j) = 1$and $N(b_i) = b_j \in M$. Then $\langle V_2 \cup V_1 \rangle$ is connected. Hence $|V_1 \cup V_2| = \gamma_{cRB}(G)$.

Let $B_i \in A$ is a smallest block with $p = 3$ vertices in G. We consider the following cases.

Case-1: Suppose $B_i \in E^{\mathrm{I}} \subseteq A$. We consider following subcases.

Subcase 1.1: Assume $B_i \in E^{\mathrm{I}} = A$. Then at least one v_i of $B_i \in D$ and D_c. The corresponding block graph $B(G)$ is a complete graph and corresponding block vertex $b_i \in \{V_2\}$. Then $|V_2| \leq |D| + |D_c|$ gives the result.

Subcase 1.2: Assume $B_i \in E^{\mathrm{I}} \subset A$. Then at least one v_i of $B_i \in D$ and D_c. The corresponding block vertex $b_i \in \{V_0\}$ in $B(G)$. Then $\forall\ b_j \in \{V_2 \cup V_1\}$ in $B(G)$ form γ_{cRB}- set and $|V_2 \cup V_1| \leq |D| + |D_c|$ gives result.

Case-2: Suppose $B_i \in M^{\mathrm{I}} \subset A$. Then at least two vertices of B_i belong to D_c and at leas one vertex of B_i belongs to D. The corresponding block vertex $b_i \in M$ in $B(G)$ with $w(b_i) = 1$ or 2. Hence $b_i \in \gamma_{cRB}$– set and $|V_2 \cup V_1| \leq |D| + |D_c|$. Hence $\gamma_{cRB}(G) \leq \gamma(G) + \gamma_C(G)$.

In the following Theorems we provide upper bounds for $\gamma_{cRB}(G)$ in terms of number of blocks n and number of vertices p of G.

Theorem 6: For any connected graph G, $\gamma_{cRB}(G) \leq n$ where n is number of blocks of G.

Proof: We consider the following cases.

Case-1: Suppose G is non – separable. Then by Theorem 1, $\gamma_{cRB}(G) = 1 = n$.

Case-2: Suppose G is separable. We consider the following subcases of case 2.

Subcase 2.1: Assume G has exactly one cutvertex. Then by Theorem 1, $\gamma_{cRB}(G) = 2 \leq n$.

Subcase 2.2: Assume G has more than one cutvertex. Let $A = \{B_1, B_2, \ldots \ldots, B_n\}$ be the set of all blocks of G, $M^{I} = \{B_1, B_2, \ldots \ldots B_k\}, k < n$ be the set of all non - endblocks of G. Let $H = \{b_1, b_2, \ldots \ldots, b_n\}$ be the corresponding block vertex set of A such that $V(B(G)) = H$ and $M = \{b_1, b_2, \ldots \ldots b_k\}, k < n$ be the set of all cut vertices corresponding to M^{I} respectively in B(G). $\forall\, b_i \in M, w(b_i) = 1$ *or* 2 such that $|M| = |V_1| + |V_2|$. Then M forms γ_{cRB}- set and $|M| = |D_{cRB}| = \gamma_{cRB}(G)$. Since $B(G)$ contains at most $n - 2$ cutvertices, $\gamma_{cRB}(G) \leq n$. Hence the result.

Theorem7: For any (p, q) graph G, $\gamma_{cRB}(G) \leq p - 1$.

Proof: By Theorem 6, $\gamma_{cRB}(G) \leq n$ where n is number of blocks of G and since any graph contains at most $n = p - 1$ blocks, the result follows.

Following Theorem relates $\gamma_{cRB}(G)$ and $\gamma_c(G)$.

Theorem 8: For any connected graph G, $\gamma_{cRB}(G) \leq 2\,\gamma_C(G)$. Equality holds for a graph G with exactly one cutvertex adjacent to all the vertices of all the blocks.

Proof: From Theorem C and Theorem 5, $\gamma_{cRB}(G) \leq \gamma(G) + \gamma_C(G)$

$$\leq \gamma_C(G) + \gamma_C(G) = 2\,\gamma_C(G)$$

Further Theorem gives relation between $\gamma_{cRB}(G)$ and $\gamma_R(G)$.

Theorem 9: For any (p, q) graph G, $\gamma_{cRB}(G) \leq 2\,\gamma_R(G) - 3$.

Proof: Let $V = \{v_1, v_2, \ldots \ldots, v_p\}$ be the set of all vertices of G. Let $f = (V_0^{I}, V_1^{I}, V_2^{I})$ be a γ_R – function in G and suppose $D_R = \{v_1, v_2, \ldots \ldots \ldots, v_s\}, s < p$ be a minimal γ_R – set of G. Let $A = \{B_1, B_2, \ldots \ldots, B_n\}$ be the set of all blocks of G, $M^{I} = \{B_1, B_2, \ldots \ldots B_k\}, k < n$ be the set of all non-endblocks of G. Let $H = \{b_1, b_2, \ldots \ldots, b_n\}$ and $M = \{b_1, b_2, \ldots \ldots b_k\}, k < n$ be the corresponding block vertex sets of A and M^{I} respectively in $B(G)$. Consider $\{v_i\} \subseteq V - D_R$ such that $w(v_i) = 0$ and v_i is incident with any $B_i \in M^{I}$ in G. Then $\forall\, B_i$ containing v_i there exist a corresponding block vertex set $\{b_i\} \in V_2 \cup V_1$ and $\{b_i\} \subseteq M$ such that M forms γ_{cRB} – set in $B(G)$. Since in G, for any block B_i, $\sum_{v_i \in B_i} w(v_i)$ is at most 2 and the weight of corresponding block vertex in $B(G)$ is $w(b_i) = 1$ *or* 2, it is clear that $\frac{\gamma_{cRB}(G)}{2} + \frac{3}{2} \leq \gamma_R(G)$.

Hence $\gamma_{cRB}(G) \leq 2\,\gamma_R(G) - 3$.

In next Theorem we establish an upper bound for $\gamma_{cRB}(G)$ in terms of diameter of G.

Theorem10: For any connected graph G, $\gamma_{cRB}(G) \leq 2p - \frac{2}{3}[1 + diam\ G]$.

Proof: From Theorem B and Theorem 9,

$$\begin{aligned}\gamma_{cRB}(G) &\leq 2\,\gamma_R(G) - 3 \\ &\leq 2\,\gamma_R(G) \\ &\leq 2\left[p - \left[\frac{1 + diam\ G}{3}\right]\right] \\ &\leq 2p - \frac{2}{3}[1 + diam\ G].\end{aligned}$$

Hence the result.

The following Theorem relates $\gamma_{cRB}(T)$ with $\gamma_{ss}(T)$.

Theorem 11: For any non – trivial tree T, $\gamma_{cRB}(T) \geq \gamma_{ss}(T) + 1$ if and only if $B(T)$ has at least two cut vertices.

Proof: For necessary condition, suppose $\gamma_{cRB}(T) \geq \gamma_{ss}(T) + 1$.

Let $B(T)$ has exactly one cutvertex. Then T has at least 3 blocks and $\gamma_{ss}(T) + 1 \geq 3 > 2 = \gamma_{cRB}(T)$, a contradiction. Hence $B(T)$ has at least two cutvertices.

For sufficient condition, suppose $B(T)$ has at least two cutvertices.

Let $V = \{v_1, v_2, \ldots\ldots, v_p\}$ be the set of all vertices and $V_c^{\mathrm{I}} = \{v_1, v_2, \ldots.., v_r\}$ be the set of all cutvertices adjacent to endvertices in T. Let $E_e = \{e_1, e_2, \ldots\ldots, e_n\}$ be the set of all endedges and $E_c = \{e_1, e_2, \ldots\ldots, e_c\}, c \leq n$ be the set of all edges adjacent to endedges in T. Let V_{nc} be the set of all minimum number of vertices that covers all the edges of $\langle E(G) \setminus E_c \cup E_e \rangle$. Then $V_c^{\mathrm{I}} \cup V_{nc}$ forms γ_{ss}- set and $|V_c^{\mathrm{I}}| + |V_{nc}| = \gamma_{ss}(T)$.$\forall\, B_i \in E(G) \setminus E_e$, $\exists$ corresponding block vertices $b_i \in V_2 \cup V_1$ in $B(T)$ such that $\langle V_2 \cup V_1 \rangle$ is connected. Then $\{b_i\}$ forms γ_{cRB} – set and $|b_i| = |V_1| + |V_2| = |D_{cRB}| = \gamma_{cRB}(T)$.

Obviously $|V_2| + |V_1| \geq |V_c^{\mathrm{I}}| + |V_{nc}| + 1$ gives $\gamma_{cRB}(T) \geq \gamma_{ss}(T) + 1$.

Suppose $B(T)$ has no cutvertex. Then the corresponding tree T has exactly one cutvertex.

Clearly $\gamma_{cRB}(T) = 2 = \gamma_{ss}(T) + 1$.

In the following Theorems we obtain lower bounds for $\gamma_{cRB}(T)$ in terms of p, q, Δ, δ

Theorem 12: For any tree T with $p \geq 3$, $\gamma_{cRB}(T) \geq p - \frac{q}{\delta} + 1$.

Proof: We consider the following cases.

Case-1: Suppose the tree T has exactly one cutvertex. Then $B(T)$ is a complete graph and $\gamma_{cRB}(T) = 2$. Since for any tree T, $q = p - 1$ and $\delta(T) = 1$

Clearly $p - \frac{q}{\delta} + 1 = p - \frac{(p-1)}{1} + 1 = 2$ gives the result.

Case-2: Suppose T has more than one cutvertex. Let $V_c = \{v_1, v_2, v_3, \ldots\ldots, v_c\}$ be the set of all cutvertices in T. For every non – endedge containing vertices $v_i \in V_c$ there exist a corresponding block vertex set$\{b_i\}$such that$|b_i| = |V_1 + V_2|$ in $B(T)$. Hence $\{b_i\}$ forms γ_{cRB} – set and $|\, b_i \,| = |\, V_1 + V_2 \,| = |\, D_{cRB} \,| \geq 2 = p - \frac{q}{\delta} + 1$. Hence the result.

Theorem 13: For any tree T, $\left\lfloor \frac{p}{\Delta(G)+1} \right\rfloor \leq \gamma_{cRB}(T)$.

Proof: Let D_b be a minimal dominating set of $B(T)$ and $D_1 = V(B(T)) \setminus D_b$.

Suppose $D_2 \subseteq D_1$ and $\forall\, b_i \in D_b$, $w(b_i) = 2\ or\ 1$, $\forall\, b_j \in D_2$, $w(b_j) = 1$ and $\forall\, b_k \in D_1 \setminus D_2$, $w(b_k) = 0$. If each b_i has weight 2 and $N(b_i) = x \in D_b$, then $\langle D_b \rangle = \langle V_2 \rangle$ which is connected and gives $|V_2| = \gamma_{cRB}(T)$ otherwise there exist $b_j \in D_2$, $w(b_j) = 1$ and $N(b_i) = b_j$. Then $\langle V_2 \cup V_1 \rangle$ is connected. Hence $|V_2 \cup V_1| = \gamma_{cRB}(T)$. For any tree T, there exist at least one vertex $u \in V(T)$ such that $\deg(u) = \Delta(T)$. Then $\left\lfloor \frac{p}{\Delta(G)+1} \right\rfloor \leq |V_1| + |V_2|$ or $|V_2|$.

Hence $\left\lfloor \frac{p}{\Delta(G)+1} \right\rfloor \leq \gamma_{cRB}(T)$.

REFERENCES

1. Cockyane E.J., Dreyer P.A., Hedetniemi S.M. and Hedetniemi S.T., Romandomination in graphs, Disc. Math. 278 (2004), 11 – 22.
2. Harary F., Graph Theory, Addison –Wesley – Reading Mass (1974).
3. Harary F., A characterization of block graphs, Canad. Math. Bull. 6 (1963), 1 – 6.
4. Mobaraky B.P. and heikholeslami S. M. S., Bounds on Roman Domination Numbers of Graphs, MATEMATицКи BECHик, 60 (2008), 247 – 253.
5. Muddebihal M.H. and Sumangala devi, Connected Roman domination in graphs, International journal of Research in Engg. Tech. Vol 2 (10), (2013), 333 – 340.
6. Muddebihal M.H. and Sumangala Devi, Roman Block Domination in Graphs, International journal of Research in science & Advanced Technologies, Nov-Dec 2013, issue 2, vol – 6, 267 – 275.
7. Sampath Kumar E. & Walikar H.B., The connected domination number of a graph, J. Math. Phys. Sci., 1979, 13: 607 – 613.

Source of support: Nil, Conflict of interest: None Declared