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K AND K*- BI NEAR SUBTRACTION SEMIGROUPS

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ABSTRACT

In this paper we introduce the notion of F- bi-near subtraction semigroup. Also we give characterizations of F- bi-near subtraction semigroup.

Mathematical subject classification: *06F35.*

***Key words**: K- bi-near subtraction semigroup, strong S_1- bi-near subtraction semigroup, strong S_2- bi-near subtraction semigroup, S_1- bi-near subtraction semigroup, S_2- bi-near subtraction semigroup, Nil near subtraction semigroup, idempotent, Nolpotent, Zero devisors, Mate function, Boolean.*

1. INTRODUCTION

In 2007, Dheena [1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz[5]. Zekiye Seydali Fathima *et.al* [3, 4] introduced the notation of S_1-near subtraction semigroup and S_2-near subtraction semigroup. Recently Firthous *et.al* [2] introduced the notation of F- Bi near subtraction semigroup. In this paper we shall obtained equivalent conditions for regularity in terms of K- Bi near subtraction semigroup.

2. PRELIMINARIES

A non-empty subset X together with two binary operations "–" and "." is said to be ***subtraction semigroup*** If (i) (X,–) is a subtraction algebra (ii) (X, .) is a semi group (iii) $x(y-z)=xy-xz$ and $(x-y)z= xz-yz$ for every x, y, $z\in X$.A non-empty subset X together with two binary operations "–" and "." is said to be ***near subtraction semigroup*** if (i) (X,–) is a subtraction algebra (ii) (X,.) is a semi group and (iii) $(x-y)z= xz-yz$ for every x, y, $z\in X$.. A non-empty subset $X=X_1\cup X_2$ together with two binary operations"-"and "." Is said to be ***bi-near subtraction semigroup*** (right). If (i) $(X_1,-,.)$ is a near-subtraction semigroup (ii) $(X_2,-,.)$ is a subtraction semigroup. A non-empty subset X is said to be ***S_1-near subtraction semi group*** if for every $a\in X$ there exists $x\in X^*$ such that axa=xa.. A non-empty subset X is said to be ***S_2-near subtraction semi group*** if for every $a\in X$ there exists $x\in X^*$ such that axa=ax. A non-empty subset X is said to be ***strong S_1-near subtraction semi group*** if aba=ba for all a, $b\in X$. A non-empty subset X is said to be ***strong S_2-near subtraction semi group*** if aba=ab for all a, $b\in X$. If there exists a map $f: X\rightarrow Y$ such that $a = a f(a) a$ for all a in X then f is called a ***mate function*** for X. An element $a\in X$ is said to be ***Boolean*** if $a^2 =a$. A **sub commutative near subtraction semigroup** is an intersection of S_1-near subtraction semigroup and S_2-near subtraction semigroup. that is, xa=ax. A non-empty subset X is said to be **nil-near subtraction semigroup** if there exists a positive integer $k>1$ such that $a^k=0$ Which implies that xa=0 where $x=a^{k-1}$.A non-empty subset X is said to be **zero-symmetric**. if 0-x=0, ox=0 and xo=o for all $x\in X$. A non-empty subset Y of X is closed under "-"and XY strictly contained in Y is called an **X-system**. A non-empty subset $X=X_1\cup X_2$ together with two binary operations"-"and "." is said to be *F- **bi near subtraction semigroups***. If (i) for every $a\in X_1$ there exists $x\in X_1^*$ such that axa=xa. (ii) for every $a\in X_2$ there exists $x\in X_2^*$ such that axa=ax.

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3. K AND K^*-BI NEAR SUBTRACTION SEMIGROUP

Definition: 3.1 A non-empty subset $X=X_1 \cup X_2$ together with two binary operations"-"and "." Is said to be K- **bi near subtraction semigroup**. If (i) if aba=ba for all a, $b \in X_1$. (ii) for every $a \in X_2$ there exists $x \in X_2^*$ such that axa=ax.

Example: 3.2 Let X_1={0, a,b,1} in which "–" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	a	a
b	a	0	1	b
1	0	a	b	1

Thus X_1 is a strong s_1-near subtraction semi group

Let X_1= {0, a, b, 1} in which "-"and "." be defined by

-	0	a	b	C
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

	.	0	a	b
0	0	0	0	0
a	a	0	0	0
b	b	0	0	0
1	c	0	0	0

Then X_2 is a S_2-near subtraction semi group.

Hence, $X=X_1 \cup X_2$ is a K-bi near Subtraction Semigroup.

Note: 3.3 Obviously, every K-bi near subtraction is a F- bi-near subtraction semi group. But the converse need not be true

Example: 3.4 Let X_1={0,a,b,c} in which "–" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	a	a	b	c
b	0	0	0	0
c	0	a	b	c

Thus X_1 is a strong s_1-near subtraction semi group but not s_1- near subtraction semigroup.

Let X_2= {0, a, b, c} in which "–"and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	a	a
b	a	0	1	b
1	0	a	b	1

Thus X_2 is an S_2-near subtraction semigroup

Hence, every K- bi-near subtraction semi group need not be a F-bi near subtraction semi group.

Definition: 3.5 A non-empty subset $X=X_1 \cup X_2$ together with two binary operations "-"and "." Is said to be K^*- **bi near subtraction semigroup**. If (i) if for every $a \in X_1$ there exists $x \in X_1^*$ such that axa=xa. (ii) aba=ab for all a, $b \in X_2$.

Example: 3.6 Let X_1= {0, a, b, 1} in which "-"and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	0
b	0	0	b	b
1	0	a	b	1

Then X_1 is a s_1-near-subtraction semi group

Let X_2={0, a, b, 1} in which "–" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	0	0	b	b
1	0	a	b	1

Thus X_2 is a strong S_2-near subtraction semigroup.

Hence, X= $X_1 \cup X_2$ is a strong K*-bi-near subtraction semi group.

Note: 3.7 Obviously, every K^*-bi near subtraction is a F- bi-near subtraction semi group. But the converse is not true

Example: 3.8 Let X_1={0, a, b, 1} in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	0
b	0	0	b	b
1	0	a	b	1

Then X_1 is a s_1-near-subtraction semi group

Let X_2={0, a, b, c} in which "–" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	a	b
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	a	a	b	a
b	0	0	0	0
c	0	0	0	c

Thus (X_2, -, .) is a strong S_2-near-subtraction semi group but not a S_2-near subtraction semigroup (since bab≠ab). Hence, X= $X_1 \cup X_2$ is not a k*-bi-near subtraction semi group.

4. RESULTS ON K AND K^*-BI NEAR SUBTRACTION SEMIGROUP

Proposition: 4.1 If X is a K-bi near subtraction semigroup then X is a zero-symmetric

Proof: Let X= $X_1 \cup X_2$ be a K-bi near Subtraction Semigroup where X_1 is a strong S_1-near subtraction semigroup and X_2 is a S_2-near subtraction semigroup. Since X_1 is a strong S_1-near subtraction semigroup that is, axa=xa for all $a \in X_1$ and $x \in X_1^*$. Subtituting a=0 we have 0x0=x0 for all $x \in X_1^*$. Thus X_1 is a zero-symmetric. Since X_2 is a strong S_2-near subtraction semigroup that is, axa=ax for all $a \in X_1$ and $x \in X_1^*$.Subtituting a=0 we have 0x0=x0 for all $x \in X_1^*$. Thus X_1 is a zero-symmetric. Hence, X= $X_1 \cup X_2$ is a zero-symmetric.

Remark: 4.2 The Converse of above Proposition need not be true

Example: 4.3 Let X_1={0, a, b, c} in which "–" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	0	0	0	a
b	a	0	0	b
c	0	0	0	c

Thus X_1 is a zero-symmetric but not a strong s_1-near subtraction semi group (Since cac≠ac).

Let X_2 ={0, a, b, 1} in which "–" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	0	0	b	b
1	0	a	b	1

Thus X_2 is a zero-symmetric and also a S_2-near subtraction semigroup. Hence, zero symmetric need not be a K- bi-near subtraction semi group.

Proposition: 4.4 The intersection of strong S_1-near subtraction semigroup and S_2-near subtraction semigroup is sub commutative near subtraction semigroup.

Proof: Let X_1 is a strong S_1-near subtraction semigroup. there exists $x \in X_1^*$ such that axa=xa. (1)
(by [3], Every Strong S_1-near subtraction semigroup is a S_1-near subtraction semigroup). Let X_2 is an S_2-near subtraction semigroup, there exists $x \in X_2^*$ such that axa=ax (2)

From (1) and (2), we get xa=ax. Thus, X is a sub commutative near subtraction semigroup.

Proposition: 4.5 Let X be a Sub-commutative K-bi near Subtraction Semigroup Then X has no non zero-zero divisor function if and only if X is Boolean.

Proof: Let $X = X_1 \cup X_2$ be a K-bi near Subtraction Semigroup where X_1 is a strong S_1-near subtraction semigroup and X_2 is a S_2-near subtraction semigroup. Let $a \in X_1$. Since a a a=a a. That is, $a^3=a^2$, which implies $(a^2-a)a=0$. Since X_1 has no non zero-zero divisor function, $a^2-a=0$, $a^2 = a$ Thus *X_1 is Boolean*. Let $a \in X_2$. Since X_2 is a strong S_2-near subtraction semigroup, there exists x€ X_2^* such that axa=xa. Which implies aax=ax. (Since X_2 be a Sub-commutative). That is, a^2x= ax that implies $(a^2-a)x=0$. Since X_2 has no non zero-zero divisor function, $a^2-a=0$, $a^2 = a$ Thus *X_2 is Boolean*. Therefore $X = X_1 \cup X_2$ where *X_1 is Boolean and X_2 is Boolean. Hence, X is Boolean.*

Proposition: 4.6 The intersection of S_1-near subtraction semigroup and strong S_2-near subtraction semigroup is sub commutative near subtraction semigroup.

Proof: Let X_1 is a S_1-near subtraction semigroup. there exists $x \in X_1^*$ such that axa=xa. (1)
Let X_2 is an strong S_2-near subtraction semigroup, there exists $x \in X_2^*$ such that axa=ax (2)

From (1) and (2), we get xa=ax (by [4], Every Strong S_2-near subtraction semigroup is a S_2-near subtraction semigroup). Thus, X is a sub commutative near subtraction semigroup.

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