International Journal of Mathematical Archive-6(9), 2015, 159-163
Available online through www.ijma.info ISSN 2229 – 5046

1 -NEAR MEAN CORDIAL LABELING OF QUADRILATERAL SNAKE, LADDER, UMBRELLA GRAPHS

S. SRIRAM*
D. G.Vaishnav College, Arumbakkam, Chennai
Assistant Professor, Patrician College of Arts and Science, Adyar, Chennai-20, India.

R. GOVINDARAJAN
Associate Professor And Head, P.G And U.G Department Of Mathematics,
D.G.Vaishnav College, Arumbakkam, Chennai, India.

(Received On: 06-09-15; Revised & Accepted On: 30-09-15)

ABSTRACT

Let G= (V, E) be a simple graph. A surjective function $f:V\to\{0,1,2\}$ *is said to be a 1-Near Mean Cordial labeling if for each edge uv, the induced map*

$$f(uv) = \begin{cases} 0 \text{ if } \dfrac{f(u)+f(v)}{2} \text{ is an integer} \\ 1 \quad \text{Otherwise} \end{cases}$$

Satisfies the condition $|e_f(0)-e_f(1)|\le 1$ *where* $e_f(0)$ *is the number of edges with 0 label and* $e_f(1)$ *is the number of edges with 1 label.*

G is said to be 1-Near Mean Cordial Graph if it has a 1-Near Mean Cordial Labeling. In this paper we proved that Quadrilateral Snake Q_n*, Ladder graph* $[P_n\times P_2]$*, Umbrella* $U(n,3)$ *are 1-Near Mean Cordial Graphs.*

***Keywords: 1**-Near Mean Cordial Labeling, 1-Near Mean Cordial Graph*

***2010 mathematics Subject Classification:** 05C78.*

1. INTRODUCTION

Let us consider the graphs to be finite, undirected and simple. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. The cardinality of V(G) and E(G) are respectively called order and size of G. Labeling of graphs has enormous application in many practical problems involved in circuit designing, communication network, astronomy etc. [1]. The concept of mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram in the year 2012 in [5]. Some results on Mean Cordial Labeling was discussed in [3, 4, 5, 6]. Let f be a function V (G) to $\{0,1,2\}$. For each edge uv of G assign the label $\dfrac{f(u)+f(v)}{2}$. f is called a mean cordial labeling of G if $|v_f(i)-v_f(j)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$, $i,j\in\{0,1,2\}$ where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x\ (x=0,1,2)$ respectively. A graph with a mean cordial labeling is called mean cordial graph. K.Palani, J.Rejila Jeya Surya [2] introduced a new concept called 1-Near Mean Cordial labeling and investigated the 1-Near Mean Cordial Labeling behavior of Paths, Combs, Fans and Crowns. Terms defined here are used as in F. Harary [7].

Corresponding Author: S. SRIRAM*
D. G.Vaishnav College, Arumbakkam, Chennai
Assistant Professor, Patrician College of Arts and Science, Adyar, Chennai-20, India.

2. PRELIMINARIES

K.Palani,J.Rejila Jeya Surya [2] define the concept of 1-Near Mean Cordial labeling as follows:
Let $G=(V,E)$ be a simple graph. A surjective function $f:V\to\{0,1,2\}$ is said to be 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv)=\left.\begin{array}{l} 0 \text{ if } \dfrac{f(u)+f(v)}{2} \\ = 1 \text{ otherwise}\end{array}\right\} \text{is an integer}$$

Satisfies the condition $|e_f(0)-e_f(1)|\le 1$ where $e_f(0)$ is the number of edges with zero label and $e_f(1)$ is the number of edges with one label.

G is said to be a 1-Near Mean Cordial Graph if it has a 1-Near Mean cordial labeling. G is said to be 1-Near Mean Cordial Graph if it has a 1-Near Mean Cordial Labeling. In this paper, we proved that Quadrilateral Snake Q_n, Ladder graph $[P_n\times P_2]$, Umbrella $U(n,3)$ are 1-Near Mean Cordial Graphs.

3. 1-NEAR MEAN CORDIAL ON SPECIAL TYPES OF GRAPHS

Theorem 3.1: Quadrilateral Snake Q_n is 1-Near Mean Cordial graph

Proof: Let $G=(V,E)$ be a simple graph and let G be Q_n.
Let $V(G)=\{[u_i,1\le i\le n],[v_i,1\le i\le n-1],[w_i,1\le i\le n-1]\}$,

$$E(G)=\{[(u_iv_i)\cup(u_iw_i):1\le i\le n-1]\cup[(u_{i+1}v_i)\cup(u_{i+1}w_i):1\le i\le n-1]\}$$

Define $f:V\to\{0,1,2\}$ by

$$f(u_i)=\left.\begin{array}{l}1 \text{ if } i\equiv 1\bmod 2\\ = 2 \text{ if } i\equiv 0\bmod 2\end{array}\right\}\quad 1\le i\le n$$

$$f(u_1)=1$$

$$f(v_i)=\left.\begin{array}{l}0 \text{ if } i\equiv 1\bmod 2\\ = 2 \text{ if } i\equiv 0\bmod 2\end{array}\right\} 1\le i\le n-1$$

$$f(w_i)=\left.\begin{array}{l}2 \text{ if } i\equiv 0\bmod 2\\ =1 \text{ if } i\equiv 1\bmod 2\end{array}\right\}\quad 1\le i\le n-1$$

The induced edge labeling are

$$f^*(u_1,v_1)=1$$

$$f^*(u_i,v_i)=\left.\begin{array}{l}1 \text{ When i is odd}\\ = 0 \text{ when i is even}\end{array}\right\}\quad \text{for } 1\le i\le n$$

$$f^*(u_{i+1},v_i)=\left.\begin{array}{l}0 \text{ When i is odd}\\ =1 \text{ when i is even}\end{array}\right\} \text{for } 1\le i\le n-1$$

$$f^*(u_i,w_i)=0 \text{ for all } 1\le i\le n$$

$$f^*(u_{i+1},wi)=1 \text{ for all } 1\le i\le n$$

Here, $e_f(0)=n$; $e_f(1)=n$

Hence, the graph satisfies the condition $|e_f(0)-e_f(1)|\le 1$

Therefore, the graph Quadrilateral Snake Q_n is a 1-Near Mean Cordial Graph.

Illustration:

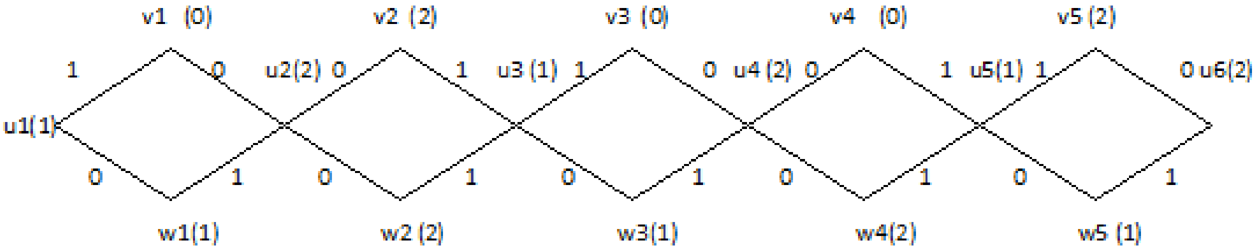

Fig. 3.1.1: Q_6

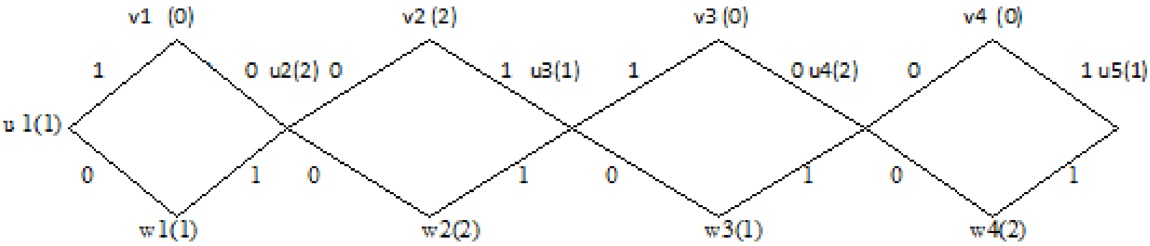

Fig.3.1.2: Q_5

Theorem 3.2: Ladder $\left[P_n \times P_2\right]$ is a 1-Near Mean Cordial Graph

Proof: Let $G = (V, E)$ be a simple graph and let G be $\left[P_n \times P_2\right]$. Let $V(G) = \left\{(u_{ij}); 1 \le i \le n, 1 \le j \le 2\right\}$ and let $E(G) = \left\{\left[(u_{i1}u_{i2}) : 1 \le i \le n\right] \cup \left[\left(u_{ij}u_{(i+1)j}\right) : 1 \le i \le n-1, 1 \le j \le 2\right]\right\}$

Define $f : V \to \{0,1,2\}$ by

For $f(u_{ij})$ when i is odd

$$f(u_{ij}) = 0 \text{ if } j \equiv 1 \bmod 2$$
$$= 2 \text{ if } j \equiv 0 \bmod 2 \quad \text{for } 1 \le i \le n$$

For $f(u_{ij})$ when i is even

$$f(u_{ij}) = 1 \text{ if } j \equiv 1 \bmod 2$$
$$= 2 \text{ if } j \equiv 0 \bmod 2 \quad \text{for } 1 \le i \le n$$

Where $1 \le i \le n$, $1 \le j \le 2$

Hence the induced edge labeling are

$$f^*(u_{i1}u_{i2}) = 0 \text{ when i is odd}$$
$$= 1 \text{ when i is even} \quad \text{for } 1 \le i \le n$$

$$f^*\left(u_{i1}u_{(i+1)1}\right) = 1 \text{ for } 1 \le i \le n$$

$$f^*\left(u_{i2}u_{(i+1)2}\right) = 0 \text{ for } 1 \le i \le n$$

Hence it satisfies the condition $\left|e_f(0) - e_f(1)\right| \le 1$

Therefore the Ladder $\left[P_n \times P_2\right]$ is a 1-Near Mean Cordial graph.

Illustration:

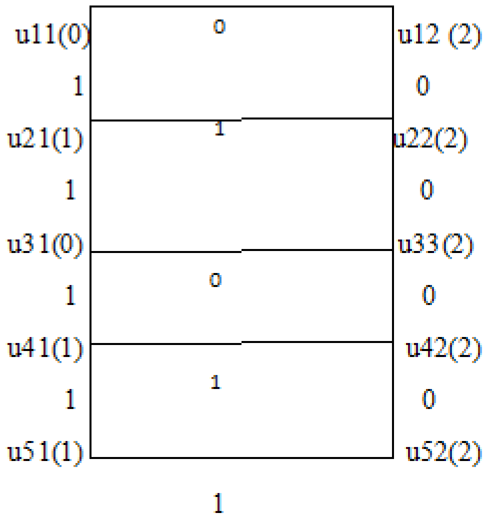

Fig. 3.2.1: $[P_5 \times P_2]$ graph

Theorem 3.3: Umbrella $U(n,3)$ is 1-Near Mean Cordial Graph

Proof: Let $G=(V,E)$ is a simple graph. Let $G=U(n,3)$ and let

For n is odd:

$V(G)=\{u,v,w,u_i : 1\le i\le n\}$;

$$E(G)=\left[\{(uu_i);1\le i\le n\}\cup\left(u_{\frac{(n+1)}{2}}v\right)\cup(vw)\cup\{(uu_{i+1});1\le i\le n\}\right]$$

For n is even:

$V(G)=\{u,v,w,u_i : 1\le i\le n\}$

$$E(G)=\left[\{(uu_i);1\le i\le n\}\cup\left(u_{\frac{n}{2}}v\right)\cup(vw)\cup\{(uu_{i+1});1\le i\le n\}\right]$$

Define $f: V\to\{0,1,2\}$

By $\left.\begin{array}{l} f(u_i)=0 \text{ if } i\equiv 1 \bmod 2 \\ \quad = 2 \text{ if } i\equiv 0 \bmod 2 \end{array}\right\} \; 1\le i\le n$

$f(u)=1$

Also define $f(v)=2; f(w)=1$

Then the induced edge labeling are

$f^*(u_iu_{i+1})=0$ For $1\le i\le n-1$

$f^*(u_iu)=1$ for $1\le i\le n$

$f^*\left(u_{\frac{n}{2}},v\right)=0$ When n is even

$f^*\left(u_{\frac{n+1}{2}},v\right)=0$ When n is odd

$f^*(v,w)=1$

Clearly it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1.
Hence Umbrella $U(n,3)$ is 1-Near Mean Cordial graph.

Illustration:

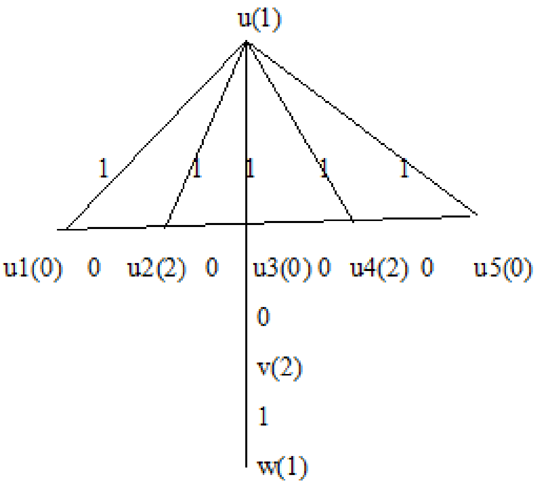

Fig.3.3.1: $U(5,3)$ Umbrella graph

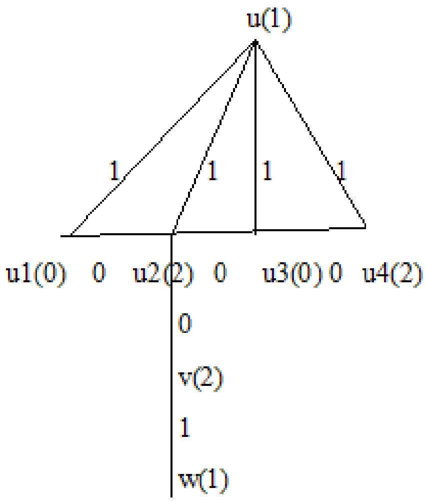

Fig.3.3.2: $U(4,3)$ Umbrella graph

REFERENCES

1. Gallian J.A, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics 6(2001)#DS6.
2. K.Palani,J.Rejila Jeya Surya, 1-Near Mean Cordial Labeling of Graphs, IJMA-6(7), July 2015 PP15-20.
3. A.Nellai Murugan and G.Esther, Path Related Mean Cordial Graphs, Journal of Global Research in Mathematical Archives, ISSN 2320-5822, Volume 2, No.3, March 2014.
4. A.Nellai Murugan and G.Esther, Some Results on Mean Cordial Graphs, International Journal of Mathematics Trends and Technology, ISSN: 2231-5373, Volume 11, No.2, July 2014.
5. Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram, Mean Cordial Labeling of Graphs, Open Journal of Discrete Mathematics 2012, 2,145-148.
6. Albert William I, Indra Rajasingh and S.Roy, Mean Cordial Labeling of Certain Graphs, Journal of Computer and Mathematical Sciences, Vol. 4, Issue 4, 31 August, 2013 Pages (202-321).
7. F.Harary, Graph Theory, Addison-Wesley Publishing Company Inc, USA, 1969.

Source of support: Nil, Conflict of interest: None Declared