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ARITHMETIC-GEOMETRIC INDICES OF AMALGAMATION OF TWO GRAPHS

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ABSTRACT

Let G be a molecular graph. The Arithmetic-Geometric (AG) indices of G are defined as AG (G)=$\sum_{uv\in E(G)}\frac{du+dv}{2\sqrt{du*dv}}$ *where du (or dv) denote the degree of the vertex u (or v), respectively. In this paper we introduce an amalgamation operation on the path graph and cycle graph such that resulting graph G:* $P_n\&C_m$ *is introduced between co-relation of the path graph and cycle graph. We investigate the topological indices like arithmetic-geometric index for the graphs* P_n & C_m*.*

Keyword: *Graph, Path graph, Cycle graph, Molecular graph, Graph Amalgamation, Arithmetic- Geometric index.*

INTRODUCTION

A graph G is a pair G=(V,E) consisting of a finite set V and set E of 2-element subsets of V (infinite graphs are also studied, here we consider only finite graphs).The elements of V are called vertices (points, nodes, junctions, or 0-simplexes) and elements of E are called edges (lines, arcs, branches or 1-simplexes).The set V is known as the vertex set of G and E as the edge set of G. If we denote the edge by e, we can then write e = uv which is an edge of G. Thus two vertices u and v of G are said to be adjacent, if an edge e joins u and v, and two edges are adjacent if they have a common vertex [3].

If in a walk W all edges are distinct, then W is called a trail and if all the vetices are distinct then W is called a **path**. A graph which forms a path with n vertices is denoted by P_n. A walk $v_0e_1v_1.....e_kv_k$ is said to be a cycle if $v_0 = v_k$ and the vertices $v_{0,}v_{1,......,}\ v_{k-1}$ are distinct from each other. A graph which forms a cycle with n vertices is denoted by C_n. If n is odd, the cycle is said to be an odd cycle, and even otherwise [3].

Chemical graph theory is a branch of graph theory that is concerned with analysis of all consequences of connectivity in a chemical graph. Chemical graph serves as a convenient model for any real or abstracted chemical system [7, 8]. It can represent different chemical objects as molecules, reactions, crystal, polymers, chester etc. The common feature of chemical systems is the presence of site and connections between them. Sites may be atoms, electrons, molecules, molecular fragments, groups of atoms, intermediates, orbitals etc. The connections between sites may represent bonds, bonded and non-bonded interactions, elementary reaction steps, rearrangements, Van derwaals force etc. chemical systems may be depicted by chemical graphs using a simple conversion rule,

Site ↔ Vertex
Connection ↔edge

The degree of a vertex v of G is the number of edges incident with v and is written deg(v).

MOLECULAR GRAPH

A molecular graph G= (V, E) is a simple graph having n=|V| nodes and m=|E| edges. The nodes $v_i \in V$ represent non-hydrogen atoms and the edges $(v_i\ ,v_j) \in E$ represent covalent bonds between the corresponding atoms. In particular, hydrocarbons are formed only by carbon and hydrogen atom and their molecular graphs represent the carbon skeleton of the molecule.

Molecular graphs are a special type of chemical graphs, which represent the constitution of molecules [7, 4, 14]. They are also called constitutional graphs. When the constitutional graph of a molecule is represented in a two- dimensional basis it is called structural graph.

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In this paper we consider simple connected graph with no self-loops and no multiple edges.

In chemical graph theory, we have many different topological index of arbitrary molecular graph G. A topological index of a graphs is a member related to a graph which is invariant under graph automorphisms, obviously, every topological index defines a counting polynomial and vice versa,

One of important connectivity topological indices is Arithmetic-Geometric (AG) index of G

$AG(G)=\sum_{uv\in E(G)}\frac{du+dv}{2\sqrt{du*dv}}$

Where, AG index is considered for distinct vertices.

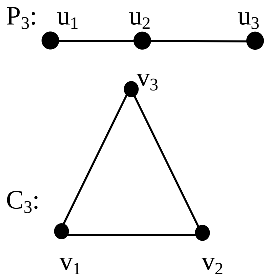
The above equation is the sum of the ratio of the Arithmetic mean and Geometric mean of u and v. where du (or dv) denote the degree of the vertex u (or v)[9,10,11].

GRAPH AMALGAMATION

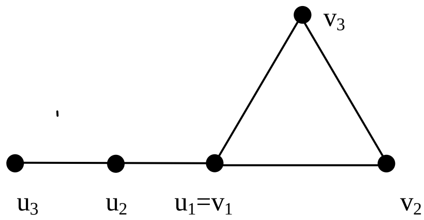
Let G_1 and G_2 be two graphs $\{u_1,u_2,\ldots,u_n\}$ and $\{v_1,v_2,\ldots,v_m\}$ on n and m vertices then amalgamation of G_1 and G_2 is denoted by G : G_1 & G_2 is obtained by fussing u_1 and v_1 [12].

Example 1: G: (P_n & C_m) are connected by amalgamation

1. When n and m are odd



G: (P_3&C_3) =

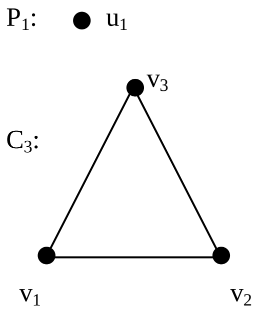


Similarly the graph structure follows when

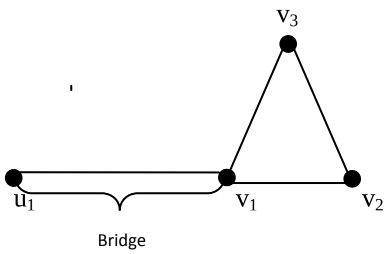
i) n and m are even
ii) n is odd and m is even
iii) n is even and m is odd
iv) n and m are equal

Example 2: G: (P_n, C_m) are connected by bridge

1. When n and m are odd



G: (P_1, C_3) =



Similarly the graph structure follows when

i) n and m are even
ii) n is odd and m is even
iii) n is even and m is odd
iv) n and m are equal

MAIN RESULTS AND DISCUSSION

Theorem 1: G: (P_n & C_m) are connected by amalgamation then AG indices of G

1. When n and m are odd

$$\sum_{l=1}^{\frac{n-1}{2}} AG\left(P_{(2l+1)} \& C_m\right)=\frac{3}{2\sqrt{2}}\sum_{l=1}^{\frac{n-1}{2}}(2l+m-2)+\sum_{l=1}^{\frac{n-1}{2}}\frac{2}{\sqrt{3}}$$

2. When n and m are even

$$\sum_{l=1}^{\frac{n}{2}} AG\left(\mathrm{P}_{2l}\ \&\ \mathrm{C_m}\right)=\frac{3}{2\sqrt{2}}\sum_{l=1}^{\frac{n}{2}}\left(2l+m-3\right)+\sum_{l=1}^{\frac{n}{2}}\frac{2}{\sqrt{3}}$$

3. When n is odd and m is even

$$\sum_{l=1}^{\frac{n-1}{2}} AG\left(P_{(2l+1)}\ \&\ C_m\right)=\frac{3}{\sqrt{2}}\sum_{l=1}^{\frac{n-1}{2}}\left(\frac{2l+m-2}{2}\right)+\sum_{l=1}^{\frac{n-1}{2}}\frac{2}{\sqrt{3}}$$

4. When n is even and m is odd

$$\sum_{l=1}^{\frac{n}{2}} AG\left(\mathrm{P}_{2l}\ \&\ \mathrm{C_m}\right)=\frac{3}{\sqrt{2}}\sum_{l=1}^{\frac{n}{2}}\left\lfloor\frac{2l+\left\lfloor\frac{m}{2}\right\rfloor}{2}\right\rfloor+\sum_{l=1}^{\frac{n}{2}}\frac{2}{\sqrt{3}}$$

5. When n and m are equal

$$\mathrm{AG}(\mathrm{P_n}\&\mathrm{C_m})=\frac{3}{2\sqrt{2}}\left[n+\left(m \bmod 3\right)\right]+\frac{2}{\sqrt{3}}$$

Or

$$\mathrm{AG}(\mathrm{P_n}\ \&\ \mathrm{C_m})=\frac{3}{2\sqrt{2}}\left[n+\left(m\%3\right)\right]+\frac{2}{\sqrt{3}}$$

Theorem 2: G: (P_n, C_m)are connected by bridge then AG indices of G

1. When n and m are odd

$$\sum_{l=1}^{\frac{n+1}{2}} AG\left(\mathrm{P}_{(2l\text{-}1)},\mathrm{C_m}\right)=\frac{3}{\sqrt{2}}\sum_{l=1}^{\frac{n+1}{2}}\left|\frac{(2l-1)+\left\lfloor\frac{m}{2}\right\rfloor}{2}\right|+\sum_{l=1}^{\frac{n+1}{2}}\frac{2}{\sqrt{3}}$$

2. When n and m are even

$$\sum_{l=1}^{\frac{n}{2}} AG\left(\mathrm{P}_{2l},\mathrm{C_m}\right)=\frac{3}{\sqrt{2}}\sum_{l=1}^{\frac{n}{2}}\left(l+m-1\right)+\sum_{l=1}^{\frac{n}{2}}\frac{2}{\sqrt{3}}$$

3. When n is odd and m is even

$$\sum_{l=1}^{\frac{n-1}{2}} AG\left(P_{(2l+1)},C_m\right)=\frac{3}{2\sqrt{2}}\sum_{l=1}^{\frac{n-1}{2}}\left((2l+1)+m-2\right)+\sum_{l=1}^{\frac{n-1}{2}}\frac{2}{\sqrt{3}}$$

4. When n is even and m is odd

$$\sum_{l=1}^{\frac{n}{2}} AG\left(P_{2l},C_m\right)=\frac{3}{2\sqrt{2}}\sum_{l=1}^{\frac{n}{2}}\left(2l+m-2\right)+\sum_{l=1}^{\frac{n}{2}}\frac{2}{\sqrt{3}}$$

5. When n and m are equal

$$\sum_{l=3}^{n} AG\left(P_l,C_l\right)=\sum_{l=3}^{n}\frac{3(l-1)}{\sqrt{2}}+\sum_{l=3}^{n}\frac{2}{\sqrt{3}}$$

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