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OPTIMIZATION MODEL WITH INTERVAL VALUED FUNCTIONS

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ABSTRACT

Optimization models have been widely applied in statistics. This paper concentrates the interval form of valued linear programming model (IVLPM). The range of the data and confidence interval of the data are considered as interval. An IVLPM is a linear programming model (LPM) with interval form of the coefficients in the objective function and all requirements. The solution of the IVLPM is analysed numerically.

Keywords: *Interval valued linear programming model, Range, Confidence interval, optimum interval and optimum solution.*

INTRODUCTION

In mathematical programming the coefficients of the models are always treated as deterministic values. Interval valued optimization model may provide an alternative choice for considering the uncertainty into the optimization model.

Consider the coefficients of the objective function and all requirements are interval form. The limits of uncertain data are easier to find by the method of estimation. The applications of interval based models are production planning, financial and corporate planning healthcare and hospital planning etc.

Definition: The function f: $R^h \longrightarrow I$.

Defined on the Education space R^n called an interval valued function.

i.e, $G(x) = G(x_1, x_2 \ldots x_n)$ is closed interval in R. The IVF G can be also written as

$$G\ (x) = \left[\underline{G}(x), \overline{G}(x)\right]$$

Where $\underline{G}(x)$ and $\overline{G}(x)$ are real valued functions defined on R^n and satisfy $\underline{G}(x) \le \overline{G}(x)$ for every $x \in R^n$

We say that the IVF is differentiable at $x_0 \in R^n$ if and only if the real valued functions $\underline{G}(x)$ and $\overline{G}(x)$ are differentiable at x_0.

Remark 1: Suppose A= $(\underline{a}, \overline{a})$ B = $(\underline{b}, \overline{b})$

Then 1) $G(A \geq B) > 0 \Leftrightarrow \overline{a} > \underline{b}$,

2) $G(A > B) > 0 \Leftrightarrow \underline{a} > \underline{b}$ (or) $\overline{a} > \overline{b}$

3) $G(A \leq B) > 0 \Leftrightarrow \underline{a} < \overline{b}$

4) $G(A < B) > 0 \Leftrightarrow \underline{a} > \underline{b}$ or $\overline{a} < \overline{b}$

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Notations
The following notations, and results are useful in our further consideration

If [a] and [b] are closed intervals in R.

$[a] = \left[\underline{a}, \overline{a}\right] \in I.$

$[b] = \left[\underline{b}, \overline{b}\right] \in I$

(i) Those intervals satisfy the additive and subtractive operations.

(ii) $[a]\,[b] = [\min\left(\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right), \max\left(\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}\overline{a}\overline{b}\right)]$

INTERVAL VALUED LINEAR PROGRAMMING MODEL

Consider the Standard Linear Programming model as

Maximize or Minimize

$$\left.\begin{array}{c} z = cx \\ subject \\ Ax = b \\ x \geq 0 \end{array}\right\} \qquad (1)$$

Where $c \in R^n$, $A \in R^{mxn}$ $b \in R^m$

The feasible solution set
$\{S^n = x \in R^n : Ax \leq b \text{ and } x \geq 0\}$
is assumed to be non empty and bounded.

By using interval coefficients, the LPM given in (1) is structured as

$$\left.\begin{array}{l} Maximize \\ \quad Z = \sum_{j=1}^{n} \left[\underline{c}_j \overline{c}_j\right] x_j \\ Subject\ to \\ \quad \sum_{j=1}^{n} \left[\underline{a}_{ij} \overline{a}_{ij}\right] x_j \leq\geq \left[\underline{b}_i, \overline{b}_i\right], x_i \geq 0 \end{array}\right\} \qquad (2)$$

Here $x = (x_1\ x_2\ \ldots..x_n)$ is a feasible solution of model (2) iff

$$\underline{b}_i \leq \sum_{j=1}^{n} a_{ij} x_j \leq \overline{b}_c \text{ , } a_j \in (\underline{a}_{ij}, \overline{a}_{ij}) \text{ , i = 1, 2\ldots..m, j= 1 2 \ldots..n}$$

It is assumed that

$\underline{b}_i = \left(\underline{b}_1, \underline{b}_2 \ldots..\underline{b}_n\right)$ and

$\overline{b}_i = \left(\overline{b}_1, \overline{b}_2 \ldots\ldots\overline{b}_n\right)$

Literature Review

Literature analysis has been studied by several researches such as Ale field and Herzbeeger (1983) Al tanu senguputa and Tapan kumar pal (2000) etc. Interval analysis has been introduced by Moore (1979). Linear programming models with interval coefficient have been analysed by many researches such as Sengupta et al. (2001). Chinneck and Ramadan (2000), Dantzig (1955), and Kuchta (2008), have computed exact range of the optimal value for linear programming problem in which input data can very in some given real compact intervals, and able to characterize the primal and dual solutions sets, the bounds of the objective function resulted from two non linear programming problem. Sengupta *et al.* (2000) have reduced the interval number LPM into a b_i objective classical LPM and then obtained an optimal solution. Suprajinto and Mohd (2008) have presented some interval linear programming models, where the coefficients and variables are in the form of intervals. Multi objective linear programming with interval coefficients have been discussed and the solution has been derived in chanas and Kuchta (1996) and Nehi and Alinezhad (2009).

Krishnamoorthy and Mathew (2004) have discussed on one sided tolerance limits in balanced and un balanced one way random effects ANOVA model. Weerahandi (1993) has introduced the concept of a generalized pivotal quantity for a scalar parameter μ and using that parameter, one can construct an interval estimate for μ. He referred to such intervals as generalized confidence intervals (GCI). Since then, several GCI have been constructed us many practical problems.

Method of Solving IVLPM

The interval coefficient objective function of model (2) is split in to three objective functions by using the limits as follows.

$$\left.\begin{array}{l} \text{Min (left limit of the interval coefficient of the objective function),} \\ \text{Min (right limits of the interval coefficient of the objective function),} \\ \text{Max (Length of the interval coefficient of the objective function),} \\ \text{Sub To (Set of feasibility constraints)} \end{array}\right\} \quad (3)$$

The concept of function Z indicates that for the maximization problem, an interval with a smaller left and right limit value is inferior to an interval with a greater left and right limit values. By using left and right limit values., reduces the LPM with three objective function to a linear bi objective functions which are given below.

$$\left.\begin{array}{ll} \text{Min} & \text{\{Left limit of the interval coefficient of the objective function\},} \\ \text{Min} & \text{\{Right limit of the interval coefficient of the objective functions\}} \\ \text{Sub} & \\ \text{To \{Set of feasibility constraints\}} & \end{array}\right\} \quad (4)$$

The length of the interval is considered as a secondary attribute. The purpose of this study is to obtain a longer interval among non dominated alternatives. An attempt is made to obtain non dominated solution through the model R,

In this stage, a weighted solution

$$\lambda_1\left(\sum_{j=1}^{n}\underline{c}_j xj\right)+\lambda_2\left(\sum_{j=1}^{n}\overline{c}_j xj\right)$$

is introduced to obtain some non dominated solutions. Here $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$, $\lambda_1+\lambda_2=1$ are the weight of the left and right end point of Z respectively.

By assuming $\lambda_1=1$ and $\lambda_2=0$ regarded as optimistic and $\lambda_1=0$, $\lambda_2=1$ regarded as pessimistic opinion of minimizing Z because the stature is best and worst respectively.

Suppose the decision maker is optimistic one can reduce the linear b_i objective programming model (4) as. Min {Left limit of the interval objective function}

Sub:

To {Set of feasibility constraints}

Now consider the inequality constraints of model (2) are modified in two way.

(i) $Ax \leq B$
(ii) $Ax \geq B$

Where $A = [\underline{a}, \overline{a}]$ and $B = [\underline{b}\overline{b}]$

An equivalent form of the interval in equality relation (1) is stated as

$$Ax \leq B \Leftrightarrow \begin{cases} G\left(Ax \geq B\right) > 0 \\ \overline{G}\left(B < Ax\right) \leq \propto \in [0,1] \end{cases} \quad (5)$$

By the use of definition (1) and remark (1) in the relation (5) we have

$$Ax \leq B \Leftrightarrow \begin{cases} \underline{a}x < \overline{b} \quad or \quad \overline{b} - \underline{a}x \geq \in \\ \overline{a}x - \overline{b} \leq \alpha\left(\overline{b} - \underline{b}\right) + \alpha\left(\overline{a} - \underline{a}\right)x, \in > 0 \end{cases} \quad (6)$$

Similarly the interval inequality relation (ii), becomes

$$Ax \geq B \Leftrightarrow \begin{cases} G\left(Ax \geq B\right) > 0 \\ \underline{G}\left(B > Ax\right) \leq \propto \in [0,1] \end{cases} \quad (7)$$

Again using the definition and remark of section (2) in (7) and get

$$Ax \geq B \Leftrightarrow \left\{ \begin{array}{l} \overline{a}x < \underline{b} \quad or \quad \overline{ax} - \underline{b} > \in, \in > 0 \\ \underline{b} - \underline{a}x \leq \alpha\left(\overline{b} - \underline{b}\right) + \alpha\left(\overline{a} - \underline{a}\right)x, \in > 0 \end{array} \right\} \tag{8}$$

From above procedure, the models may be formulated as

$$\left.\begin{array}{l} \text{Minimize } \underline{Z} = \sum_{j=1}^{n} \underline{c}_j x_j\ , \\ \text{Subject to} \\ \sum_{j=1}^{n} \overline{a}_{ij}\ x_j > \underline{b}_i \\ \forall i=1,2\ \ldots\ldots m\ v \\ \underline{b}_i - \sum_{j=1}^{n} \underline{a}_{ij}\ x_j \leq \propto (\overline{b}_i - \underline{b}_i) + \alpha \sum_{j=1}^{n} (\overline{a}_{ij} - \underline{a}_{ij})\ x_j,\ \ \forall\ i=1,2\ldots..m \\ \qquad x_j \geq 0,\ \forall j=1,2\ldots..n \end{array}\right\} \tag{9}$$

And

$$\left.\begin{array}{l} \text{Maximize} \\ \underline{Z} = \sum_{j=1}^{n} \left(\overline{c}_j - \underline{c}_j\right) x_j, \\ \text{Subject to} \\ \sum_{j=1}^{n} \underline{a}_{ij}\ xj \leq \overline{b}_i \\ \sum_{j=1}^{n} \overline{a}_{ij} xj - \overline{b}_i \leq \alpha(\overline{b}_i - \underline{b}_i) + \alpha \sum_{j=1}^{n} (\overline{a}_{ij} - \underline{a}_{ij}) x_j\ , X_j \geq 0\ \ \forall j=1,2\ldots..n \end{array}\right\} \tag{10}$$

Again, using the models (9) and (10) in the model (2) which gives a standard form Maximize

$$\left.\begin{array}{ll} Z = \sum_{j=1}^{n} \left(\overline{c}_j - \underline{c}_j\right) xj & \\ \text{Subject to} & \\ \sum_{j=1}^{n} \left(\overline{a}_{ij} - r_{ij}\alpha\right) x_j & \\ \leq \geq r_i^{'}\ \alpha + \overline{b}_i & i=1,\ 2\ldots m \\ & J=1,\ 2\ldots n \\ \alpha \in (0,1) & \end{array}\right\} \tag{11}$$

where $r_{ij} = \overline{a}_{ij} - \underline{a}_{ij}$

and $r_i^1 = \overline{b}_1 - \underline{b}_i$

Numerical Illustration:

Consider an industrial illustration there are three operating units working simultaneously. Each section has 3 machines which produces different type of products. The measurements relating to manufacturing (production) time per unit for each product (minutes) unit capacity (min) and cost of product of each and every product are collected daily from three units. After collecting the data for about one month, choose or calculate the interval format of the data. Interval may be in the forms of (i) Whole interval (ii) confidence interval.

(ii) Confidence Interval

This is a powerful technique to form an interval based on the whole observation. The confidence interval for population mean or that for population variance or for any other population is obtained. In this case, the confidence interval for population mean is computed for each product. The sample mean, the sample standard deviation are calculated the confidence interval is obtained from the relation.

$$P\left[\bar{x}-\frac{s}{\sqrt{n}}t_{\alpha/2} \le \mu \le \bar{x}+\frac{s}{\sqrt{n}}t_{\alpha/2}\right] \le 100\left(1-\alpha/2\right) \tag{12}$$

Where $s^2 = \dfrac{\sum(X-\overline{X})^2}{n-1}$

$t_{\alpha/2} = t_{(n-1)}$d.f on (α/2)

Using range based on t statistic and Welch's method, the optimization interval valued model can be formulated as follows.

Maximize

$$\left.\begin{aligned}
&Z = (32, 54)\ x_1 + (15, 30)\ x_2 + (20, 40)\ x_3 \\
&\textbf{Subject to the constraints} \\
&(14, 15.8)\ x_1 + (8.1, 9.2)\ x_2 + (10.1, 11.4)\ x_3 + \le (375, 430) \\
&(13, 14.5)\ x_1 + (9,10.2)\ x_2 + (8,10)\ x_3 \le (380, 450) \\
&(12, 14)\ x_1 + (8,10)\ x_2 + (10,12)\ x_3 + \le (300,400),\ x_1, x_2, x_3 \ge 0
\end{aligned}\right\} \tag{13}$$

Using the model (11), the optimization model can be formulated from the

IVLPM (13).

Maximize

$Z = 22\ x_1 + 15\ x_2 + 20\ x_3$

Subject to the constraints

$(15.8\text{-}1.8\alpha)\ x_1 + (9.2\text{-}1.1\ \alpha)\ x_2 + (11.4\ \text{-}1.32)\ x_3 \le 430+55\alpha$
$(14.5\text{-}1.52)\ x_1 + (10.2\text{-}1.2\alpha)\ x_2 + (10\text{-}2\alpha)\ x_3 \le 450+70\alpha$
$(14\text{-}22)\ x_1 + (10\text{-}2\alpha)\ x_2 + (12\text{-}2\alpha)\ x_3 \le 400+100\ \alpha,\ \ x_1, x_2, x_3, \ge 0$

In model $\alpha \in (0,1)$ is assumed and fined optimistic threshold by the decision maker. The obtained results from solving model (12) us presented in Table (Whole Interval).

X	X_1	X_2	X_3	Z
0.1	8.72	22.5	10.4	737.34
0.2	12.26	16.20	12.00	752.72
0.3	7.93	23.80	12.75	786.46
0.4	13.157	18.50	11.80	795.30
0.5	10.184	22.50	12.50	811.46

Table-1: Optimum Solution under whole interval

Using confidence interval based on t statistic and Welch's method can be formulated as follows.

Maximize

$Z = (6.76, 9.52)\ x_1 + (8.61, 8.96)\ x_2 + (10.07, 10.52)\ x_3$

Subject to the constraints

$(14.5, 14.90)\ x_1 + (8.451, 8.54)\ x_2 + (10.65, 10.72)\ x_3 \le (298.14, 499.46)$
$(13.639), 14.00)\ x_1 + (9.7, 9.83)\ x_2 + (8.97, 9.4)\ x_3 \le (295.0, 233.0)$
$(12.93, 13.46)\ x_1 + (8.953, 9.50)\ x_2 + (10.86, 11.34)\ x_3 \le (331.096, 353.30)$

Maximize

$Z = 2.76x_1 + 035x_2 + 0.45x_3$

Subject to the Constraints

$(14.90\text{-}\ 0.4\ \alpha)\ x_1 + (8.54 - 0.09\alpha)\ x_1 + (10.72 - 0.07\ \alpha)\ x_3 \le 499.46 + 201.32\ \alpha$
$(14.00 - 0.361\ \alpha)\ x_1 + (9.83\text{-}0.13\alpha\ )\ x_1 + (9.4\text{-}0.43\ \alpha) \le (395+71\ \alpha)$
$(13.46\text{-}0.53\ \alpha)\ x_1 + (9.50\text{-}0.97\alpha)\ x_2 + (11.34 - 0.48\ \alpha)\ x_3 \le (353.30 + 22.3\alpha),\ x_1, x_2, x_3 \ge 0$

In model $\alpha \in (0, 1)$ is assumed and final optimistic there should by the decision maker. The obtained results from solving model 13is represented in Table (Using confidence interval.)

X	X_1	X_2	X_3	Z
0.1	15.4	21.5	23.1	60.42
0.2	16.8	22.0	25.2	65.40
0.3	17.7	24.6	26.7	69.48
0.4	18.2	25.5	28.3	71.89
0.5	19.6	27.4	31.1	77.68

Table-2: Optimum Solution under Confidence interval.

CONCLUSION

Whole Interval and confidence interval of the collected information are used for the coefficients in both objective and constraints as the interval form. The values of the objective function increase in both cases for increasing α. In the case of whole interval function are increasing rapidly. But the values of objective function are slowly and steadily in the case of confidence interval. Hence based in the confidence interval LPP most opted for production analysis. In the case of profit analysis full interval LPP is most suitable model.

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