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A STUDY ON (T, S)-INTUITIONISTIC FUZZY SUBNEARRINGS OF A NEARRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of a (T, S)-intuitionistic fuzzy subnearring of a nearring.

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Key Words: *T-fuzzy subnearring, anti S-fuzzy subnearring, (T, S)-intuitionistic fuzzy subnearring, product.*

INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov[4, 5], as a generalization of the notion of fuzzy set. Azriel Rosenfeld[6] defined the fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [13, 14]. In this paper, we introduce the some Theorems in (T, S)-intuitionistic fuzzy subnearring of a nearring.

1.PRELIMINARIES:

1.1 Definition: A (T, S)-norm is a binary operations T: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ and S: $[0, 1]\times[0, 1] \rightarrow [0, 1]$ satisfying the following requirements;
(i) $T(0, x)= 0$, $T(1, x) = x$ (boundary condition)
(ii) $T(x, y) = T(y, x)$ (commutativity)
(iii) $T(x, T(y, z))= T (T(x,y), z)$(associativity)
(iv) if $x \leq y$ and $w \leq z$, then $T(x, w) \leq T (y, z)$ (monotonicity).
(v) $S(0, x) = x$, $S (1, x) = 1$ (boundary condition)
(vi) $S(x, y) = S (y, x)$(commutativity)
(vii) $S (x, S(y, z))= S (S(x, y), z)$ (associativity)
(viii) if $x \leq y$ and $w \leq z$, then $S (x, w) \leq S (y, z)$ (monotonicity).

1.2 Definition: Let (R, +, .) be a nearring. A fuzzy subset A of R is said to be a T-fuzzy subnearring (fuzzy subnearring with respect to T-norm) of R if it satisfies the following conditions:
(i) $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$
(ii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$ for all x and y in R.

1.3 Definition: Let (R, +, .) be a nearring. An intuitionistic fuzzy subset A of R is said to be an (T, S)-intuitionistic fuzzy subnearring (intuitionistic fuzzy subnearring with respect to (T, S)-norm) of R if it satisfies the following conditions:
(i) $\mu_A(x - y) \geq T (\mu_A(x), \mu_A(y))$
(ii) $\mu_A(xy) \geq T (\mu_A(x), \mu_A(y))$
(iii) $\nu_A(x - y) \leq S (\nu_A(x), \nu_A(y))$
(iv) $\nu_A(xy) \leq S (\nu_A(x), \nu_A(y))$ for all x and y in R.

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1.4 Definition: Let A and B be intuitionistic fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as $A\times B = \{\langle (x, y), \mu_{A\times B}(x, y), \nu_{A\times B}(x, y)\rangle$ / for all x in G and y in H}, where $\mu_{A\times B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $\nu_{A\times B}(x, y) = \max\{\nu_A(x), \nu_B(y)\}$.

1.5 Definition: Let A be an intuitionistic fuzzy subset in a set S, the strongest intuitionistic fuzzy relation on S, that is an intuitionistic fuzzy relation on A is V given by $\mu_V(x, y) = \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_V(x, y) = \max\{\nu_A(x), \nu_A(y)\}$ for all x and y in S.

1.6 Definition: Let (R, +, .) and (R$^{|}$, +, .) be any two nearrings. Let $f : R \to R^{|}$ be any function and A be an (T, S)-intuitionistic fuzzy subnearring in R, V be an (T, S)-intuitionistic fuzzy subnearring in f(R) = R$^{|}$, defined by $\mu_V(y) = \sup\limits_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf\limits_{x\in f^{-1}(y)} \nu_A(x)$ for all x in R and y in R$^{|}$. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, ·) and a in R. Then the pseudo (T, S)-intuitionistic fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$ and $((a\nu_A)^p)(x) = p(a)\nu_A(x)$ for every x in R and for some p in P.

2- PROPERTIES

2.1 Theorem: Intersection of any two (T, S)-intuitionistic fuzzy subnearrings of a nearring R is a (T, S)-intuitionistic fuzzy subnearring of a nearring R.

Proof: Let A and B be any two (T, S)-intuitionistic fuzzy subnearrings of a nearring R and x and y in R. Let A = {(x, $\mu_A(x)$, $\nu_A(x)$) / x∈R} and B = {(x, $\mu_B(x)$, $\nu_B(x)$) / x∈R} and also let C = A∩B = {(x, $\mu_C(x)$, $\nu_C(x)$) / x∈R} where $\min\{\mu_A(x), \mu_B(x)\} = \mu_C(x)$ and $\max\{\nu_A(x), \nu_B(x)\} = \nu_C(x)$. Now $\mu_C(x-y) = \min\{\mu_A(x-y), \mu_B(x-y)\} \ge \min\{T(\mu_A(x), \mu_A(y)), T(\mu_B(x), \mu_B(y))\} \ge T(\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}) = T(\mu_C(x), \mu_C(y))$. Therefore $\mu_C(x-y) \ge T(\mu_C(x), \mu_C(y))$ for all x and y in R. And $\mu_C(xy) = \min\{\mu_A(xy), \mu_B(xy)\} \ge \min\{T(\mu_A(x), \mu_A(y)), T(\mu_B(x), \mu_B(y))\} \ge T(\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}) = T(\mu_C(x), \mu_C(y))$. Therefore $\mu_C(xy) \ge T(\mu_C(x), \mu_C(y))$ for all x and y in R. Now $\nu_C(x-y) = \max\{\nu_A(x-y), \nu_B(x-y)\} \le \max\{S(\nu_A(x), \nu_A(y)), S(\nu_B(x), \nu_B(y))\} \le S(\max\{\nu_A(x), \nu_B(x)\}, \max\{\nu_A(y), \nu_B(y)\}) = S(\nu_C(x), \nu_C(y))$. Therefore $\nu_C(x-y) \le S(\nu_C(x), \nu_C(y))$ for all x and y in R. And $\nu_C(xy) = \max\{\nu_A(xy), \nu_B(xy)\} \le \max\{S(\nu_A(x), \nu_A(y)), S(\nu_B(x), \nu_B(y))\} \le S(\max\{\nu_A(x), \nu_B(x)\}, \max\{\nu_A(y), \nu_B(y)\}) = S(\nu_C(x), \nu_C(y))$. Therefore $\nu_C(xy) \le S(\nu_C(x), \nu_C(y))$ for all x and y in R. Therefore C is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.2 Theorem: The intersection of a family of (T, S)-intuitionistic fuzzy subnearrings of nearring R is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

Proof: It is trivial.

2.3 Theorem: If A and B are any two (T, S)-intuitionistic fuzzy subnearrings of the nearrings R_1 and R_2 respectively, then A×B is an (T, S)-intuitionistic fuzzy subnearring of $R_1\times R_2$.

Proof: Let A and B be two (T, S)-intuitionistic fuzzy subnearrings of the nearrings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1\times R_2$. Now $\mu_{A\times B}[(x_1, y_1)-(x_2, y_2)] = \mu_{A\times B}(x_1-x_2, y_1-y_2) = \min\{\mu_A(x_1-x_2), \mu_B(y_1-y_2)\} \ge \min\{T(\mu_A(x_1), \mu_A(x_2)), T(\mu_B(y_1), \mu_B(y_2))\} \ge T(\min\{\mu_A(x_1), \mu_B(y_1)\}, \min\{\mu_A(x_2), \mu_B(y_2)\}) = T(\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2))$. Therefore $\mu_{A\times B}[(x_1, y_1) -(x_2, y_2)] \ge T(\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2))$. Also $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \mu_{A\times B}(x_1x_2, y_1y_2) = \min\{\mu_A(x_1x_2), \mu_B(y_1y_2)\} \ge \min\{T(\mu_A(x_1), \mu_A(x_2)), T(\mu_B(y_1), \mu_B(y_2))\} \ge T(\min\{\mu_A(x_1), \mu_B(y_1)\}, \min\{\mu_A(x_2), \mu_B(y_2)\}) = T(\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2))$. Therefore $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)] \ge T(\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2))$. Now $\nu_{A\times B}[(x_1, y_1) -(x_2, y_2)] = \nu_{A\times B}(x_1-x_2, y_1- y_2) = \max\{\nu_A(x_1-x_2), \nu_B(y_1-y_2)\} \le \max\{S(\nu_A(x_1), \nu_A(x_2)), S(\nu_B(y_1), \nu_B(y_2))\} \le S(\max\{\nu_A(x_1), \nu_B(y_1)\}, \max\{\nu_A(x_2), \nu_B(y_2)\}) = S(\nu_{A\times B}(x_1, y_1), \nu_{A\times B}(x_2, y_2))$. Therefore $\nu_{A\times B}[(x_1, y_1)-(x_2, y_2)] \le S(\nu_{A\times B}(x_1, y_1), \nu_{A\times B}(x_2, y_2))$. Also $\nu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \nu_{A\times B}(x_1x_2, y_1y_2) = \max\{\nu_A(x_1x_2), \nu_B(y_1y_2)\} \le \max\{S(\nu_A(x_1), \nu_A(x_2)), S(\nu_B(y_1), \nu_B(y_2))\} \le S(\max\{\nu_A(x_1), \nu_B(y_1)\}, \max\{\nu_A(x_2), \nu_B(y_2)\}) = S(\nu_{A\times B}(x_1, y_1), \nu_{A\times B}(x_2, y_2))$. Therefore $\nu_{A\times B}[(x_1, y_1)(x_2, y_2)] \le S(\nu_{A\times B}(x_1, y_1), \nu_{A\times B}(x_2, y_2))$. Hence A×B is an (T, S)-intuitionistic fuzzy subnearring of nearring of $R_1\times R_2$.

2.4 Theorem: If A is a (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +,·), then $\mu_A(x) \leq \mu_A(0)$ and $\nu_A(x) \geq \nu_A(0)$ for x in R, the identity element 0 in R.

Proof: For x in R and 0 is the identity element of R. Now $\mu_A(0) = \mu_A(x-x) \geq T(\mu_A(x), \mu_A(x)) \geq \mu_A(x)$ for all x in R. So $\mu_A(x) \leq \mu_A(0)$. And $\nu_A(0) = \nu_A(x-x) \leq S(\nu_A(x), \nu_A(x)) \leq \nu_A(x)$ for all x in R. So $\nu_A(x) \geq \nu_A(0)$.

2.5 Theorem: Let A and B be (T, S)-intuitionistic fuzzy subnearring of the nearrings R_1 and R_2 respectively. Suppose that 0 and 0_l are the identity element of R_1 and R_2 respectively. If A×B is an (T, S)-intuitionistic fuzzy subnearring of $R_1 \times R_2$, then at least one of the following two statements must hold. (i) $\mu_B(0_l) \geq \mu_A(x)$ and $\nu_B(0_l) \leq \nu_A(x)$ for all x in R_1 (ii) $\mu_A(0) \geq \mu_B(y)$ and $\nu_A(0) \leq \nu_B(y)$ for all y in R_2.

Proof: Let A×B be an (T, S)-intuitionistic fuzzy subnearring of $R_1 \times R_2$. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in R_1 and b in R_2 such that $\mu_A(a) > \mu_B(0_l)$, $\nu_A(a) < \nu_B(0_l)$ and $\mu_B(b) > \mu_A(0)$, $\nu_B(b) < \nu_A(0)$. We have $\mu_{A\times B}(a, b) = \min\{\mu_A(a), \mu_B(b)\} > \min\{\mu_B(0_l), \mu_A(0)\} = \min\{\mu_A(0), \mu_B(0_l)\} = \mu_{A\times B}(0, 0_l)$. And $\nu_{A\times B}(a, b) = \max\{\nu_A(a), \nu_B(b)\} < \max\{\nu_B(0_l), \nu_A(0)\} = \max\{\nu_A(0), \nu_B(0_l)\} = \nu_{A\times B}(0, 0_l)$. Thus A×B is not an (T, S)-intuitionistic fuzzy subnearring of $R_1 \times R_2$. Hence either $\mu_B(0_l) \geq \mu_A(x)$ and $\nu_B(0_l) \leq \nu_A(x)$ for all x in R_1 or $\mu_A(0) \geq \mu_B(y)$ and $\nu_A(0) \leq \nu_B(y)$ for all y in R_2.

2.6 Theorem: Let A and B be two intuitionistic fuzzy subsets of the nearrings R_1 and R_2 respectively and A×B is an (T, S)-intuitionistic fuzzy subnearring of $R_1 \times R_2$. Then the following are true:
(i) if $\mu_A(x) \leq \mu_B(0_l)$ and $\nu_A(x) \geq \nu_B(0_l)$, then A is an (T, S)-intuitionistic fuzzy subnearring of R_1.
(ii) if $\mu_B(x) \leq \mu_A(0)$ and $\nu_B(x) \geq \nu_A(0)$, then B is an (T, S)-intuitionistic fuzzy subnearring of R_2.
(iii) either A is an (T, S)-intuitionistic fuzzy subnearring of R_1 or B is an (T, S)-intuitionistic fuzzy subnearring of R_2.

Proof: Let A×B be an (T, S)-intuitionistic fuzzy subnearring of $R_1 \times R_2$ and x and y in R_1 and 0_l in R_2. Then $(x, 0_l)$ and $(y, 0_l)$ are in $R_1 \times R_2$. Now using the property that $\mu_A(x) \leq \mu_B(0_l)$ and $\nu_A(x) \geq \nu_B(0_l)$ for all x in R_1. We get $\mu_A(x-y) = \min\{\mu_A(x-y), \mu_B(0_l-0_l)\} = \mu_{A\times B}((x-y), (0_l-0_l)) = \mu_{A\times B}[(x, 0_l) -(y, 0_l)] \geq T(\mu_{A\times B}(x, 0_l), \mu_{A\times B}(y, 0_l)) = T(\min\{\mu_A(x), \mu_B(0_l)\}, \min\{\mu_A(y), \mu_B(0_l)\}) = T(\mu_A(x), \mu_A(y))$. Therefore $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$ for all x and y in R_1. Also $\mu_A(xy) = \min\{\mu_A(xy), \mu_B(0_l0_l)\} = \mu_{A\times B}((xy), (0_l0_l)) = \mu_{A\times B}[(x, 0_l)(y, 0_l)] \geq T(\mu_{A\times B}(x, 0_l), \mu_{A\times B}(y, 0_l)) = T(\min\{\mu_A(x), \mu_B(0_l)\}, \min\{\mu_A(y), \mu_B(0_l)\}) = T(\mu_A(x), \mu_A(y))$. Therefore $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$, for all x and y in R_1. And $\nu_A(x-y) = \max\{\nu_A(x-y), \nu_B(0_l-0_l)\} = \nu_{A\times B}((x-y), (0_l-0_l)) = \nu_{A\times B}[(x, 0_l)-(y, 0_l)] \leq S(\nu_{A\times B}(x, 0_l), \nu_{A\times B}(y, 0_l)) = S(\max\{\nu_A(x), \nu_B(0_l)\}, \max\{\nu_A(y), \nu_B(0_l)\}) = S(\nu_A(x), \nu_A(y))$. Therefore $\nu_A(x-y) \leq S(\nu_A(x), \nu_A(y))$ for all x and y in R_1. Also $\nu_A(xy) = \max\{\nu_A(xy), \nu_B(0_l0_l)\} = \nu_{A\times B}((xy), (0_l0_l)) = \nu_{A\times B}[(x, 0_l)(y, 0_l)] \leq S(\nu_{A\times B}(x, 0_l), \nu_{A\times B}(y, 0_l)) = S(\max\{\nu_A(x), \nu_B(0_l)\}, \max\{\nu_A(y), \nu_B(0_l)\}) = S(\nu_A(x), \nu_A(y))$. Therefore $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$, for all x and y in R_1. Hence A is an (T, S)-intuitionistic fuzzy subnearring of R_1. Thus (i) is proved. Now using the property that $\mu_B(x) \leq \mu_A(0)$ and $\nu_B(x) \geq \nu_A(0)$, for all x in R_2, let x and y in R_2 and 0 in R_1. Then (0, x) and (0, y) are in $R_1 \times R_2$.We get $\mu_B(x-y) = \min\{\mu_B(x-y), \mu_A(0-0)\} = \min\{\mu_A(0-0), \mu_B(x-y)\} = \mu_{A\times B}((0-0), (x-y)) = \mu_{A\times B}[(0, x) -(0, y)] \geq T(\mu_{A\times B}(0, x), \mu_{A\times B}(0, y)) = T(\min\{\mu_A(0), \mu_B(x)\}, \min\{\mu_A(0), \mu_B(y)) = T(\mu_B(x), \mu_B(y))$. Therefore $\mu_B(x-y) \geq S(\mu_B(x), \mu_B(y))$ for all x and y in R_2. Also $\mu_B(xy) = \min\{\mu_B(xy), \mu_A(00)\} = \min\{\mu_A(00), \mu_B(xy)\} = \mu_{A\times B}((00), (xy)) = \mu_{A\times B}[(0, x)(0, y)] \geq T(\mu_{A\times B}(0, x), \mu_{A\times B}(0, y)) = T(\min\{\mu_A(0), \mu_B(x)\}, \min\{\mu_A(0), \mu_B(y)\}) = T(\mu_B(x), \mu_B(y))$. Therefore $\mu_B(xy) \geq T(\mu_B(x), \mu_B(y))$ for all x and y in R_2. And $\nu_B(x-y) = \max\{\nu_B(x-y), \nu_A(0-0)\} = \max\{\nu_A(0-0), \nu_B(x-y)\} = \nu_{A\times B}((0-0), (x-y)) = \nu_{A\times B}[(0, x) -(0, y)] \leq S(\nu_{A\times B}(0, x), \nu_{A\times B}(0, y)) = S(\max\{\nu_A(0), \nu_B(x)\}, \max\{\nu_A(0), \nu_B(y)\}) = S(\nu_B(x), \nu_B(y))$. Therefore $\nu_B(x-y) \leq S(\nu_B(x), \nu_B(y))$ for all x and y in R_2. Also $\nu_B(xy) = \max\{\nu_B(xy), \nu_A(00)\} = \max\{\nu_A(00), \nu_B(xy)\} = \nu_{A\times B}((00), (xy)) = \nu_{A\times B}[(0, x)(0, y)] \leq S(\nu_{A\times B}(0, x), \nu_{A\times B}(0, y)) = S(\max\{\nu_A(0), \nu_B(x)\}, \max\{\nu_A(0), \nu_B(y)\}) = S(\nu_B(x), \nu_B(y))$. Therefore $\nu_B(xy) \leq S(\nu_B(x), \nu_B(y))$, for all x and y in R_2. Hence B is an (T, S)-intuitionistic fuzzy subnearring of a nearring R_2. Thus (ii) is proved. (iii) is clear.

2.7 Theorem: Let A be an intuitionistic fuzzy subset of a nearring R and V be the strongest intuitionistic fuzzy relation of R. Then A is an (T, S)-intuitionistic fuzzy subnearring of R if and only if V is an (T, S)-intuitionistic fuzzy subnearring of R×R.

Proof: Suppose that A is an (T, S)-intuitionistic fuzzy subnearring of a nearring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R. We have $\mu_V(x-y) = \mu_V[(x_1, x_2) -(y_1, y_2)] = \mu_V(x_1-y_1, x_2-y_2) = \min\{\mu_A(x_1-y_1), \mu_A(x_2-y_2)\} \geq \min\{T(\mu_A(x_1), \mu_A(y_1)), T(\mu_A(x_2), \mu_A(y_2))\} \geq T(\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\mu_V(x), \mu_V(y))$. Therefore $\mu_V(x-y) \geq T(\mu_V(x), \mu_V(y))$, for all x and y in R×R. And $\mu_V(xy) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(x_1y_1, x_2y_2) = \min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} \geq \min\{T(\mu_A(x_1), \mu_A(y_1)), T(\mu_A(x_2), \mu_A(y_2))\} \geq T(\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\mu_V(x), \mu_V(y))$. Therefore, $\mu_V(xy) \geq T(\mu_V(x), \mu_V(y))$, for all x and y in R×R. We have $\nu_V(x-y) = \nu_V[(x_1, x_2)-(y_1, y_2)] = \nu_V(x_1-y_1, x_2-y_2) = \max\{\nu_A(x_1-y_1), \nu_A(x_2-y_2)\} \leq \max\{S(\nu_A(x_1), \nu_A(y_1)), S(\nu_A(x_2), \nu_A(y_2))\} \leq S(\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}) = S(\nu_V(x_1, x_2), \nu_V(y_1, y_2)) = S(\nu_V(x), \nu_V(y))$. Therefore $\nu_V(x-y) \leq S(\nu_V(x), \nu_V(y))$, for all x and y in R×R. And $\nu_V(xy) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(x_1y_1, x_2y_2) =$

max $\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} \leq \max\{S(\nu_A(x_1), \nu_A(y_1)), S(\nu_A(x_2), \nu_A(y_2))\} \leq S(\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\}) = S(\nu_V(x_1, x_2), \nu_V(y_1, y_2)) = S(\nu_V(x), \nu_V(y))$. Therefore, $\nu_V(xy) \leq S(\nu_V(x), \nu_V(y))$, for all x and y in R×R. This proves that V is an (T, S)-intuitionistic fuzzy subnearring of R×R. Conversely assume that V is an (T, S)-intuitionistic fuzzy subnearring of R×R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R, we have $\min\{\mu_A(x_1-y_1), \mu_A(x_2-y_2)\} = \mu_V(x_1-y_1, x_2-y_2) = \mu_V[(x_1, x_2) -(y_1, y_2)] = \mu_V(x-y) \geq T(\mu_V(x), \mu_V(y)) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\})$. If $x_2 = 0$, $y_2 = 0$, we get, $\mu_A(x_1- y_1) \geq T(\mu_A(x_1), \mu_A(y_1))$, for all x_1 and y_1 in R. And $\min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} = \mu_V(x_1y_1, x_2y_2) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(xy) \geq T(\mu_V(x), \mu_V(y)) = T(\mu_V(x_1, x_2), \mu_V(y_1, y_2)) = T(\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\})$. If $x_2 = 0$, $y_2 = 0$, we get $\mu_A(x_1y_1) \geq T(\mu_A(x_1), \mu_A(y_1))$, for all x_1 and y_1 in R. We have $\max\{\nu_A(x_1-y_1), \nu_A(x_2-y_2)\} = \nu_V(x_1- y_1, x_2- y_2) = \nu_V[(x_1, x_2) -(y_1, y_2)] = \nu_V(x-y) \leq S(\nu_V(x), \nu_V(y)) = S(\nu_V(x_1, x_2), \nu_V(y_1, y_2)) = S(\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\})$. If $x_2 = 0$, $y_2 = 0$, we get $\nu_A(x_1-y_1) \leq S(\nu_A(x_1), \nu_A(y_1))$ for all x_1 and y_1 in R. And $\max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} = \nu_V(x_1y_1, x_2y_2) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(xy) \leq S(\nu_V(x), \nu_V(y)) = S(\nu_V(x_1, x_2), \nu_V(y_1, y_2)) = S(\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_A(y_1), \nu_A(y_2)\})$. If $x_2 = 0$, $y_2 = 0$, we get $\nu_A(x_1y_1) \leq S(\nu_A(x_1), \nu_A(y_1))$, for all x_1 and y_1 in R. Therefore A is an (T, S)-intuitionistic fuzzy subnearring of R.

2.8 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, .), then $H = \{x \,/\, x\in R: \mu_A(x) = 1, \nu_A(x) = 0\}$ is either empty or is a subnearring of R.

Proof: It is trivial.

2.9 Theorem: If A be an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, .), then (i) if $\mu_A(x-y) = 0$, then either $\mu_A(x) = 0$ or $\mu_A(y) = 0$ for all x and y in R. (ii) if $\mu_A(xy) = 0$, then either $\mu_A(x) = 0$ or $\mu_A(y) = 0$ for all x and y in R. (iii) if $\nu_A(x-y) = 1$, then either $\nu_A(x) = 1$or $\nu_A(y) = 1$ for all x and y in R. (iv) if $\nu_A(xy) = 1$, then either $\nu_A(x) = 1$or $\nu_A(y) = 1$ for all x and y in R.

Proof: It is trivial.

2.10 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring (R,+, .), then $\Box A$ is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R. Consider $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle\}$, for all x in R, we take $\Box A = B = \{\langle x, \mu_B(x), \nu_B(x)\rangle\}$, where $\mu_B(x) = \mu_A(x)$, $\nu_B(x) = 1- \mu_A(x)$. Clearly $\mu_B(x-y) \geq T(\mu_B(x), \mu_B(y))$ for all x and y in R and $\mu_B(xy) \geq T(\mu_B(x), \mu_B(y))$ for all x and y in R. Since A is an (T, S)-intuitionistic fuzzy subnearring of R, we have $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$ for all x and y in R, which implies that $1- \nu_B(x-y) \geq T((1- \nu_B(x)), (1- \nu_B(y)))$, which implies that $\nu_B(x-y) \leq 1- T((1- \nu_B(x)), (1- \nu_B(y))) \leq S(\nu_B(x), \nu_B(y))$. Therefore $\nu_B(x-y) \leq S(\nu_B(x), \nu_B(y))$, for all x and y in R. And $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$ for all x and y in R, which implies that $1- \nu_B(xy) \geq T((1-\nu_B(x)), (1-\nu_B(y)))$ which implies that $\nu_B(xy) \leq 1- T((1- \nu_B(x)), (1- \nu_B(y))) \leq S(\nu_B(x), \nu_B(y))$. Therefore $\nu_B(xy) \leq S(\nu_B(x), \nu_B(y))$ for all x and y in R. Hence $B = \Box A$ is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.11 Theorem: If A is an (T, S)-intuitionistic fuzzy subnearring of a nearring (R, +, .), then $\Diamond A$ is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R.That is $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle\}$ for all x in R. Let $\Diamond A = B = \{\langle x, \mu_B(x), \nu_B(x)\rangle\}$ where $\mu_B(x) =1-\nu_A(x)$, $\nu_B(x) = \nu_A(x)$. Clearly $\nu_B(x-y) \leq S(\nu_B(x), \nu_B(y))$ for all x and y in R and $\nu_B(xy) \leq S(\nu_B(x), \nu_B(y))$ for all x and y in R. Since A is an (T, S)-intuitionistic fuzzy subnearring of R, we have $\nu_A(x-y) \leq S(\nu_A(x), \nu_A(y))$ for all x and y in R, which implies that $1-\mu_B(x-y) \leq S((1- \mu_B(x)), (1- \mu_B(y)))$ which implies that $\mu_B(x-y) \geq 1- S((1- \mu_B(x)), (1- \mu_B(y))) \geq T(\mu_B(x), \mu_B(y))$. Therefore $\mu_B(x-y) \geq T(\mu_B(x), \mu_B(y))$ for all x and y in R. And $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$ for all x and y in R, which implies that $1-\mu_B(xy) \leq S((1-\mu_B(x)), (1-\mu_B(y)))$ which implies that $\mu_B(xy) \geq 1-S((1-\mu_B(x)), (1-\mu_B(y))) \geq T(\mu_B(x), \mu_B(y))$. Therefore $\mu_B(xy) \geq T(\mu_B(x), \mu_B(y))$ for all x and y in R. Hence $B = \Diamond A$ is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.12 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearing (R, +, .), then the pseudo (T, S)-intuitionistic fuzzy coset $(aA)^p$ is an (T, S)-intuitionistic fuzzy subnearring of a nearring R, for every a in R.

Proof: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring R.
For every x and y in R, we have $((a\mu_A)^p)(x-y) = p(a)\mu_A(x-y) \geq p(a)T((\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Therefore $((a\mu_A)^p)(x-y) \geq T(((a\,\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Now $((a\mu_A)^p)(xy) = p(a)\mu_A(xy) \geq p(a)T(\mu_A(x), \mu_A(y)) = T(p(a)\mu_A(x), p(a)\mu_A(y)) = T(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Therefore $((a\mu_A)^p)(xy) \geq T(((a\,\mu_A)^p)(x), ((a\mu_A)^p)(y))$. For every x and y in R, we have $((a\nu_A)^p)(x-y) = p(a)\nu_A(x-y) \leq p(a)S((\nu_A(x), \nu_A(y)) = S(p(a)\nu_A(x), p(a)\nu_A(y)) = S(((a\nu_A)^p)(x), ((a\nu_A)^p)(y))$. Therefore $((a\nu_A)^p)(x-y) \leq S(((a\nu_A)^p)(x), ((a\nu_A)^p)(y))$.

Now $((a\nu_A)^p)(xy) = p(a)\nu_A(xy) \le p(a)\ S(\nu_A(x), \nu_A(y)) = S(p(a)\nu_A(x), p(a)\nu_A(y)) = S(((a\nu_A)^p)(x), ((a\nu_A)^p)(y))$. Therefore $((a\nu_A)^p)(xy) \le S(((a\nu_A)^p)(x), ((a\nu_A)^p)(y))$. Hence $(aA)^p$ is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

In the following Theorem ◦ is the composition operation of functions:

2.13 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H and f is an isomorphism from a nearring R onto H. Then A◦f is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let x and y in R and A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H. Then we have $(\mu_A\circ f)(x-y) = \mu_A(f(x-y)) = \mu_A(f(x) - f(y)) \ge T(\mu_A(f(x)), \mu_A(f(y))) = T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$ which implies that $(\mu_A\circ f)(x-y) \ge T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$. And $(\mu_A\circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) \ge T(\mu_A(f(x)), \mu_A(f(y))) = T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$ which implies that $(\mu_A\circ f)(xy) \ge T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$. Then we have $(\nu_A\circ f)(x-y) = \nu_A(f(x-y)) = \nu_A(f(x) - f(y)) \le S(\nu_A(f(x)), \nu_A(f(y))) = S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$ which implies that $(\nu_A\circ f)(x-y) \le S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$. And $(\nu_A\circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(x)f(y)) \le S(\nu_A(f(x)), \nu_A(f(y))) = S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$ which implies that $(\nu_A\circ f)(xy) \le S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$. Therefore (A◦f) is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.14 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H and f is an anti-isomorphism from a nearring R onto H. Then A◦f is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let x and y in R and A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H. Then we have $(\mu_A\circ f)(x-y) = \mu_A(f(x-y)) = \mu_A(f(y)-f(x)) \ge T(\mu_A(f(x)), \mu_A(f(y))) = T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$ which implies that $(\mu_A\circ f)(x-y) \ge T(\mu_A\circ f)(x), (\mu_A\circ f)(y))$. And $(\mu_A\circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) \ge T(\mu_A(f(x)), \mu_A(f(y))) = T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$ which implies that $(\mu_A\circ f)(xy) \ge T((\mu_A\circ f)(x), (\mu_A\circ f)(y))$. Then we have $(\nu_A\circ f)(x-y) = \nu_A(f(x-y)) = \nu_A(f(y)-f(x)) \le S(\nu_A(f(x)), \nu_A(f(y))) = S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$ which implies that $(\nu_A\circ f)(x-y) \le S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$.
And $(\nu_A\circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(y)f(x)) \le S(\nu_A(f(x)), \nu_A(f(y))) = S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$, which implies that $(\nu_A\circ f)(xy) \le S((\nu_A\circ f)(x), (\nu_A\circ f)(y))$. Therefore A◦f is an (T, S)-intuitionistic fuzzy subnearring of the nearring R.

2.15 Theorem: Let (R, +, .) and $(R^I, +, .)$ be any two nearrings. The homomorphic image of an (T, S)-intuitionistic fuzzy subnearring of R is an (T, S)-intuitionistic fuzzy subnearring of R^I.

Proof: Let (R, +, .) and $(R^I, +, .)$ be any two nearrings. Let $f : R \to R^I$ be a homomorphism. Let V = f(A) where A is an (T, S)-intuitionistic fuzzy subnearring of R. We have to prove that V is an (T, S)-intuitionistic fuzzy subnearring of R^I. Now for f(x), f(y) in R^I, $\mu_v(f(x)-f(y)) = \mu_v(f(x-y)) \ge \mu_A(x-y) \ge T(\mu_A(x), \mu_A(y))$ which implies that $\mu_v(f(x)-f(y)) \ge T(\mu_v(f(x)), \mu_v(f(y)))$. Again $\mu_v(f(x)f(y)) = \mu_v(f(xy)) \ge \mu_A(xy) \ge T(\mu_A(x), \mu_A(y))$ which implies that $\mu_v(f(x)f(y)) \ge T(\mu_v(f(x)), \mu_v(f(y)))$. And $\nu_v(f(x)-f(y)) = \nu_v(f(x-y)) \le \nu_A(x-y) \le S(\nu_A(x), \nu_A(y))$. Therefore $\nu_v(f(x)-f(y)) \le S(\nu_v(f(x)), \nu_v(f(y)))$. Again $\nu_v(f(x)f(y))=\nu_v(f(xy)) \le \nu_A(xy) \le S(\nu_A(x), \nu_A(y))$ which implies that $\nu_v(f(x)f(y)) \le S(\nu_v(f(x)), \nu_v(f(y)))$. Hence V is an (T, S)-intuitionistic fuzzy subnearring of R^I.

2.16 Theorem: Let (R, +, .) and $(R^I, +, .)$ be any two nearrings. The homomorphic preimage of an (T, S)-intuitionistic fuzzy subnearring of R^I is a (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let V = f(A), where V is an (T, S)-intuitionistic fuzzy subnearring of R^I. We have to prove that A is an (T, S)-intuitionistic fuzzy subnearring of R. Let x and y in R. Then $\mu_A(x-y) = \mu_v(f(x-y)) = \mu_v(f(x)-f(y)) \ge T(\mu_v(f(x)), \mu_v(f(y))) = T(\mu_A(x), \mu_A(y))$ which implies that $\mu_A(x-y) \ge T(\mu_A(x), \mu_A(y))$. Again $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(x)f(y)) \ge T(\mu_v(f(x)), \mu_v(f(y))) = T(\mu_A(x), \mu_A(y))$ which implies that $\mu_A(xy) \ge T(\mu_A(x), \mu_A(y))$. And $\nu_A(x-y) = \nu_v(f(x-y)) = \nu_v(f(x)-f(y)) \le S(\nu_v(f(x)), \nu_v(f(y))) = S(\nu_A(x), \nu_A(y))$ which implies that $\nu_A(x-y) \le S(\nu_A(x), \nu_A(y))$. Again $\nu_A(xy) = \nu_v(f(xy)) = \nu_v(f(x)f(y)) \le S(\nu_v(f(x)), \nu_v(f(y))) = S(\nu_A(x), \nu_A(y))$ which implies that $\nu_A(xy) \le S(\nu_A(x), \nu_A(y))$. Hence A is an (T, S)-intuitionistic fuzzy subnearring of R.

2.17 Theorem: Let (R, +, .) and $(R^I, +, .)$ be any two nearrings. The anti-homomorphic image of an (T, S)-intuitionistic fuzzy subnearring of R is an (T, S)-intuitionistic fuzzy subnearring of R^I.

Proof: Let (R, +, .) and $(R^I, +, .)$ be any two nearrings. Let $f : R \to R^I$ be an anti-homomorphism. Then f(x+y) = f(y) + f(x) and f(xy) = f(y)f(x) for all x and y in R. Let V = f(A) where A is an (T, S)-intuitionistic fuzzy subnearring of R. We have to prove that V is an (T, S)-intuitionistic fuzzy subnearring of R^I. Now for f(x), f(y) in R^I, $\mu_v(f(x)-f(y)) = \mu_v(f(y-x)) \ge \mu_A(y-x) \ge T(\mu_A(y), \mu_A(x)) = T(\mu_A(x), \mu_A(y))$, which implies that $\mu_v(f(x)-f(y)) \ge T(\mu_v(f(x)), \mu_v(f(y)))$. Again $\mu_v(f(x)f(y)) = \mu_v(f(yx)) \ge \mu_A(yx) \ge T(\mu_A(y), \mu_A(x)) = T(\mu_A(x), \mu_A(y))$ which implies that $\mu_v(f(x)f(y)) \ge T(\mu_v(f(x)), \mu_v(f(y)))$. And $\nu_v(f(x)-f(y)) = \nu_v(f(y-x)) \le \nu_A(y-x) \le S(\nu_A(y), \nu_A(x)) = S(\nu_A(x), \nu_A(y))$ which implies that $\nu_v(f(x)-f(y)) \le S(\nu_v(f(x)), \nu_v(f(y)))$. Again $\nu_v(f(x)f(y)) = \nu_v(f(yx)) \le \nu_A(yx) \le S(\nu_A(y), \nu_A(x)) = S(\nu_A(x), \nu_A(y))$ which implies that $\nu_v(f(x)f(y)) \le S(\nu_v(f(x)), \nu_v(f(y)))$. Hence V is an (T, S)-intuitionistic fuzzy subnearring of R^I.

2.18 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two nearrings. The anti-homomorphic preimage of an (T, S)-intuitionistic fuzzy subnearring of R^{l} is an (T, S)-intuitionistic fuzzy subnearring of R.

Proof: Let V = f(A), where V is an (T, S)-intuitionistic fuzzy subnearring of R^{l}. We have to prove that A is an (T, S)-intuitionistic fuzzy subnearring of R. Let x and y in R. Then $\mu_A(x-y)=\mu_v(f(x-y)) = \mu_v(f(y)-f(x)) \geq T(\mu_v(f(y)), \mu_v(f(x))) = T(\mu_v(f(x)), \mu_v(f(y))) = T(\mu_A(x), \mu_A(y))$ which implies that $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$. Again $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(y)f(x)) \geq T(\mu_v(f(y)), \mu_v(f(x))) = T(\mu_v(f(x)), \mu_v(f(y)))= T(\mu_A(x), \mu_A(y))$ which implies that $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$. And $\nu_A(x-y) = \nu_v(f(x-y)) = \nu_v(f(y)-f(x)) \leq S(\nu_v(f(y)), \nu_v(f(x))) = S(\nu_v(f(x)), \nu_v(f(y))) = S(\nu_A(x), \nu_A(y))$ which implies that $\nu_A(x-y) \leq S(\nu_A(x), \nu_A(y))$. Again $\nu_A(xy) = \nu_v(f(xy)) = \nu_v(f(y)f(x)) \leq S(\nu_v(f(y)), \nu_v(f(x))) = S(\nu_v(f(x)), \nu_v(f(y))) = S(\nu_A(x), \nu_A(y))$ which implies that $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$. Hence A is an (T, S)-intuitionistic fuzzy subnearring of R.

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