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BIPOLAR-VALUED MULTI FUZZY SUBSEMIRINGS OF A SEMIRING

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ABSTRACT

In this paper, we study some of the properties of bipolar-valued multi fuzzy subsemiring of a semiring and prove some results on these.

Key Words: *Bipolar-valued fuzzy subset, bipolar-valued multi fuzzy subset, bipolar-valued multi fuzzy subsemiring.*

INTRODUCTION

In 1965, Zadeh [13] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [6]. Lee [8] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [–1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [–1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [8, 9]. Anitha.M.S., Muruganantha Prasad & K.Arjunan[1] defined as Bipolar-valued fuzzy subgroups of a group. We introduce the concept of bipolar-valued multi fuzzy subsemiring and established some results.

1. PRELIMINARIES

1.1 Definition: A bipolar-valued fuzzy set (BVFS) A in X is defined as an object of the form A = {< x, $A^+(x)$, $A^-(x)$ >/ x∈X}, where $A^+: X\to [0, 1]$ and $A^-: X\to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued fuzzy set A. If $A^+(x) \neq 0$ and $A^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^+(x) = 0$ and $A^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.2 Example: A = {< a, 0.5, –0.3 >, < b, 0.1, –0.7 >, < c, 0.5, –0.4 >} is a bipolar-valued fuzzy subset of X= {a, b, c}.

1.3 Definition: A bipolar-valued multi fuzzy set (BVMFS) A in X is defined as an object of the form A = {< x, $A_i^+(x)$, $A_i^-(x)$ >/ x∈X}, where $A_i^+: X\to [0, 1]$ and $A_i^- : X\to [-1, 0]$. The positive membership degrees $A_i^+(x)$ denote the satisfaction degree of an element x to the property corresponding to a bipolar-valued multi fuzzy set A and the negative membership degrees $A_i^-(x)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued multi fuzzy set A. If $A_i^+(x) \neq 0$ and $A_i^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A_i^+(x) = 0$ and $A_i^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A_i^+(x) \neq 0$ and $A_i^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X, where i = 1 to n.

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1.4 Example: A = {< a, 0.5, 0,6, 0.3, –0.3, –0.6, –0.5 >, < b, 0.1, 0.4, 0.7, –0.7, –0.3, –0.6 >, < c, 0.5, 0.3, 0.8, –0.4, –0.5, –0.3 >} is a bipolar-valued multi fuzzy subset of X = {a, b, c}.

1.5 Definition: Let R be a semiring. A bipolar-valued multi fuzzy subset A of R is said to be a bipolar-valued multi fuzzy subsemiring of R (BVMFSSR) if the following conditions are satisfied,
(i) $A_i^+(x+y) \geq \min\{A_i^+(x), A_i^+(y)\}$
(ii) $A_i^+(xy) \geq \min\{A_i^+(x), A_i^+(y)\}$
(iii) $A_i^-(x+y) \leq \max\{A_i^-(x), A_i^-(y)\}$
(iv) $A_i^-(xy) \leq \max\{A_i^-(x), A_i^-(y)\}$ for all x and y in R.

1.6 Example: Let R = Z_3 = {0, 1, 2} be a semiring with respect to the ordinary addition and multiplication. Then A = {< 0, 0.5, 0.8, 0.6, – 0.6, – 0.5, – 0.7 >, < 1, 0.4, 0.7, 0.5, – 0.5, – 0.4, –0.6 >, < 2, 0.4, 0.7, 0.5, – 0.5, –0.4,–0.6 >} is a bipolar-valued multi fuzzy subsemiring of R.

1.7 Definition: Let A = $\langle A_i^+, A_i^-\rangle$ and B = $\langle B_i^+, B_i^-\rangle$ be any two bipolar-valued multi fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as A×B = $\{\langle (x, y), (A_i\times B_i)^+(x, y), (A_i\times B_i)^-(x, y)\rangle$ / for all x in G and y in H} where $(A_i\times B_i)^+(x, y) = \min\{A_i^+(x), B_i^+(y)\}$ and $(A_i\times B_i)^-(x, y) = \max\{A_i^-(x), B_i^-(y)\}$ for all x in G and y in H.

1.8 Definition: Let A = $\langle A_i^+, A_i^-\rangle$ be a bipolar-valued multi fuzzy subset in a set S, the strongest bipolar-valued multi fuzzy relation on S, that is a bipolar-valued multi fuzzy relation on A is V = $\{\langle (x, y), V_i^+(x, y), V_i^-(x, y)\rangle$/ x and y in S} given by $V_i^+(x, y) = \min\{A_i^+(x), A_i^+(y)\}$ and $V_i^-(x, y) = \max\{A_i^-(x), A_i^-(y)\}$ for all x and y in S.

1.9 Definition: Let A be a bipolar valued multi fuzzy subset of X. Then the following operations are defined as
(i) $?(A) = \{\langle x, \min\{½, A_i^+(x)\}, \max\{-½, A_i^-(x)\}\rangle$ / for all $x\in X$}.
(ii) $!(A) = \{\langle x, \max\{½, A_i^+(x)\}, \min\{-½, A_i^-(x)\}\rangle$ / for all $x\in X$}.
(iii) $Q_{\alpha,\beta}(A)=\{\langle x, \min\{\alpha, A_i^+(x)\}, \max\{\beta, A_i^-(x)\}\rangle$/ for all $x\in X$ and α in [0, 1], β in [–1, 0]}.
(iv) $P_{\alpha,\beta}(A)=\{\langle x, \max\{\alpha, A_i^+(x)\}, \min\{\beta, A_i^-(x)\}\rangle$/ for all $x\in X$ and α in [0, 1], β in [–1, 0]}.
(v) $G_{\alpha,\beta}(A) =\{\langle x, \alpha A_i^+(x), -\beta A_i^-(x)\}\rangle$ / for all $x\in X$ and α in [0, 1] and β in [–1, 0]}.

1.10 Definition: Let A = $\langle A_i^+, A_i^-\rangle$ be a bipolar valued multi fuzzy subsemiring of a semiring R and a in R. Then the **pseudo bipolar valued multi fuzzy coset** $(aA)^p = \langle (aA_i^+)^p, (aA_i^-)^p\rangle$ is defined by $(aA_i^+)^p(x) = p(a)A_i^+(x)$ and $(aA_i^-)^p(x) = p(a)A_i^-(x)$, for every x in R and for some p in P.

2. PROPERTIES

2.1 Theorem: Let A = $\langle A_i^+, A_i^-\rangle$ be a bipolar-valued multi fuzzy subsemiring of a semiring R. (i) If $A_i^+(x+y) = 0$ then either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ for x and y in R.
(ii) If $A_i^+(xy) = 0$ then either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ for x and y in R.
(iii) If $A_i^-(x+y) = 0$ then either $A_i^-(x) = 0$ or $A_i^-(y) = 0$ for x and y in R.
(iv) If $A_i^-(xy) = 0$ then either $A_i^-(x) = 0$ or $A_i^-(y) = 0$ for x and y in R.

Proof: Let x and y be in R. (i) By the definition $A_i^+(x+y) \geq \min\{A_i^+(x), A_i^+(y)\}$ which implies that $0 \geq \min\{A_i^+(x), A_i^+(y)\}$. Therefore either $A_i^+(x) = 0$ or $A_i^+(y) = 0$. (ii) By the definition $A_i^+(xy) \geq \min\{A_i^+(x), A_i^+(y)\}$ which implies that $0 \geq \min\{A_i^+(x), A_i^+(y)\}$. Therefore either $A_i^+(x) = 0$ or $A_i^+(y) = 0$. (iii) By the definition $A_i^-(x+y) \leq \max\{A_i^-(x), A_i^-(y)\}$ which implies that $0 \leq \max\{A_i^-(x), A_i^-(y)\}$. Therefore either $A_i^-(x) = 0$ or $A_i^-(y) = 0$. (iv) By the definition $A_i^-(xy) \leq \max\{A_i^-(x), A_i^-(y)\}$ which implies that $0 \leq \max\{A_i^-(x), A_i^-(y)\}$. Therefore either $A_i^-(x) = 0$ or $A_i^-(y) = 0$.

2.2 Theorem: If A = $\langle A_i^+, A_i^-\rangle$ is a bipolar-valued multi fuzzy subsemiring of a semiring R then H = $\{x\in R \mid A_i^+(x) = 1, A_i^-(x) = -1\}$ is either empty or is a subsemiring of R.

Proof: If no element satisfies this condition then H is empty. If x and y in H then $A_i^+(x+y) \geq \min\{A_i^+(x), A_i^+(y)\}$ = min {1, 1} = 1. Therefore $A_i^+(x+y) = 1$. And $A_i^+(xy) \geq \min\{A_i^+(x), A_i^+(y)\}$ = min {1, 1} = 1. Therefore $A_i^+(xy) = 1$. Also $A_i^-(x+y) \leq \max\{A_i^-(x), A_i^-(y)\}$= max{–1, –1} = –1. Therefore $A_i^-(x+y) = -1$. And $A_i^-(xy) \leq \max\{A_i^-(x), A_i^-(y)\}$= max {–1, –1} = –1. Therefore $A_i^-(xy) = -1$. That is $x+y\in H$ and $xy\in H$. Hence H is a subsemiring of R. Hence H is either empty or a subsemiring of R.

2.3 Theorem: If A = $\langle A_i^+, A_i^-\rangle$ and B = $\langle B_i^+, B_i^-\rangle$ are two bipolar-valued multi fuzzy subsemirings of a semiring R, then their intersection $A\cap B$ is a bipolar-valued multi fuzzy subsemiring of R.

Proof: Let A = $\{< x, A_i^+(x), A_i^-(x) > / x \in G\}$, B = $\{< x, B_i^+(x), B_i^-(x) > / x \in G\}$. Let C = A$\cap$B and C = $\{< x, C_i^+(x), C_i^-(x) > / x \in G\}$. Now $C_i^+(x+y) = \min\{A_i^+(x+y), B_i^+(x+y)\} \geq \min\{\min\{A_i^+(x), A_i^+(y)\}, \min\{B_i^+(x), B_i^+(y)\}\} \geq \min\{\min\{A_i^+(x), B_i^+(x)\}, \min\{A_i^+(y), B_i^+(y)\}\} = \min\{C_i^+(x), C_i^+(y)\}$. Therefore $C_i^+(x+y) \geq \min\{C_i^+(x), C_i^+(y)\}$. And $C_i^+(xy) = \min\{A_i^+(xy), B_i^+(xy)\} \geq \min\{\min\{A_i^+(x), A_i^+(y)\}, \min\{B_i^+(x), B_i^+(y)\}\} \geq \min\{\min\{A_i^+(x), B_i^+(x)\}, \min\{A_i^+(y), B_i^+(y)\}\} = \min\{C_i^+(x), C_i^+(y)\}$. Therefore $C_i^+(xy) \geq \min\{C_i^+(x), C_i^+(y)\}$. Also $C_i^-(x+y) = \max\{A_i^-(x+y), B_i^-(x+y)\} \leq \max\{\max\{A_i^-(x), A_i^-(y)\}, \max\{B_i^-(x), B_i^-(y)\}\} \leq \max\{\max\{A_i^-(x), B_i^-(x)\}, \max\{A_i^-(y), B^-(y)\}\} = \max\{C_i^-(x), C_i^-(y)\}$. Therefore $C_i^-(x+y) \leq \max\{C_i^-(x), C_i^-(y)\}$. And $C_i^-(xy) = \max\{A_i^-(xy), B_i^-(xy)\} \leq \max\{\max\{A_i^-(x), A_i^-(y)\}, \max\{B_i^-(x), B_i^-(y)\}\} \leq \max\{\max\{A_i^-(x), B_i^-(x)\}, \max\{A_i^-(y), B_i^-(y)\}\} = \max\{C_i^-(x), C_i^-(y)\}$. Therefore $C_i^-(xy) \leq \max\{C_i^-(x), C_i^-(y)\}$. Hence A$\cap$B is a bipolar-valued multi fuzzy subsemiring of R.

2.4 Theorem: The intersection of a family of bipolar-valued multi fuzzy subsemirings of a semiring R is a bipolar-valued multi fuzzy subsemiring of R.

Proof: The theorem can easily prove by **Theorem 2.3.**

2.5 Theorem: If $A = \langle A_i^+, A_i^-\rangle$ and $B = \langle B_i^+, B_i^-\rangle$ are any two bipolar-valued multi fuzzy subsemirings of the semirings R_1 and R_2 respectively, then $A\times B = \langle (A_i\times B_i)^+, (A_i\times B_i)^-\rangle$ is a bipolar-valued multi fuzzy subsemiring of $R_1\times R_2$.

Proof: Let A and B be two bipolar-valued multi fuzzy subsemirings of the semirings R_1 and R_2 respectively. Let x_1, x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1\times R_2$. Now, $(A_i\times B_i)^+[(x_1, y_1) + (x_2, y_2)] = (A_i\times B_i)^+(x_1+x_2, y_1+y_2) = \min\{A_i^+(x_1+x_2), B_i^+(y_1+y_2)\} \geq \min\{\min\{A_i^+(x_1), A_i^+(x_2)\}, \min\{B_i^+(y_1), B_i^+(y_2)\}\} = \min\{\min\{A_i^+(x_1), B_i^+(y_1)\}, \min\{A_i^+(x_2), B_i^+(y_2)\}\} = \min\{(A_i\times B_i)^+(x_1, y_1), (A_i\times B_i)^+(x_2, y_2)\}$. Therefore $(A_i\times B_i)^+[(x_1, y_1) + (x_2, y_2)] \geq \min\{(A_i\times B_i)^+(x_1, y_1), (A_i\times B_i)^+(x_2, y_2)\}$. And $(A_i\times B_i)^+[(x_1, y_1)(x_2, y_2)] = (A_i\times B_i)^+(x_1x_2, y_1y_2) = \min\{A_i^+(x_1x_2), B_i^+(y_1y_2)\} \geq \min\{\min\{A_i^+(x_1), A_i^+(x_2)\}, \min\{B_i^+(y_1), B_i^+(y_2)\}\} = \min\{\min\{A_i^+(x_1), B_i^+(y_1)\}, \min\{A_i^+(x_2), B_i^+(y_2)\}\} = \min\{(A_i\times B_i)^+(x_1, y_1), (A_i\times B_i)^+(x_2, y_2)\}$. Therefore, $(A_i\times B_i)^+[(x_1, y_1)(x_2, y_2)] \geq \min\{(A_i\times B_i)^+(x_1, y_1), (A_i\times B_i)^+(x_2, y_2)\}$. Also $(A_i\times B_i)^-[(x_1, y_1) + (x_2, y_2)] = (A_i\times B_i)^-(x_1+x_2, y_1+y_2) = \max\{A_i^-(x_1+x_2), B_i^-(y_1+y_2)\} \leq \max\{\max\{A_i^-(x_1), A_i^-(x_2)\}, \max\{B_i^-(y_1), B_i^-(y_2)\}\} = \max\{\max\{A_i^-(x_1), B_i^-(y_1)\}, \max\{A_i^-(x_2), B_i^-(y_2)\}\} = \max\{(A_i\times B_i)^-(x_1, y_1), (A_i\times B_i)^-(x_2, y_2)\}$. Therefore $(A_i\times B_i)^-[(x_1, y_1)+(x_2, y_2)] \leq \max\{(A_i\times B_i)^-(x_1, y_1), (A_i\times B_i)^-(x_2, y_2)\}$. And $(A_i\times B_i)^-[(x_1, y_1)(x_2, y_2)] = (A_i\times B_i)^-(x_1x_2, y_1y_2) = \max\{A_i^-(x_1x_2), B_i^-(y_1y_2)\} \leq \max\{\max\{A_i^-(x_1), A_i^-(x_2)\}, \max\{B_i^-(y_1), B_i^-(y_2)\}\} = \max\{\max\{A_i^-(x_1), B_i^-(y_1)\}, \max\{A_i^-(x_2), B_i^-(y_2)\}\} = \max\{(A_i\times B_i)^-(x_1, y_1), (A_i\times B_i)^-(x_2, y_2)\}$. Therefore $(A_i\times B_i)^-[(x_1, y_1)(x_2, y_2)] \leq \max\{(A_i\times B_i)^-(x_1, y_1), (A_i\times B_i)^-(x_2, y_2)\}$. Hence A×B is a bipolar-valued multi fuzzy subsemiring of $R_1\times R_2$.

2.6 Theorem: Let $A = \langle A_i^+, A_i^-\rangle$ be a bipolar-valued multi fuzzy subset of a semiring R and $V = \langle V_i^+, V_i^-\rangle$ be the strongest bipolar-valued multi fuzzy relation of R. If A is a bipolar-valued multi fuzzy subsemiring of R, then V is a bipolar-valued multi fuzzy subsemiring of R×R.

Proof: Suppose that A is a bipolar-valued multi fuzzy subsemiring of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R. We have $V_i^+(x+y) = V_i^+[(x_1, x_2)+(y_1, y_2)] = V_i^+(x_1+y_1, x_2+y_2) = \min\{A_i^+(x_1+y_1), A_i^+(x_2+y_2)\} \geq \min\{\min\{A_i^+(x_1), A_i^+(y_1)\}, \min\{A_i^+(x_2), A_i^+(y_2)\}\} = \min\{\min\{A_i^+(x_1), A_i^+(x_2)\}, \min\{A_i^+(y_1), A_i^+(y_2)\}\} = \min\{V_i^+(x_1, x_2), V_i^+(y_1, y_2)\} = \min\{V_i^+(x), V_i^+(y)\}$. Therefore $V_i^+(x+y) \geq \min\{V_i^+(x), V_i^+(y)\}$ for all x and y in R×R. And $V_i^+(xy) = V_i^+[(x_1, x_2)(y_1, y_2)] = V_i^+(x_1y_1, x_2y_2) = \min\{A_i^+(x_1y_1), A_i^+(x_2y_2)\} \geq \min\{\min\{A_i^+(x_1), A_i^+(y_1)\}, \min\{A_i^+(x_2), A_i^+(y_2)\}\} = \min\{\min\{A_i^+(x_1), A_i^+(x_2)\}, \min\{A_i^+(y_1), A_i^+(y_2)\}\} = \min\{V_i^+(x_1, x_2), V_i^+(y_1, y_2)\} = \min\{V_i^+(x), V_i^+(y)\}$. Therefore $V_i^+(xy) \geq \min\{V_i^+(x), V_i^+(y)\}$ for all x and y in R×R. Also we have $V_i^-(x+y) = V_i^-[(x_1, x_2)+(y_1, y_2)] = V_i^-(x_1+y_1, x_2+y_2) = \max\{A_i^-(x_1+y_1), A_i^-(x_2+y_2)\} \leq \max\{\max\{A_i^-(x_1), A_i^-(y_1)\}, \max\{A_i^-(x_2), A_i^-(y_2)\}\} = \max\{\max\{A_i^-(x_1), A_i^-(x_2)\}, \max\{A_i^-(y_1), A_i^-(y_2)\}\} = \max\{V_i^-(x_1, x_2), V_i^-(y_1, y_2)\} = \max\{V_i^-(x), V_i^-(y)\}$. Therefore $V_i^-(x+y) \leq \max\{V_i^-(x), V_i^-(y)\}$ for all x, y in R×R. And $V_i^-(xy) = V_i^-[(x_1, x_2)(y_1, y_2)] = V_i^-(x_1y_1, x_2y_2) = \max\{A_i^-(x_1y_1), A_i^-(x_2y_2)\} \leq \max\{\max\{A_i^-(x_1), A_i^-(y_1)\}, \max\{A_i^-(x_2), A_i^-(y_2)\}\} = \max\{\max\{A_i^-(x_1), A_i^-(x_2)\}, \max\{A_i^-(y_1), A_i^-(y_2)\}\} = \max\{V_i^-(x_1, x_2), V_i^-(y_1, y_2)\} = \max\{V_i^-(x), V_i^-(y)\}$. Therefore $V_i^-(xy) \leq \max\{V_i^-(x), V_i^-(y)\}$ for all x, y in R×R. This proves that V is a bipolar-valued multi fuzzy subsemiring of R×R.

2.7 Theorem: Let $A = \langle A_i^+, A_i^-\rangle$ be a bipolar valued multi fuzzy subsemiring of a semiring R. Then the pseudo bipolar valued multi fuzzy coset $(aA)^p = \langle (a\,A_i^+)^p, (a\,A_i^-)^p\rangle$ is a bipolar valued multi fuzzy subsemiring of the semiring R, for every a in R and p in P.

Proof: Let A be a bipolar valued multi fuzzy subsemiring of the semiring R. For every x and y in R, we have $(aA_i^+)^p(x+y) = p(a)A_i^+(x+y) \geq p(a)\min\{A_i^+(x), A_i^+(y)\} = \min\{p(a)A_i^+(x), p(a)A_i^+(y)\} = \min\{(a\,A_i^+)^p(x), (a\,A_i^+)^p(y)\}$. Therefore $(aA_i^+)^p(x+y) \geq \min\{(aA_i^+)^p(x), (aA_i^+)^p(y)\}$ for x and y in R. And $(aA_i^+)^p(xy) = p(a)A_i^+(xy) \geq p(a)\min\{A_i^+(x), A_i^+(y)\} = \min\{p(a)A_i^+(x), p(a)A_i^+(y)\} = \min\{(aA_i^+)^p(x), (aA_i^+)^p(y)\}$. Therefore $(aA_i^+)^p(xy) \geq \min\{(a\,A_i^+)^p(x), (aA_i^+)^p(y)\}$ for x and y in R. Also $(aA_i^-)^p(x+y) = p(a)A_i^-(x+y) \leq p(a)\max\{A_i^-(x), A_i^-(y)\} = \max\{p(a)A_i^-(x), p(a)A_i^-(y)\} = \max\{(aA_i^-)^p(x), (aA_i^-)^p(y)\}$. Therefore $(aA_i^-)^p(x+y) \leq \max\{(aA_i^-)^p(x), (aA_i^-)^p(y)\}$ for x and y in R. And $(aA_i^-)^p(xy) = p(a)A_i^-(xy) \leq p(a)\max\{A_i^-(x), A_i^-(y)\} = \max\{p(a)A_i^-(x), p(a)A_i^-(y)\} = \max\{(aA_i^-)^p(x), (aA_i^-)^p(y)\}$. Therefore

$(aA_i^-)^p(xy) \leq \max\{(aA_i^-)^p(x), (aA_i^-)^p(y)\}$ for x and y in R. Hence $(aA)^p$ is a bipolar valued multi fuzzy subsemiring of the semiring R.

2.8 Theorem: If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar valued multi fuzzy subsemiring of a semiring R, then $?(A) = \langle ?A_i^+, ?A_i^- \rangle$ is a bipolar valued multi fuzzy subsemiring of R.

Proof: For every x and y in R, we have $?A_i^+(x+y) = \min\{½, A_i^+(x+y)\} \geq \min\{½, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{\min\{½, A_i^+(x)\}, \min\{½, A_i^+(y)\}\} = \min\{?A_i^+(x), ?A_i^+(y)\}$. Therefore $?A_i^+(x+y) \geq \min\{?A_i^+(x), ?A_i^+(y)\}$ for all x and y in R. Also $?A_i^+(xy) = \min\{½, A_i^+(xy)\} \geq \min\{½, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{\min\{½, A_i^+(x)\}, \min\{½, A_i^+(y)\}\} = \min\{?A_i^+(x), ?A_i^+(y)\}$. Therefore $?A_i^+(xy) \geq \min\{?A_i^+(x), ?A_i^+(y)\}$ for all x and y in R. And $?A_i^-(x+y) = \max\{-½, A_i^-(x+y)\} \leq \max\{-½, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{\max\{-½, A_i^-(x)\}, \max\{-½, A_i^-(y)\}\} = \max\{?A_i^-(x), ?A_i^-(y)\}$. Therefore $?A_i^-(x+y) \leq \max\{?A_i^-(x), ?A_i^-(y)\}$ for all x and y in R. Also $?A_i^-(xy) = \max\{-½, A_i^-(xy)\} \leq \max\{-½, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{\max\{-½, A_i^-(x)\}, \max\{-½, A_i^-(y)\}\} = \max\{?A_i^-(x), ?A_i^-(y)\}$. Therefore $?A_i^-(xy) \leq \max\{?A_i^-(x), ?A_i^-(y)\}$ for all x and y in R. Hence ?A is a bipolar-valued multi fuzzy subsemiring of R.

2.9 Theorem: If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar-valued multi fuzzy subsemiring of a semiring R, then $!(A) = \langle !A_i^+, !A_i^- \rangle$ is a bipolar-valued multi fuzzy subsemiring of R.

Proof: For every x and y in R, we have $!A_i^+(x+y) = \max\{½, A_i^+(x+y)\} \geq \max\{½, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{\max\{½, A_i^+(x)\}, \max\{½, A_i^+(y)\}\} = \min\{!A_i^+(x), !A_i^+(y)\}$. Therefore $!A_i^+(x+y) \geq \min\{!A_i^+(x), !A_i^+(y)\}$ for all x and y in R. And $!A_i^+(xy) = \max\{½, A_i^+(xy)\} \geq \max\{½, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{\max\{½, A_i^+(x)\}, \max\{½, A_i^+(y)\}\} = \min\{!A_i^+(x), !A_i^+(y)\}$. Therefore $!A_i^+(xy) \geq \min\{!A_i^+(x), !A_i^+(y)\}$ for all x and y in R. Also $!A_i^-(x+y) = \min\{-½, A_i^-(x+y)\} \leq \min\{-½, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{\min\{-½, A_i^-(x)\}, \min\{-½, A_i^-(y)\}\} = \max\{!A_i^-(x), !A_i^-(y)\}$. Therefore $!A_i^-(x+y) \leq \max\{!A_i^-(x), !A_i^-(y)\}$ for all x and y in R. And $!A_i^-(xy) = \min\{-½, A_i^-(xy)\} \leq \min\{-½, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{\min\{-½, A_i^-(x)\}, \min\{-½, A_i^-(y)\}\} = \max\{!A_i^-(x), !A_i^-(y)\}$.

Therefore $!A_i^-(xy) \leq \max\{!A_i^-(x), !A_i^-(y)\}$ for all x and y in R. Hence !A is a bipolar-valued multi fuzzy subsemiring of R.

2.10 Theorem: If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar-valued multi fuzzy subsemiring of a semiring R, then $Q_{\alpha,\beta}(A) = \langle Q_{\alpha,\beta}(A_i)^+, Q_{\alpha,\beta}(A_i)^- \rangle$ is a bipolar-valued multi fuzzy subsemiring of R.

Proof: For every x and y in R, α in [0, 1] and β in [–1, 0], we have $Q_{\alpha,\beta}(A_i)^+(x+y) = \min\{\alpha, A_i^+(x+y)\} \geq \min\{\alpha, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{\min\{\alpha, A_i^+(x)\}, \min\{\alpha, A_i^+(y)\}\} = \min\{Q_{\alpha,\beta}(A_i)^+(x), Q_{\alpha,\beta}(A_i)^+(y)\}$. Therefore $Q_{\alpha,\beta}(A_i)^+(x+y) \geq \min\{Q_{\alpha,\beta}(A_i)^+(x), Q_{\alpha,\beta}(A_i)^+(y)\}$ for all x and y in R. And $Q_{\alpha,\beta}(A_i)^+(xy) = \min\{\alpha, A_i^+(xy)\} \geq \min\{\alpha, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{\min\{\alpha, A_i^+(x)\}, \min\{\alpha, A_i^+(y)\}\} = \min\{Q_{\alpha,\beta}(A_i)^+(x), Q_{\alpha,\beta}(A_i)^+(y)\}$. Therefore $Q_{\alpha,\beta}(A_i)^+(xy) \geq \min\{Q_{\alpha,\beta}(A_i)^+(x), Q_{\alpha,\beta}(A_i)^+(y)\}$ for all x and y in R. Also $Q_{\alpha,\beta}(A_i)^-(x+y) = \max\{\beta, A_i^-(x+y)\} \leq \max\{\beta, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{\max\{\beta, A_i^-(x)\}, \max\{\beta, A_i^-(y)\}\} = \max\{Q_{\alpha,\beta}(A_i)^-(x), Q_{\alpha,\beta}(A_i)^-(y)\}$. Therefore $Q_{\alpha,\beta}(A_i)^-(x+y) \leq \max\{Q_{\alpha,\beta}(A_i)^-(x), Q_{\alpha,\beta}(A_i)^-(y)\}$ for all x, y in R. And $Q_{\alpha,\beta}(A_i)^-(xy) = \max\{\beta, A_i^-(xy)\} \leq \max\{\beta, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{\max\{\beta, A_i^-(x)\}, \max\{\beta, A_i^-(y)\}\} = \max\{Q_{\alpha,\beta}(A_i)^-(x), Q_{\alpha,\beta}(A_i)^-(y)\}$. Therefore $Q_{\alpha,\beta}(A_i)^-(xy) \leq \max\{Q_{\alpha,\beta}(A_i)^-(x), Q_{\alpha,\beta}(A_i)^-(y)\}$ for all x and y in R. Hence $Q_{\alpha,\beta}(A)$ is a bipolar-valued multi fuzzy subsemiring of R.

2.11 Theorem: If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar-valued multi fuzzy subsemiring of a semiring R, then $P_{\alpha,\beta}(A) = \langle P_{\alpha,\beta}(A_i)^+, P_{\alpha,\beta}(A_i)^- \rangle$ is a bipolar-valued multi fuzzy subsemiring of R.

Proof: For every x and y in R, α in [0, 1] and β in [–1, 0], we have $P_{\alpha,\beta}(A_i)^+(x+y) = \max\{\alpha, A_i^+(x+y)\} \geq \max\{\alpha, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{\max\{\alpha, A_i^+(x)\}, \max\{\alpha, A_i^+(y)\}\} = \min\{P_{\alpha,\beta}(A_i)^+(x), P_{\alpha,\beta}(A_i)^+(y)\}$. Therefore $P_{\alpha,\beta}(A_i)^+(x+y) \geq \min\{P_{\alpha,\beta}(A_i)^+(x), P_{\alpha,\beta}(A_i)^+(y)\}$ for all x and y in R. And $P_{\alpha,\beta}(A_i)^+(xy) = \max\{\alpha, A_i^+(xy)\} \geq \max\{\alpha, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{\max\{\alpha, A_i^+(x)\}, \max\{\alpha, A_i^+(y)\}\} = \min\{P_{\alpha,\beta}(A_i)^+(x), P_{\alpha,\beta}(A_i)^+(y)\}$. Therefore $P_{\alpha,\beta}(A_i)^+(xy) \geq \min\{P_{\alpha,\beta}(A_i)^+(x), P_{\alpha,\beta}(A_i)^+(y)\}$ for all x and y in R. Also $P_{\alpha,\beta}(A_i)^-(x+y) = \min\{\beta, A_i^-(x+y)\} \leq \min\{\beta, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{\min\{\beta, A_i^-(x)\}, \min\{\beta, A_i^-(y)\}\} = \max\{P_{\alpha,\beta}(A_i)^-(x), P_{\alpha,\beta}(A_i)^-(y)\}$. Therefore $P_{\alpha,\beta}(A_i)^-(x+y) \leq \max\{P_{\alpha,\beta}(A_i)^-(x), P_{\alpha,\beta}(A_i)^-(y)\}$ for all x and y in R. And $P_{\alpha,\beta}(A_i)^-(xy) = \min\{\beta, A_i^-(xy)\} \leq \min\{\beta, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{\min\{\beta, A_i^-(x)\}, \min\{\beta, A_i^-(y)\}\} = \max\{P_{\alpha,\beta}(A_i)^-(x), P_{\alpha,\beta}(A_i)^-(y)\}$. Therefore $P_{\alpha,\beta}(A_i)^-(xy) \leq \max\{P_{\alpha,\beta}(A_i)^-(x), P_{\alpha,\beta}(A_i)^-(y)\}$ for all x and y in R. Hence $P_{\alpha,\beta}(A)$ is a bipolar-valued multi fuzzy subsemiring of R.

2.12 Theorem: If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar-valued multi fuzzy subsemiring of a semiring R, then $G_{\alpha,\beta}(A) = \langle G_{\alpha,\beta}(A_i)^+, G_{\alpha,\beta}(A_i)^- \rangle$ is a bipolar-valued multi fuzzy subsemiring of R.

Proof: For every x and y in R, α in [0, 1] and β in [−1, 0], we have $G_{\alpha,\beta}(A_i)^+(x+y) = \alpha A_i^+(x+y) \geq \alpha (\min\{A_i^+(x), A_i^+(y)\}) = \min\{\alpha A_i^+(x), \alpha A_i^+(y)\} = \min\{G_{\alpha,\beta}(A_i)^+(x), G_{\alpha,\beta}(A_i)^+(y)\}$. Therefore $G_{\alpha,\beta}(A_i)^+(x+y) \geq \min\{G_{\alpha,\beta}(A_i)^+(x), G_{\alpha,\beta}(A_i)^+(y)\}$ for all x and y in R. And $G_{\alpha,\beta}(A_i)^+(xy) = \alpha A_i^+(xy) \geq \alpha (\min\{A_i^+(x), A_i^+(y)\}) = \min\{\alpha A_i^+(x), \alpha A_i^+(y)\} =$ min

$\{G_{\alpha,\beta}(A_i)^+(x), G_{\alpha,\beta}(A_i)^+(y)\}$. Therefore $G_{\alpha,\beta}(A_i)^+(xy) \geq \min\{G_{\alpha,\beta}(A_i)^+(x), G_{\alpha,\beta}(A_i)^+(y)\}$ for all x and y in R. Also $G_{\alpha,\beta}(A_i)^-(x+y) = -\beta A_i^-(x+y) \leq -\beta (\max\{A_i^-(x), A_i^-(y)\}) = \max\{-\beta A_i^-(x), -\beta A_i^-(y)\} = \max\{G_{\alpha,\beta}(A_i)^-(x), G_{\alpha,\beta}(A_i)^-(y)\}$. Therefore $G_{\alpha,\beta}(A_i)^-(x+y) \leq \max\{G_{\alpha,\beta}(A_i)^-(x), G_{\alpha,\beta}(A_i)^-(y)\}$ for all x and y in R. And $G_{\alpha,\beta}(A_i)^-(xy) = -\beta A_i^-(xy) \leq -\beta (\max\{A_i^-(x), A_i^-(y)\}) = \max\{-\beta A_i^-(x), -\beta A_i^-(y)\} = \max\{G_{\alpha,\beta}(A_i)^-(x), G_{\alpha,\beta}(A_i)^-(y)\}$. Therefore $G_{\alpha,\beta}(A_i)^-(xy) \leq \max\{G_{\alpha,\beta}(A_i)^-(x), G_{\alpha,\beta}(A_i)^-(y)\}$ for all x and y in R. Hence $G_{\alpha,\beta}$ (A) is a bipolar-valued multi fuzzy subsemiring of R.

2.13 Theorem: If *A* and *B* are bipolar-valued multi fuzzy subsemirings of a semiring R, then !(*A*∩*B*) = !(*A*) ∩!(*B*) is also a bipolar-valued multi fuzzy subsemiring of R.

Proof: By Theorem 2.3 and 2.9, it is true.

2.14 Theorem: If *A* and *B* are bipolar-valued multi fuzzy subsemirings of a semiring R, then ?(*A*∩*B*) = ?(*A*) ∩ ?(*B*) is also a bipolar-valued multi fuzzy subsemiring of R.

Proof: By Theorem 2.3 and 2.8, it is true.

2.15 Theorem: If A is a bipolar-valued multi fuzzy subsemiring of a semiring R, then!(?(*A*)) = ?(!(*A*)) is also a bipolar-valued multi fuzzy subsemiring of R.

Proof: By Theorem 2.8 and 2.9, it is true.

2.16 Theorem: If *A* and *B* are bipolar-valued multi fuzzy subsemirings of a semiring R, then $P_{\alpha,\beta}(A \cap B) = P_{\alpha,\beta}(A) \cap P_{\alpha,\beta}(B)$ is also a bipolar-valued multi fuzzy subsemiring of R.

Proof: By Theorem 2.3 and 2.11, it is true.

2.17 Theorem: If *A* and *B* are bipolar-valued multi fuzzy subsemirings of a semiring R, then $Q_{\alpha,\beta}(A \cap B) = Q_{\alpha,\beta}(A) \cap Q_{\alpha,\beta}(B)$ is also a bipolar-valued multi fuzzy subsemiring of R.

Proof: By Theorem 2.3 and 2.10, it is true.

2.18 Theorem: If A is a bipolar-valued multi fuzzy subsemiring of a semiring R, then $P_{\alpha,\beta}(Q_{\alpha,\beta}(A)) = Q_{\alpha,\beta}(P_{\alpha,\beta}(A))$ is also a bipolar-valued multi fuzzy subsemiring of R.

Proof: By Theorem 2.10 and 2.11, it is true.

2.19 Theorem: If *A* and *B* are bipolar-valued multi fuzzy subsemirings of a semiring R, then $G_{\alpha,\beta}(A \cap B) = G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B)$ is also a bipolar-valued multi fuzzy subsemiring of R.

Proof: By Theorem 2.3 and 2.12, it is true.

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