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ON SEMI PRECLOSED WEAKLY GENERALIZED STAR SEMI CLOSED (βwg^*s) SET IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce the concept of βwg^*s closed sets and investigate some of its properties in topological spaces. We also define βwg^*s continuity and some of its fundamental properties are given.

***Key words**: βwg^*s closed set, βwg^*s continuous map.*

1. INTRODUCTION

In 1970, Levine [12] introduced the concept of generalized closed set in topological spaces. Biswas [6] defined semi closed sets. Crossley and Hildebrand [7] defined semi closure of a set. Abd El-Monsef, El-Deeb and Mahmoud [2] introduced the concept of β open sets and β continuous mappings. Sundaram and Sheik John [22] introduced the concept of weakly closed set. Veerakumar [23] introduced g^*closed sets. Dhanapakyam, Subashini and Indirani [9] introduced the concept of βwg^* closed set in topological spaces. In this paper we define a βwg^*s closed set and βwg^*s continuity. We also study their fundamental properties.

2. PRELIMINARIES

Definition: 2.1 A subset A of X is called generalized closed (briefly g-closed) [12] set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition: 2.2 A subset A of X is called regular open (briefly r-open) [21] set if $A = int(cl(A))$ and regular closed (briefly r-closed) [21] set if $A = cl\big(int(A)\big)$.

Definition: 2.3 A subset A of X is called semi open [13] set if $A \subseteq cl(int(A))$and semi closed [6] set if $int\big(cl(A)\big) \subseteq A$.

Definition: 2.4 A subset A of X is called α-open [19] set if $A \subseteq int(cl(int(A)))$ and α-closed [15] set if $cl(int(cl(A))) \subseteq A$.

Definition: 2.5 A subset A of X is called β-open [2] set if $A \subseteq cl(int(cl(A)))$ and β-closed [11] set if $int(cl(int(A))) \subseteq A$.

Definition: 2.6 A subset A of X is called θ-closed [24] set if $A = cl_\theta(A)$
where $cl_\theta(A) = \{x \in X | cl(U) \cap A \neq \emptyset, U \in \tau, x \in U\}$.

Definition: 2.7 A subset A of X is called δ-closed [24] set if $A = cl_\delta(A)$
where $cl_\delta(A) = \{x \in X | int(cl(U)) \cap A \neq \emptyset, U \in \tau, x \in U\}$.

Definition: 2.8 A subset A of X is called Generalized α- closed (briefly $g\alpha$-closed) [15] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.

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Definition: 2.9 A subset A of X is called α-generalized closed (briefly αg-closed) [16] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.10 A subset A of X is called strongly generalized closed (briefly g^*-closed) [18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

Definition: 2.11 A subset A of X is called weakly closed (briefly w-closed) [22] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

Definition: 2.12 A subset A of X is called weakly generalized closed (briefly wg-closed) [18] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.13 A subset A of X is called Generalized regular closed (briefly gr-closed) [20] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.14 A subset A of X is called Regular generalized closed (briefly rg-closed) [20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition: 2.15 A subset A of X is called Regular weakly closed (briefly rw-closed) [4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X.

Definition: 2.16 A subset A of X is called βwg^*closed set [9] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is β -open in X.

Definition: 2.17 A subset A of X is called Generalized pre regular closed (brieflygpr-closed) [11] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition: 2.18 A subset A of X is called Generalized semi pre closed (brieflygsp-closed) [10] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition: 2.19 A subset A of X is called $w\alpha$ closed [5] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is w open in X.

3. ON (βwg^*s) CLOSED SET

Definition: 3.1 A subset A of a topological space (X, τ) is called Semi pre closed weakly generalized star semi closed (briefly βwg^*s-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is βwg^* open in X.

Theorem: 3.2 Every closed set is βwg^*s closed set.

Proof: Let A be a closed set in X such that $A \subseteq U$ where U is βwg^* open. Since $scl(A) \subseteq cl(A) = A$. Therefore $scl(A) \subseteq U$. Hence A is a βwg^*s closed set.

The converse of above theorem need not be true.

Example: 3.3 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{a, b\}$ is βwg^*s closed but not closed.

Remark: 3.4 Every θ-closed, π-closed, δ-closed set is closed. Therefore every θ-closed,π-closed, δ-closed set is βwg^*s closed.

Theorem: 3.5 Every regular closed set is βwg^*s closed.

Proof: Let A be a regular closed set such that $A \subseteq U$ where U is βwg^*open. Every regular closed set is closed. By theorem 3.2, every regular closed set is βwg^*s closed.

The converse of above theorem need not be true.

Example: 3.6 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{b, c\}$ is βwg^*s closed set but not regular closed.

Theorem: 3.7 Every semi closed set is βwg^*s closed.

Proof: Let A be a semi closed set such that $A \subseteq U$ where U is βwg^*open. Since A is semi closed. $scl(A) = A$. Therefore $scl(A) \subseteq U$. Hence A is βwg^*s closed.

The converse of above theorem need not be true.

Example: 3.8 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{a, c, d\}$ is $\beta wg^{*}s$ closed set but not semi closed.

Theorem: 3.9 Every α closed set is $\beta wg^{*}s$ closed.

Proof: Let A be a α closed set such that $A \subseteq U$ where U is βwg^{*} open. Since A is α closed. $scl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $scl(A) \subseteq U$. Hence A is $\beta wg^{*}s$ closed.

The converse of above theorem need not be true.

Example: 3.10 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{a\}$ is $\beta wg^{*}s$ closed but not α closed.

Theorem: 3.11 Every g-closed set is $\beta wg^{*}s$ closed.

Proof: Let A be a g closed set such that $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Then $scl(A) \subseteq cl(A) \subseteq U$. Therefore $scl(A) \subseteq U$ and U is open. Since every open set is βwg^{*} open. Hence A is $\beta wg^{*}s$ closed.

The converse of above theorem need not be true.

Example: 3.12 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{c\}$ is $\beta wg^{*}s$ closed but not g closed.

Theorem: 3.13 Every g^{*} closed set is $\beta wg^{*}s$ closed.

Proof: Let A be a g^{*} closed set such that $A \subseteq U$ and U is βwg^{*} open. Since every g^{*} closed set is g closed. By theorem 3.11, every g^{*} closed set is $\beta wg^{*}s$ closed.

The converse of above theorem need not be true.

Example: 3.14 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{a, b\}$ is $\beta wg^{*}s$ closed but not g^{*} closed.

Theorem: 3.15 Every αg closed set is $\beta wg^{*}s$ closed.

Proof: Let A be a αg closed set such that $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Then $scl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Since every open set is βwg^{*} open. Hence A is $\beta wg^{*}s$ closed set.

The converse of above theorem need not be true.

Example: 3.16 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{a\}$ is $\beta wg^{*}s$ closed but not αg closed.

Theorem: 3.17 Every gr closed set is $\beta wg^{*}s$ closed set.

Proof: Let A be a gr closed set such that $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Then $scl(A) \subseteq rcl(A) \subseteq U$. Therefore $scl(A) \subseteq U$ and U is open. Since every open set is βwg^{*} open. Hence A is $\beta wg^{*}s$ closed.

The converse of above theorem need not be true.

Example: 3.18 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{b, c\}$ is $\beta wg^{*}s$ closed but not gr closed.

Theorem: 3.19 Every $w\alpha$ closed set is $\beta wg^{*}s$ closed set.

Proof: Let A be a $w\alpha$ closed set such that $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is w open. Then $scl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $scl(A) \subseteq U$ and U is w open. Since every w open set is βwg^{*} open. Hence A is $\beta wg^{*}s$ closed set.

The converse of above theorem need not be true.

Example: 3.20 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{c\}$ is βwg^*s closed but not $w\alpha$ closed.

Remark: 3.21 βwg^*s closed set and gpr closed set are independent to each other as seen from the following example.

Example: 3.22 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{a\}$ is βwg^*s closed but not gpr closed. Also $A = \{a, b, c\}$ is gpr closed but not βwg^*s closed.

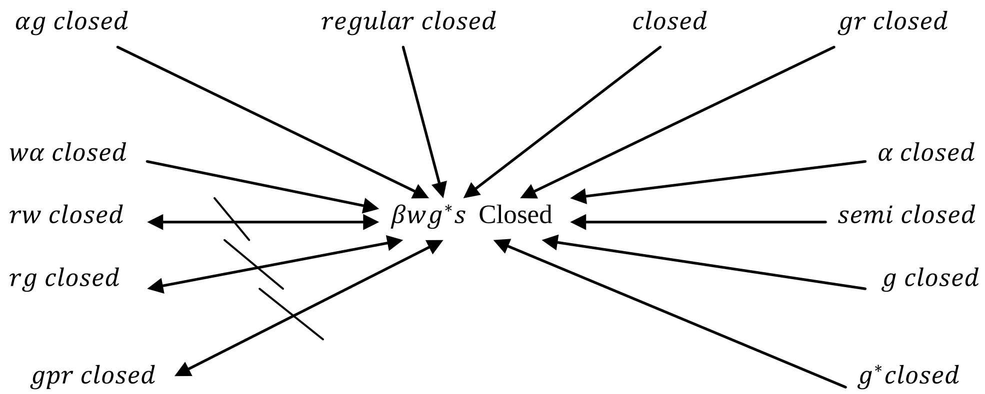
Remark: 3.23 βwg^*s closed set and rw closed set are independent to each other as seen from the following example.

Example: 3.24 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{c\}$ is βwg^*s closed but not rw closed. Also $A = \{a, c\}$ is rw closed but not βwg^*s closed.

Remark: 3.25 βwg^*s closed set and rg closed set are independent to each other as seen from the following example.

Example: 3.26 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{c\}$ is βwg^*s closed but not rg closed. Also $A = \{a, c\}$ is rg closed but not βwg^*s closed.

Remark: 3.27 The above discussions are summarized in the following diagram.



$A \longrightarrow B$ Means A implies B but not conversely.
$A \not\longleftrightarrow B$ Means A and B are independent of each other.

4. CHARACTERISTICS OF βwg^*s CLOSED SET

Theorem: 4.1 If A and B are βwg^*s closed sets, then $A \cap B$ is also βwg^*s closed in (X, τ).

Proof: Let A and B be any two βwg^*s closed sets in X such that $A \subseteq U$ and $B \subseteq U$, where U is βwg^* is open in X and so $A \cap B \subseteq U$. Since A and B are βwg^*s closed, $Scl(A) \subseteq U$ and $Scl(A) \subseteq U$ and hence $Scl(A \cap B) \subseteq Scl(A) \cap Scl(B) \subseteq U$. Thus $A \cap B$ is βwg^*s closed in(X, τ).

Remark: 4.2 The union of two βwg^*s closed sets in X is generally not an βwg^*s closed set in X.

Example: 4.3 Let $X = \{a, b, c, d\}$; $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b, c\}, X, \emptyset\}$. Here $A = \{a\}$ and $B = \{c\}$ be two βwg^*s closed subsets of X. But $A \cup B = \{a, c\}$ is not a βwg^*s closed.

Theorem: 4.4 If A is a βwg^*s closed set in X if and only if $scl(A) - A$ does not contain any non-empty βwg^* closed set.

Proof: Let F be a βwg^* closed set such that $F \subset scl(A) - A$. Then $F \subset X - A$ implies $A \subset X - F$. A is βwg^*s closed set and $X - F$ is βwg^* open. Therefore $scl(A) \subset X - F$ that is $F \subset X - scl(A)$. Hence $F \subset scl(A) \cap \big(X - scl(A)\big) = \emptyset$. Hence $F = \emptyset$.

Conversely let us assume that $scl(A) - A$ contains no non-empty βwg^* closed set. Let $A \subseteq U$ and U is βwg^* open. Suppose that $scl(A) - A$ is not contained in U. Then $scl(A) \cap U^c$ is a non-empty βwg^* closed set of $scl(A) - A$. which is a contradiction. Hence A is βwg^*s closed.

Theorem: 4.5 If A is βwg^*s closed and $A \subseteq B \subseteq scl(A)$, then B is βwg^*s closed.

Proof: Since $B \subseteq scl(A)$, we have $scl(B) \subseteq scl(A)$ and $scl(B) - B \subseteq scl(A) - A$. But A is βwg^*s closed. Hence $scl(A) - A$ has no non-empty βwg^*closed, neither does $scl(B) - B$. By theorem 4.4, B is βwg^*s closed.

Theorem: 4.6 The union of semi closed set and gr closed set is a βwg^*s closed.

Proof: Let A be a semi closed set and B be a gr closed set. Let $A \cup B \subseteq U$ and U be βwg^* open. Since A be semi closed, we have $scl(A) = A \subseteq U$. Since B is gr closed, we have $rcl(B) \subseteq U$, U is open. But $scl(B) \subseteq rcl(B) \subseteq U$ and also we know that every open set is βwg^* open. Therefore $A \cup B$ is βwg^*s closed set.

Theorem: 4.7 If A is βwg^* open and βwg^*s closed in X then A is semi closed set in X.

Proof: Let A be βwg^* open and βwg^*s closed in X. Then $scl(A) \subseteq A$. But $A \subseteq scl(A)$. Therefore $A = scl(A)$. Hence A is semi closed in X.

5. On βwg^*s Continuity Set

Definition: 5.1 A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called,

1. Continuous [13] if $f^{-1}(V)$ is closed set in (X, τ) for every closed set V of (Y, σ).
2. Regular continuous [1] if $f^{-1}(V)$ is regular closed set in (X, τ) for every closed set V of (Y, σ).
3. α continuous [17] if $f^{-1}(V)$ is α closed set in (X, τ) for every closed set V of (Y, σ).
4. Semi continuous [13] if $f^{-1}(V)$ is Semi closed set in (X, τ) for every closed set V of (Y, σ).
5. g continuous [3] if $f^{-1}(V)$ is g closed set in (X, τ) for every closed set V of (Y, σ).
6. g^* continuous [23] if $f^{-1}(V)$ is g^* closed set in (X, τ) for every closed set V of (Y, σ).
7. gr continuous [14] if $f^{-1}(V)$ is gr closed set in (X, τ) for every closed set V of (Y, σ).
8. αg continuous [8] if $f^{-1}(V)$ is αg closed set in (X, τ) for every closed set V of (Y, σ).

Definition: 5.2 A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called βwg^*s continuous if $f^{-1}(V)$ is βwg^*s closed set in (X, τ) for every closed set V of (Y, σ).

Theorem: 5.3 Every continuous map is βwg^*s continuous, but not conversely.

Proof: The proof follows from the fact that every closed set is βwg^*s closed set.

Example: 5.4 Let $X = Y = \{a, b, c, d\}$; $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}\}$ and $\sigma = \{X, \emptyset, \{b\}, \{b, d\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = a, f(b) = c, f(c) = d, f(d) = d$. This map is βwg^*s continuous, but not continuous. Since for the closed set $\{a, b, d\}$ in Y. $f^{-1}\{a, b, d\} = \{a, c, d\}$ is not closed set in X.

Theorem: 5.5 Every regular continuous map is βwg^*s continuous, but not conversely.

Proof: The proof follows from the fact that every regular closed set is βwg^*s closed set.

Example: 5.6 Let $X = Y = \{a, b, c, d\}$; $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}\}$ and $\sigma = \{X, \emptyset, \{b\}, \{b, d\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = a, f(b) = c, f(c) = d, f(d) = d$. This map is βwg^*s continuous, but not regular continuous. Since for the closed set $\{d\}$ in Y. $f^{-1}\{d\} = \{c, d\}$ is not regular closed set in X.

Theorem: 5.7 Every semi continuous map is βwg^*s continuous, but not conversely.

Proof: The proof follows from the fact that every semi closed set is βwg^*s closed set.

Example: 5.8 Let $X = Y = \{a, b, c, d\}$; $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}\}$ and $\sigma = \{X, \emptyset, \{b\}, \{b, d\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = a, f(b) = c, f(c) = d, f(d) = d$ This map is βwg^*s continuous, but not semi continuous. Since for the closed set $\{a, b, d\}$ in Y. $f^{-1}\{a, b, d\} = \{a, c, d\}$ is not semi closed set in X.

Theorem: 5.9 Every gr continuous map is βwg^*s continuous, but not conversely.

Proof: The proof follows from the fact that every gr closed set is βwg^*s closed set.

Example: 5.10 Let $X = Y = \{a, b, c, d\}$; $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}\}$ and $\sigma = \{X, \emptyset, \{b\}, \{b, d\}\}$. Define a map $f : X \rightarrow Y$ by $f(a) = a, f(b) = d, f(c) = c, f(d) = b$. This map is βwg^*s continuous, but not gr continuous. Since for the closed set $\{d\}$ in Y. $f^{-1}\{d\} = \{b\}$ is not gr closed set in X.

Remark: 5.11

1. Every g closed set is βwg^*s closed. Therefore every g continuous map is βwg^*s continuous.
2. Every g^* closed set is βwg^*s closed. Therefore everyg^*continuous map is βwg^*s continuous.
3 Every αg closed set is βwg^*s closed. Therefore every αg continuous map is βwg^*s continuous.
4 Every α closed set is βwg^*s closed. Therefore every α continuous map is βwg^*s continuous.
5 Every $w\alpha$ closed set is βwg^*s closed. Therefore every $w\alpha$ continuous map is βwg^*s continuous.

Theorem: 5.12 If $f : X \rightarrow Y$ is βwg^*s continuous and $g : Y \rightarrow Z$ is continuous then their composition $f \circ g: X \rightarrow Z$ is βwg^*s continuous.

Proof: Let $f : X \rightarrow Y$ is βwg^*s continuous and $g : Y \rightarrow Z$ is continuous. Let U be a closed set in Z.Therefore $g^{-1}(U)$ is closed in Y and $f^{-1}(g^{-1}(U))$ is βwg^*s closed in X. Therefore $f \circ g$ is βwg^*s continuous.

Theorem: 5.13 Let $X = A \cup B$, where A and B are closed in X. Let $f : A \rightarrow Y$ and $g : B \rightarrow Y$ be continuous. If $f(x) = g(x)$ for every $x \in A \cap B$ then f and g are combine to give βwg^*s continuous function $h : X \rightarrow Y$ defined by $h(x) = f(x)$ if $x \in A$, and $h(x) = g(x)$ if $x \in B$.

Proof: Let C be a closed subset of Y. Now $h^{-1}(C) = f^{-1}(C) \cup g^{-1}(C)$. Since f is continuous, $f^{-1}(C)$ is closed in A and therefore closed in X. Similarly $g^{-1}(C)$ is closed in B and therefore closed in X. Their union $h^{-1}(C)$ is also closed in X. Therefore h is continuous. By theorem (5.3) h is βwg^*s continuous.

Theorem: 5.14 Let $f : (X, \tau) \rightarrow (Y, \sigma)$be a function where X and Y be topological spaces. Then the following are equivalent.

a. f is βwg^*s continuous.
b. The inverse of each open set in Y is βwg^*s open in X.
c. For each subset A of X, $f(\beta wg^*s\, cl(A)) \subseteq cl\big(f(A)\big)$.

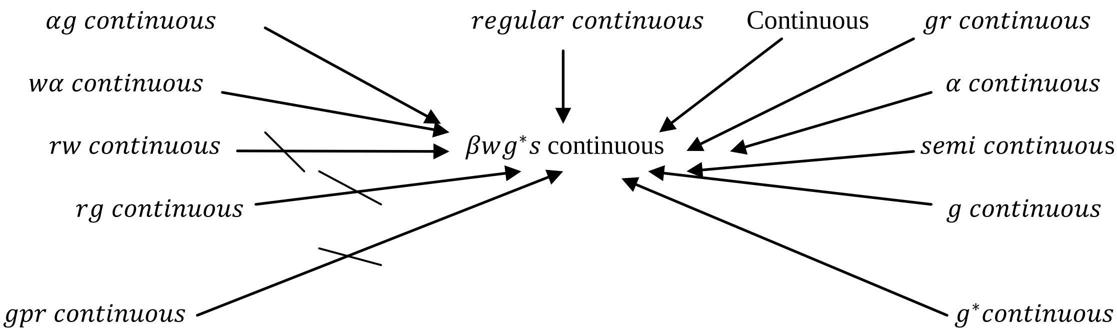
Proof:
$(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$: Let B be an open subset of Y. Then $Y - B$ is closed in Y. Since f is βwg^*s continuous,$f^{-1}(Y - B)$ is βwg^*s closed in X. That is,$X - f^{-1}(B)$ is βwg^*s closed in X. Hence $f^{-1}(B)$ is βwg^*s open in X.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$: Let G be a closed subset of Y. Then $Y - G$ is open in Y. Then$f^{-1}(Y - G)$ is βwg^*s open in X. That is $X - f^{-1}(G)$ is βwg^*s open in X. Hence $f^{-1}(G)$ is βwg^*s closed in X , which implies that f is βwg^*s continuous.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{iii})$: Let A be a subset of X. Since $A \subset f^{-1}\big(f(A)\big)$, $A \subset f^{-1}\left(cl\big(f(A)\big)\right)$. Now $cl\big(f(A)\big)$ is a closed set in Y. Then by (ii), $f^{-1}\left(cl\big(f(A)\big)\right)$ is βwg^*s closed in X containing A. But $\beta wg^*s\, cl(A)$ is the smallest βwg^*s closed set in X containing A. Therefore $\beta wg^*s\, cl(A) \subseteq f^{-1}\left(cl\big(f(A)\big)\right)$. Hence $f\big(\beta wg^*s\, cl(A)\big) \subseteq cl\big(f(A)\big)$.

$(\boldsymbol{iii}) \Rightarrow (\boldsymbol{ii})$: Let B be a closed subset of Y. Then $f^{-1}(B)$ is a subset of X. By (iii) $f\left(\beta wg^*s\, cl\left(f^{-1}(B)\right)\right) \subseteq cl\left(f\left(f^{-1}(B)\right)\right) \subseteq cl(B) = B$. This implies $\beta wg^*s\, cl\left(f^{-1}(B)\right) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq \beta wg^*s\, cl\left(f^{-1}(B)\right)$. Hence $f^{-1}(B) = \beta wg^*s\, cl\left(f^{-1}(B)\right)$ and $f^{-1}(B)$ is βwg^*s closed in X. This implies that f is βwg^*s continuous.

Remark: 5.15 The above discussions are summarized in the following diagram.



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