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ON μp$\hat{g}$ SET AND CONTINUITY IN TOPOLOGICAL SPACES

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ABSTRACT

*The aim of this paper is to introduce the concept of **μp$\hat{g}$** closed and open set and to introduce the **μp$\hat{g}$** continuous map and their relations. Various properties and characterizations of **μp$\hat{g}$** continuous map and study their basic properties in topological spaces.*

***Keywords:** **μp$\hat{g}$** closed set, **μp$\hat{g}$** open set, regular open, **μp$\hat{g}$** continuous map.*

1. INTRODUCTION

In 2000, M. K. R. S. Veera kumar introduced the concept of **μp –** closed sets in topological spaces. Later he introduced $\hat{g}$ closed sets in topological spaces. In this paper I introduce the some properties of **μp$\hat{g}$** closed set and continuity in topological spaces.

2. PRELIMINARIES

Definition 2.1: A subset A of X is called generalized closed (briefly g-closed) [3]set if $cl(A) \subseteq U$ whenever $A \subseteq U$ *and U is open* in X

Definition 2.2: A subset A of X is called regular open(briefly r-open) [5] set if $A = int(cl(A))$ and regular closed(briefly r-closed) set if $A = cl(int(A))$.

Definition 2.3: A subset A of X is called pre-open [7] set if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$

Definition 2.4: A subset A of X is called $\alpha - open$ [8] if $A \subseteq int(cl(int(A)))$ *and* $\alpha - closed$ if $cl(int(cl(A))) \subseteq A$.

Definition 2.5: A subset A of X is called θ-closed [13] if A= cl_θ (A), where cl_θ (A)={x∈ $X: cl(U) \cap A \neq U \in \tau$}.

Definition 2.6: A subset A of X is called δ- closed [13] if A=cl_δ(A),where cl_δ(A)=$x \in X: int(cl(U)) \cap A \neq U \in \tau\}$

Definition 2.7:A subset A of X is called δ-generalized closed (briefly δ-g-closed) [12] if cl_δ(A)⊆U whenever A⊆ U and U is open in X.

Definition 2.8: A subset A of X is called $g\alpha^\star$ closed set [6] if αcl(A)⊆ $int(U)$, whenever A⊆ U and U is α open in X.

Definition 2.9: A subset A of X is called $\hat{g}$ closed set [15] if cl(A)⊆ U, whenever A⊆ U and U is semi open in X.

Definition 2.10: A subset A of X is called $g^\star$ closed set [14] if cl(A)⊆ U, whenever A⊆ U and U is g open in X.

Definition 2.11: A subset A of X is called gr closed set [10] if rcl(A)⊆ U, whenever A⊆ U and U is open in X.

Definition 2.12: A subset A of X is called midly g closed set [9] if cl(int(A)) ⊆ U, whenever A⊆ U and U is g open in X.

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Definition 2.13: A subset A of X is called * g closed set [17s] if cl(A)⊆ U, whenever A⊆ U and U is $\boldsymbol{g}$ open in X.

Definition 2.14: A subset A of X is called μ**p** closed set[16] if **p**cl(A)⊆ U, whenever A⊆ U and U is $g\alpha^{\star}$ open in X.

3. *On* μp$\hat{g}$ Closed set

Definition 3.1: A subset A of a topological space (X,τ) is called **μp$\hat{g}$** closed set if μpcl(A)⊆ U, whenever A⊆ U and U is $\hat{g}$ open in X.

Theorem 3.2: Every closed set is **μp$\hat{g}$** closed set, but not conversely.

Proof: Let A be closed set such that A⊆ U and U is $\hat{g}$ open set. Every closed set is **μp** closed set. A= Cl (A)⊆ U ⇒ μpcl(A)⊆ U. Hence μpcl (A)⊆ U, whenever A⊆ $U$and U is $\hat{g}$ open. Therefore A is **μp$\hat{g}$** closed set.

Example 3.3: Let X={a, b, c, d}, τ={X,ϕ , {a},{b},{a,b},{a,b,c}} here A={c} is **μp$\hat{g}$** closed but not closed set in X.

Theorem 3.4: Every midly g closed set is **μp$\hat{g}$** closed set.

Proof: Let A be midly g closed set such that cl (int (A)) ⊆ U whenever A⊆ U and U is g open. A = cl (int (A)) ⊆ cl (A) ⊆ U ⇒ μpcl(A)⊆ U. Every g open set is $\hat{g}$ open. Therefore A is **μp$\hat{g}$** closed set.

Theorem 3.5: Every g closed set is μ**p$\hat{g}$** closed set, but not conversely.

Proof: Let A be g closed set such that cl (A) ⊆U, whenever A⊆U and U is open. Then cl(A) ⊆ U ⇒ μ$\boldsymbol{pcl(A)}$ ⊆ U. Every open set is $\hat{g}$ open. Therefore A is μ**p$\hat{g}$** closed set.

Example 3.6: Let X={a, b, c, d} ,τ={X,ϕ,{a},{b},{a,b},{a,b,c}}. Let A={c} is μ**p$\hat{g}$** closed but not g closed set in X.

Theorem 3.7: Every g* closed set is μ**p$\hat{g}$** closed set, but not conversely.

Proof: Let A be g* closed set. Every g* closed set is g closed. By theorem 3.5, therefore **A** is **μp$\hat{g}$** closed set.

Example 3.8: Let X={a, b, c, d}, τ = {X,ϕ,{a},{b},{a,b},{a,b,c}}. Let A={c} is **μp$\hat{g}$** closed but not g* closed set in X.

Theorem 3.9: Every gr closed set is μ**p$\hat{g}$** closed set, but not conversely.

Proof: Let A be gr closed set. Every gr closed set is g closed. By theorem 3.5, A is **μp$\hat{g}$** closed set.

Example 3.10: Let X={a, b, c, d} ,τ={X,ϕ,{a},{b},{a,b},{a,b,c}}. Let A={c} is μ**p$\hat{g}$** closed but not gr closed set in X.

Theorem 3.11: Every *g closed set is μ**p$\hat{g}$** closed set, but not conversely.

Proof: Let A be *g closed set such that cl(A)) ⊆U, whenever A⊆U and U is $\hat{g}$ open. Then cl(A) ⊆ U ⇒ μ**p**cl(A) ⊆ U. Therefore A is μ**p$\hat{g}$** closed set.

Example 3.12: Let X={a, b, c, d} ,τ={X,ϕ,{a},{b},{a,b},{a,b,c}}. Let A={c} is μ**p$\hat{g}$** closed but not * g closed set in X.

Theorem 3.13: Every regular closed set is μ**p$\hat{g}$** closed, but not conversely.

Proof: Let A be a regular closed set, such that A⊆ U and U is $\hat{g}$open set, Every regular closed set is closed. By theorem 3.2 A is μ**p$\hat{g}$** closed set.

Example 3.14: Let X={a ,b, c, d} ,τ={X,ϕ,{a},{b},{a,b},{a,b,c}}.Let A={a, b, d} is μ**p$\hat{g}$** closed but not regular closed.

Remark: Every θ-closed and δ- closed is closed. Therefore every θ-closed and δ- closed is μ**p$\hat{g}$** closed

Theorem 3.15: The Union of two μ**p$\hat{g}$** closed subsets of X is also an μ**p$\hat{g}$** closed subsets of X.

Proof: Assume that A and B are μ**p$\hat{g}$** closed sets in X, such that A⊂ U and B⊂ $U$and U is $\hat{g}$ open. Since A and B are μ**p$\hat{g}$** closed set, therefore μ**p**cl(A)⊂ U and μ**p**cl(B)⊂ U**.** Hence μ**p**cl(A∪B)= μ**p**cl(A) ∪ μ**p**cl(B)⊂U.That is A∪B is μ**p$\hat{g}$** closed set.

Theorem 3.16: The intersection of two $\mu p\hat{g}$ closed subsets of X is also an $\mu p\hat{g}$ closed subsets of X.

Proof: Assume that A and B are $\mu p\hat{g}$ closed sets in X, such that A⊂ U and B⊂ $U$and U is $\hat{g}$ open. Since A and B are $\mu p\hat{g}$ closed set, therefore μ**p**cl(A)⊂ U and μ**p**cl(B)⊂ U**.** Hence μ**p**cl(A∩B)= μ**p** cl(A)∩ μ**p**cl(B)⊂U.That is A∩B is $\mu p\hat{g}$ closed set.

Theorem 3.17: Let A⊆ B⊆ μ**p**cl(A) and A is a $\mu p\hat{g}$ closed subset of (X,τ) then B is also a $\mu p\hat{g}$ closed subset of (X,τ).

Proof: Since A is a $\mu p\hat{g}$ closed subset of (X,τ), So μ**p**cl(A) ⊆ U,whenever A ⊆ U, U is $\hat{g}$ open subset of X. Let A⊆ B⊆ μ**p**cl(A). That is μ**p**cl(A)= μ**p**cl(B).Let if possible there exists an $\hat{g}$ open subset V of X such that B ⊆ V.So A ⊆ V and A being $\mu p\hat{g}$ closed subset of X, μ**p** cl(A) ⊆ V. That is μ**p**cl(B) ⊆ V. Hence B is also a $\mu p\hat{g}$ closed subset of X.

Theorem 3.18: Let A⊆ B⊆X, where B is $\hat{g}$ open in X. If A is $\mu p\hat{g}$ closed in X, then A is $\mu p\hat{g}$ closed in B.

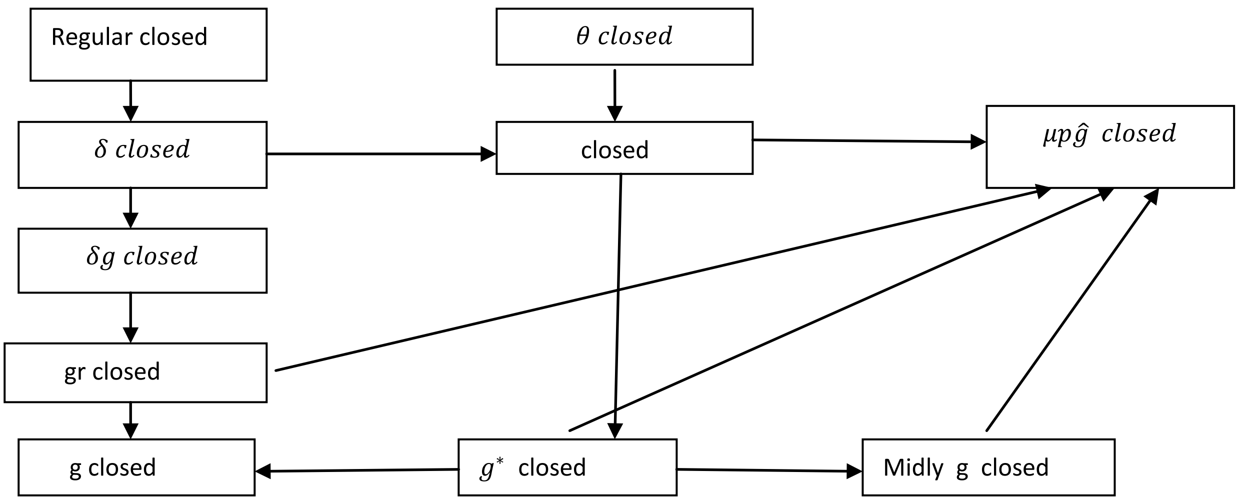
Proof: Let A ⊆ U, where U is $\hat{g}$ open set of X. Since U=V∩B, for Some $\hat{g}$ open set V of X and B is $\hat{g}$ open in X. Using assumption A is $\mu p\hat{g}$ closed in X. We have μ**p**cl(A)⊆U and so μ**p**cl(A)=cl(A)∩B ⊆ U∩B ⊆ U. Hence A is $\mu p\hat{g}$ closed in B.

Theorem 3.19: A subset A of X is $\mu p\hat{g}$ closed sets iff μ**p**cl(A) ∩ A^c contains no non-zero closed set in X.

Proof: Let A be a $\mu p\hat{g}$ closed subset of X. Also if possible let M be closed subset of X such that M ⊆ μ**p** cl(A) ∩ A^c. That is M ⊆ μ**p**cl(A) and M ⊆ A^c. Since M is a closed subset of X, M^c is an open subset of X ⊆ A, and A being $\mu p\hat{g}$ open subset of X, μ**p**cl(A) ⊆ M^c. But M⊆ μ**p**cl(A).So we get a contradiction. Therefore M=∅. So the condition is true. Conversely, let A ⊆ N, and N is a open subset of X. Then N^c ⊆A^c, And N^c is a closed subset of X. Let if possible μ**p**cl(A) ⊆ N .Then μ**p**cl(A) ∩ N^c is a nonzero closed subset of μ**p**cl(A) ∩ A^c, which is a contradiction .Hence A is a $\mu p\hat{g}$ closed subset of X.

Theorem3.20: A subset A of X is $\mu p\hat{g}$ closed set in X iff μ**p**cl(A)-A contain no non-empty $\hat{g}$ closed set in X.

Proof: Suppose that F is a non-empty $\hat{g}$ closed subset if μ**p**cl(A)-A. Now F ⊆ μ**p**cl(A)-A. Then F ⊆ μ**p**cl(A) ∩ A^c. Therefore F⊆ A^c. Since F^c is $\hat{g}$ open set and A is $\mu p\hat{g}$ closed, μ**p** cl(A) ⊆ F^c. That is F⊆ μ**p** cl(A)^c. Hence F⊆ μ**p**cl(A) ∩[μ**p**cl(A)]^c=∅. That is F=∅. Thus μ**p**cl(A)-A contains no non empty $\hat{g}$ closed set. Conversely assume that μ**p**cl(A)-A contains no nonempty $\hat{g}$ closed set. Let A ⊆ U, U is $\hat{g}$ open. Suppose that μ**p**cl(A) is not contained in U. Then μ**p** cl(A) ∩ U^c is a non-empty $\hat{g}$ closed set and contained in μ**p** cl(A)-A, which is a contradiction. Therefore μ**p**cl(A) ⊆ U and hence A is $\mu p\hat{g}$ closed set.



4. On $\mu p\hat{g}$ open set

Definition 4.1: A subset A of a topological space X is called $\mu p\hat{g}$ open sets if A^c is $\mu p\hat{g}$ closed.

Theorem 4.2: A subset A of a topological space (X,τ) is $\mu p\hat{g}$ open if and only if B ⊆ μ**p** int(A) whenever B is $\hat{g}$ closed in X and B⊆A.

Proof: Necessity: Suppose B ⊆ μ**p** (int(A)) where B is $\hat{g}$ closed in (X,τ) and B⊆A. Let A^c ⊆M where M is $\hat{g}$ open. Hence M^c ⊆A, where M^c is $\hat{g}$ closed. Hence by assumption M^c ⊆ μ**p** (int(A)) which implies(μ**p**(int(A)))^c ⊆M. Therefore μ**p** (cl(A^c)) ⊆M. Thus A^c is μ**p**$\hat{g}$ closed, implies A is μ**p**$\hat{g}$ open.

Sufficiency: Let A is μ**p**$\hat{g}$ open in X with N ⊆A, Where N is $\hat{g}$ closed. We have A^c is μ**p**$\hat{g}$ closed with A^c⊆N^c where N^c is $\hat{g}$ open. Then we have μ**p** (cl(A^c)) ⊆N^c implies N⊆X- μ**p** (cl(A^c))= μ**p** (int(X-A^c))= μ**p** (int(A))

Theorem 4.3: If μ**p** (int(A)) ⊆B ⊆A and A is μ**p**$\hat{g}$ open subset of (X,τ) then B is also μ**p**$\hat{g}$ open subset of (X,τ).

Proof: Let μ**p** (int(A)) ⊆B ⊆A implies A^c ⊆B^c ⊆ μ**p** (cl(A^c)). Given A^c isμ**p**$\hat{g}$ closed. By theorem 3.17, B^c is μ**p**$\hat{g}$ closed. Therefore B is μ**p**$\hat{g}$ open.

Theorem 4.4: If a subset A of a topological space (X,τ) is μ**p**$\hat{g}$ open in X then F=X, whenever F is regular open and μ**p** (int(A)) ⊆ A^c ⊆ F.

Proof: Let A be a μ**p**$\hat{g}$ open and F be $\hat{g}$ open, μ**p** (int(A))∪ A^c ⊆ F. This gives F^c ⊆ (X- μ**p** (int(A)))∩A=μ**p** (cl(A^c)) ∩A= μ**p** (cl(A^c))-A^c. Since F^c is $\hat{g}$ closed and A^c is μ**p**$\hat{g}$ closed. By theorem 3.19, we have F^c=∅. Thus F=X.

Theorem 4.5: If a subset A of a topological space (X,τ) is μ**p**$\hat{g}$ closed, then μ**p** (cl(A))-A is μ**p**$\hat{g}$ open.

Proof: Let A⊆ X be a μ**p**$\hat{g}$ closed and let F be $\hat{g}$ closed such that F⊆ μ**p** (cl(A))-A. By theorem 3.19, we have F=∅. So $\emptyset$ = F⊆ μ**p**(int(μ**p**(cl(A))-A)).Therefore μ**p**(cl(A))-A is μ**p**$\hat{g}$ open.

Theorem 4.6: If A and B are μ**p**$\hat{g}$ open sets in X then A∩B is also μ**p**$\hat{g}$ open sets in X.

Proof: Let A and B be two μ**p**$\hat{g}$ open sets in X. Then A^c and B^c are μ**p**$\hat{g}$ closed sets in X. By theorem3.15, A^c∪B^c is a μ**p**$\hat{g}$ closed in X. That is (A∩B)^c is a μ**p**$\hat{g}$ closed in X. Therefore (A∩B) is μ**p**$\hat{g}$ open set in X.

Theorem 4.7: If A and B are μ**p**$\hat{g}$ open sets in X then A∪B also μ**p**$\hat{g}$ open set in X.

Proof: Let A and B be two μ**p**$\hat{g}$ open sets in X. Then A^c and B^c are μ**p**$\hat{g}$ closed sets in X. By theorem 3.16, A^c∩B^c is a μ**p**$\hat{g}$ closed in X. That is (A∩B)^c is a μ**p**$\hat{g}$ closed in X. Therefore A∪B is μ**p**$\hat{g}$ open sets in X.

Theorem 4.8: A × B is a μ**p**$\hat{g}$ open subset of (X ×Y,τ × σ), iff A is a μ**p**$\hat{g}$ open subset in (X,τ) and B is a μ**p**$\hat{g}$ open subset in (Y, σ) .

Proof: Let A × B be a μ**p**$\hat{g}$ open subset of (X ×Y,τ × σ). Let H be a closed subset of (X,τ) and G be a closed subset of (Y,σ) such that H⊆A,G⊆B.Then H×G is closed in (X ×Y,τ × σ) such that H×G⊆A×B.By assumption A×B is a μ**p**$\hat{g}$ open subset of (X ×Y,τ × σ) and so H×G⊆ μ**p** (int(A×B)) ⊆ μ**p** (int(A))× μ**p** (int(B)).That is H⊆ μ**p** (int(A)),G⊆ μ**p**(int(B)) and hence A is a μ**p**$\hat{g}$ open subset in (X,τ) and B is a μ**p**$\hat{g}$ open subset in (Y,σ).Conversely , let M be a closed subset of (X×Y,τ × σ) such that M⊆A×B.For each (x,y) ⊆ M, cl(X) ×cl(Y) ⊆cl(M)=M⊆A × B. Then the two closed sets cl(X) and cl(Y) are contained in A and B respectively. By assumption cl(X) ⊆ μ**p** (int(A)) and cl(Y) ⊆ μ**p** (int(B)) hold. This implies that for each (x,y) ⊆M, (x,y) ⊆ μ**p** (int(A×B)).Thus AxB is a μ**p**$\hat{g}$ open subset of (X ×Y,τ × σ).

5. On μp$\hat{g}$ continuity

Definition 5.1: A map f: (X,τ) → (Y,σ) is called

1. Continuous [3] if f^{-1}(V) is closed subset in (X,τ) for every closed subset V in (Y,σ).
2. Midly g continuous [9] if f^{-1}(V) is midly g closed subset in (X,τ) for every closed subset V in (Y,σ).
3. g continuous [2] if f^{-1}(V) is g closed subset in (X,τ) for every closed subset V in (Y,σ).
4. *g continuous [17] if f^{-1}(V) is *g closed subset in (X,τ) for every closed subset V in (Y,σ).
5. g* continuous [13] if f^{-1}(V) is g* closed subset in (X,τ) for every closed subset V in (Y,σ).
6. Regular continuous [1] if f^{-1}(V) is r closed subset in (X,τ) for every closed subset V in (Y,σ).
7. gr continuous [5] if f^{-1}(V) is gr closed subset in (X,τ) for every closed subset V in (Y,σ).

Definition 5.2: A function f:(X,τ) →(Y,σ) is called μ**p**$\hat{g}$ continuous if f^{-1}(V) is μ**p**$\hat{g}$ closed subset of (X,τ) for every closed subset V of (Y,σ).

Theorem 5.3: Every continuous map is μ**p**$\hat{g}$ continuous, but not conversely.

Proof: The proof follows from the fact that every closed set is μ**p**$\hat{g}$ closed set.

Example 5.4: Let X=Y={a, b, c, d}, τ = {X,ϕ,{a},{b},{a,b},{a,b,c}} and σ ={X, ϕ ,{b},{a, b}} .define a map f : X→Y by f(a) = a, f(b) = c, f(c) = d, f(d) = c. This map is μ**p**$\hat{g}$ continuous, but not continuous. Since for the closed set U = { d} in Y. f^{-1}(U) = {css} is not closed in X.

Theorem 5.5: Every regular continuous map is μ**p**$\hat{g}$ continuous, but not conversely.

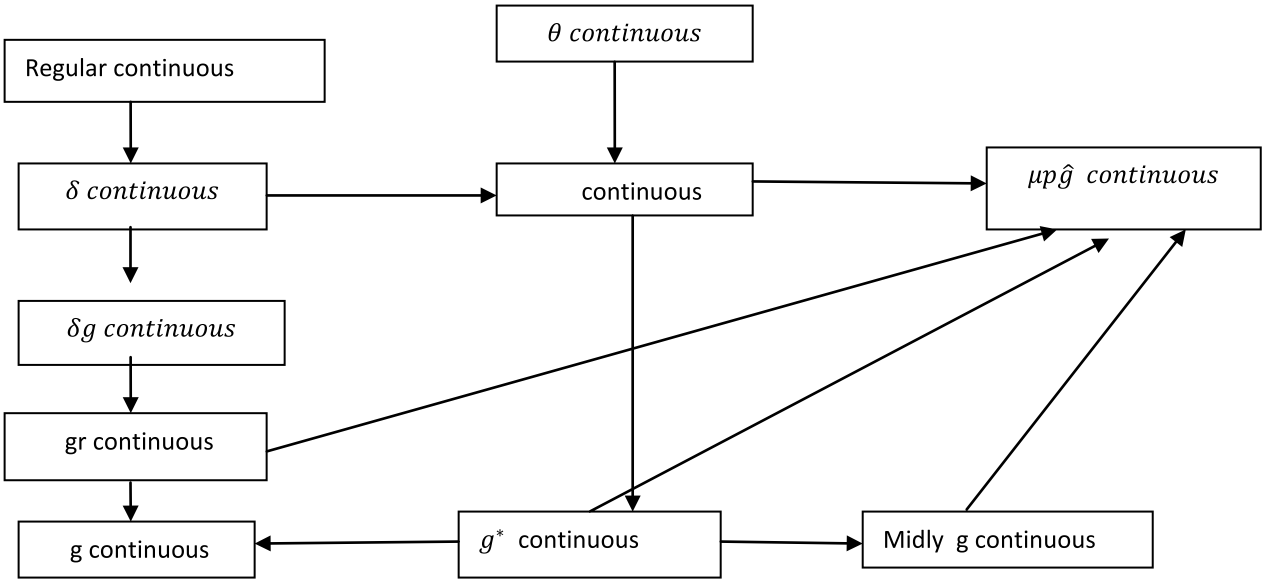
Proof: The proof follows from the fact that every regular closed set is μ**p**$\hat{g}$ closed set.

Example 5.6: Let X=Y={a, b, c, d}, τ ={X,ϕ , {a},{b},{a, b},{a, b, c}} and σ ={X,ϕ , {b},{b, d}} .define a map f : X→Y by f(a) = a, f(b) = c, f(c) = d, f(d) = c. This map is μ**p**$\hat{g}$ continuous, but not regular continuous. Since for the closed set U={d} in Y. f^{-1}(U) = {c} is not regular closed in X.

Theorem 5.7: Every g continuous map is μ**p**$\hat{g}$ continuous, but not conversely.

Proof: The proof follows from the fact that every g closed set is μ**p**$\hat{g}$ closed set.

Example 5.8: Let X=Y={a, b, c, d}, τ={X,ϕ,{a},{b},{a,b},{a,b, c}} and σ ={X,ϕ, {b},{b, d}}. define a map f : X→Y by f(a) = b, f(b) = a, f(c) = d, f(d) = c. This map is μ**p**$\hat{g}$ continuous, but not g continuous. Since for the closed set U={d} in Y. f^{-1}(U)={c} is not g closed in X.



Theorem 5.9: If f: X→Y is μ**p**$\hat{g}$ continuous and g:Y→Z is continuous then their composition f∘g : X→Z is μ**p**$\hat{g}$ continuous.

Proof: Let f: X→Y is μ**p**$\hat{g}$ continuous and g:Y→Z is continuous. Let U be a closed set in Z Therefore g^{-1}(U) is closed in Y and f^{-1}(g^{-1}(U)) is μ**p**$\hat{g}$ closed in X . ∴ f∘g is μ**p**$\hat{g}$ continuous.

Theorem 5.10: Let X and Y be topological spaces .Let f : (X,τ) →(Y,σ).Then the following are equivalent.

(i) (i).f is μ**p**$\hat{g}$ continuous.
(ii) (ii).for every subset A of X, one has f $\overline{(A)} \subset \overline{f(A)}$.
(iii) (iii).for every closed set B of Y, the set f^{-1}(B) is closed in X.
(iv) for each x∈ X and each neighborhood V of f(x), there is a neighborhood U of x such that f(U) ⊂ V.

Proof:
(i)⟹(ii): Assume that f is μ**p**$\hat{g}$ continuous. Let A be a subset of X. Let V be a neighborhood of f(x), then f^{-1}(V) is an open set of X containing x, it must intersect A in some point y. Then V intersects f(A) in the point f(y). So that f(x) ∈ $\overline{f(A)}$

(ii)⟹(iii): Let B be closed in Y and let A=f^{-1}(B).Prove that, A is closed in X and we show that $\bar{A}$ = A. By elementary set theory, we have f(A)=f(f^{-1}(A)) ⊂B, If x∈ $\bar{A}$, f(x) ∈ f $\overline{(A)}$ ⊂ $\overline{f(A)}$ ⊂ $\bar{B}$=B. f(x) ∈B, so that x ∈f^{-1}(B)=A. Thus $\bar{A} \subset A$, So that $\bar{A} = A$.

(iii)⟹(i): Let V be an open set of Y set B=Y-V. Then f^{-1}(B) = f^{-1}(Y-V) = f^{-1}(Y)-f^{-1}(V)=X-f^{1}(V). Now B is a closed set of Y. Then f^{-1}(B) is closed in X by hypothesis so that f^{-1}(V) is open in X.

(i)⟹(iv): Let x∈X and let V be a neighborhood of f(x). Then the set U= f^{-1}(V) is a neighborhood of x such that f (U) ⊂V.

(iv)⟹(i): Let V be an open set of Y. Let x be a point of f^{-1}(V).Then f(x) ∈V, so that by hypothesis there is a neighborhood U_x of x such that f(U_x) ⊂ V. Then U_x ⊂ f^{-1}(V), and hence f^{1}(V) = $\bigcup_{x\in f-1(V)}$ Ux .Therefore f is continuous⟹f is μp$\hat{g}$ continuous.

Theorem 5.16: Let X=A∪ B, where A and B are closed in X. Let f : A→ Y and g : B→ Y be continuous. If f(x) = g(x) for every x∈ A ∩ B then f and g combine to give a μp$\hat{g}$ continuous function h: X→Y defined by setting h(x) = f(x) if x ∈ A, and h(x) = g(X) if x ∈ B.

Proof: Let c be a closed subset of Y. Now h^{-1}(c)= f^{-1}(c) ∪g^{-1}(c).Since f is continuous, f^{-1}(c) is closed in A and therefore closed in X. Similarly g^{-1}(c) is closed in B and therefore closed in X. Their union h^{-1}(c) is also closed in X. Therefore h is continuous. By theorem 5.3, h is μp$\hat{g}$ continuous.

Theorem 5.17: A function f: (X,τ) → (Y,σ) from a topological space *X* into a topological space *Y* is μp$\hat{g}$ continuous if and only if $f^{1}(V)$ is μp$\hat{g}$ open set in *X* for every open set *V* in *Y*

Proof: It is obvious

Theorem 5.18: Let f: (X,τ) → (Y,σ) be a function from a topological space *X* into a topological space Y. If f: (X,τ) → (Y,σ)is continuous then *f*(μp$\hat{g}$cl*(A))*⊆ $cl(f(A)$ for every open subset *A* of *X*.

Proof: Since $f(A) \subseteq cl(f(A) \Rightarrow A \subseteq f^{-1}(cl(f(A)))$. Since $cl(f(A)$ is closed set in *Y* and *f* is μp$\hat{g}$ continuous, then $f^{1}(cl(f(A)))$ is a μp$\hat{g}$ closed set in *X* containing *A*. Hence μp$\hat{g}$*cl(A)*⊆ $f^{-1}(cl(f(A)))$. Therefore *f*(μp$\hat{g}$*cl(A))*⊆ $cl(f(A)$

Theorem 5.19: Let f: (X,τ) → (Y,σ)be a function from a topological space *X* into a topological space Y. Then the following statements are equivalent.

(i) For each point *x* in *X* and each open set *V* in *Y* with *f(x)*∈*V*, there is a μp$\hat{g}$ open set *U* in *X* such that *x*∈*U* and *f(U)*⊆ *V*
(ii) For each subset *A* of *x*, f(μp$\hat{g}$*cl(A))* ⊆ $cl(f(A)$
(iii) For each subset *B* of *y*, μp$\hat{g}$cl(f^{-1}(B)) ⊆f^{-1}(cl(B))

Proof:
(i) ⇒ **(ii):** Suppose that (i) holds and let y ∈ f (μp$\hat{g}$cl(A)) and let V be any open neighborhood of y. Since y∈f(μp$\hat{g}$cl(A)) ⇒ ∃ x∈ μp$\hat{g}$cl(A) such that f(x) = y. Since f(x) ∈ V, then by (i) ∃ a μp$\hat{g}$ open set U in x such that x∈U and f(U)⊆V. Since x∈ μp$\hat{g}$cl(A) then for any x∈ X x∈ μp$\hat{g}$cl*(A)*if and only if U∩A≠ ∅for every μp$\hat{g}$ open set U containing x, and hence f(A) ∩V≠ ∅. Therefore we have y = f(x) ∈cl(f(A)). Hence f(μp$\hat{g}$cl(A)) ⊆cl(f(A)).

(ii) ⇒ **(i):** If (ii) holds and let x∈X and V be any open set in Y containing f(x). Let A=f^{-1}(V^c) ⇒ x∉A. Since f(μp$\hat{g}$cl(A)) ⊆cl(f(A)) ⊆V^c⇒ μp$\hat{g}$cl(A) ⊆f^{-1}(V^c)=A. Since x∉A ⇒ x∉ μp$\hat{g}$cl(A) then for any x∈ X, x∈ μp$\hat{g}$cl*(A)*if and only if U∩A≠ ∅,there exists a μp$\hat{g}$ open set U containing x such that U ∩A= ∅ and hence f(U) ⊆f(A^c) ⊆V.

(ii) ⇒**(iii):** Suppose that (ii) holds and let B be any subset of Y. Replacing A by f^{-1}(B) we get from (ii), f(μp$\hat{g}$cl(f^{-1}(B)) ⊆cl(f(f^{-1}(B))) ⊆cl(B). Hence μp$\hat{g}$(f^{-1}(B)) ⊆f^{-1}(cl(B)).

(iii) ⇒**(ii):** Suppose that (iii) holds. Let B=f(B) where A is a subset of X. then we get from (iii) μp$\hat{g}$cl(A) ⊆ μp$\hat{g}$cl(f^{-1}(f(A)) ⊆f^{-1}(cl(f(A)). Therefore f(μp$\hat{g}$cl(A)) ⊆cl(f(A)).

Theorem 5.20: Let f: (X,τ) → (Y,σ) be a function. Then the following are equivalent.

(i) f is μp$\hat{g}$ continuous.
(ii) The inverse image of each open set in Y is μp$\hat{g}$ open in X.
(iii) The inverse image of each closed set in Y is μp$\hat{g}$ closed in X.

Proof: (i) ⇒**(ii):** Let G be any open set in Y. Then Y-G is closed in Y. Since f is μp$\hat{g}$continuous, f^{-1}(Y-G) is closed in X. But f^{-1}(Y-G)=X- f^{-1}(G) is μp$\hat{g}$ closed in X. Therefore f^{-1}(G) is μp$\hat{g}$ open in X.

(ii) ⇒ **(iii)** and **(iii)** ⇒ **(i)** are obvious.

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