International Journal of Mathematical Archive-6(9), 2015, 18-21
IJMA Available online through www.ijma.info ISSN 2229 – 5046

STRING COSMOLOGY IN SAEZ-BALLESTER THEORY OF GRAVITATION

*PUND A. M., **NIMKAR A. S.

*Department of Mathematics,
Science College, Congress Nagar, Nagpur (M.S.), India.

**Department of Mathematics,
Shri. Dr. R. G. Rathod Arts & Science College, Murtizapur, Dist. Akola, (M.S.), India.

(Received On: 10-08-15; Revised & Accepted On: 24-09-15)

ABSTRACT

Ruban's space time is considered in the presence of a cosmic string source in the frame work of scalar-tensor theory of gravitation proposed by Saez-Ballester [5]. To get the deterministic solution assume the relation between metric potential and Nambu string. Some physical and kinematical properties of the models are also discussed.

Keywords: *Ruban's metric, cosmic string, Saez-Ballester theory.*

1. INTRODUCTION

In recent year there has been considerable interest in scalar–tensor theories of gravitation proposed by [1-5]. Brans-Dicke theory includes a long range scalar field interacting equally with all forms of matter (with the exception of electromagnetism) while in Saez-Ballester scalar-tensor theory the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields.

The field equations given by [5] for the combined Scalar and tensor fields are

$$G_{ij} - \omega\phi^{n}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1}$$

$$2\phi^{n}\phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, R_{ij} is the Ricci tensor, R is the scalar curvature, n an arbitrary constant , ω is a dimensionless coupling constant and T_{ij} is the matter energy-momentum tensor. Here comma and semicolon denote partial and covariant differentiation respectively.

The equation of motion

$$T^{ij}_{;j} = 0, \tag{3}$$

is a consequence of field equation (1) and (2).

A detailed discussion of Saez - Ballester cosmological model is contained in the work of [6-11]. The cosmic string plays an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature drops down below some critical temperature as predicted by grand unified theories [12-17]. It is thought that cosmic strings cause density perturbations leading to the formation of galaxies [18]. These cosmic strings have stress-energy and couple with the gravitational field. Therefore it is interesting to study the gravitational effects that arise from strings. The general relativistic treatment of strings was started by [19-21]. Exact solutions of string cosmology in various space-times have been studied by several authors [18-23].

*Corresponding Author: *Pund A. M., *Department of Mathematics,
Science College, Congress Nagar, Nagpur (M.S.), India.*

The purpose of the present work is to obtained Ruban's cosmological model in a scalar tensor theory of gravitation proposed by Saez and Ballester in presence of a cosmic string. Our paper is organized as follows. In section 2, we derive the field equation in presence cosmic string with the aid of Ruban's space time. Section 3, deals with solution of field Equations, section 4 is mainly concerned with the physical and kinematical properties of the model. The last section contains some conclusion.

2. THE METRIC AND FIELD EQUATIONS

We consider the space- time of Ruban's [24] in the form

$$ds^2 = dt^2 - Q^2(x,t)dx^2 - R^2(t)(dy^2 + h^2 dz^2) \tag{4}$$

where $h(y) = \dfrac{\sin\sqrt{k}y}{\sqrt{k}} = \begin{cases} \sin y & if \quad k=1 \\ y & if \quad k=0 \\ \sinh y & if \quad k=-1 \end{cases}$

and k is the curvature parameter of the homogeneous 2-spaces t and x constants. The functions Q and R are free and will be determined.

The energy momentum tensor for cosmic strings is given by

$$T_i^j = \rho\, u_i u^j - \lambda\, x_i x^j \tag{5}$$
,

Here ρ is the rest energy density of cloud of strings with particles attached to them, $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the tension density of the system of strings. As pointed out by [11] λ may be positive or negative, u^i describes the system four-velocity and x^i represents a direction of anisotropy, i.e. the direction of the strings.

We have

$$u^i u_i = -x^i x_i = -1 \text{ , and } u^i x_i = 0 \tag{6}$$

we consider $\rho = \rho_p + \lambda$ (7)

where ρ_p is the rest energy density of the particles attached to the string. Here ρ and λ are the functions of t only.

The energy-momentum tensor T_{ij} in comoving co-ordinates for cosmic string is given by

$$T_1^1 = \lambda,\ T_2^2 = T_3^3 = 0,\ T_4^4 = \rho \text{ and } T_j^i = 0 \text{ for } i \neq j. \tag{8}$$

The field equations (1-3) for (4) with the help of (5-8) reduce to

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} - \frac{\omega}{2}\phi^n \dot{\phi}^2 = \lambda \tag{9}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} - \frac{\omega}{2}\phi^n \dot{\phi}^2 = 0 \tag{10}$$

$$2\frac{\dot{R}\dot{Q}}{RQ} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} + \frac{\omega}{2}\phi^n \dot{\phi}^2 = \rho \tag{11}$$

$$\ddot{\phi} + \left(\frac{\dot{Q}}{Q} + 2\frac{\dot{R}}{R}\right)\dot{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0, \tag{12}$$

$$\dot{\rho} + (\rho - \lambda)\frac{\dot{Q}}{Q} + 2\rho\frac{\dot{R}}{R} = 0 \tag{13}$$

Here over head dot represent partial differentiation with respect to t.

© 2015, IJMA. All Rights Reserved

3. SOLUTION AND THE MODEL

Here we have four independent field equations (9-12) connecting five unknown R, Q, λ, ρ and ϕ. Therefore in order to obtain exact solutions, we must need one more relation connecting the unknown quantities. We assume the relations as the equation of state for strings $\rho = \lambda$ .Since the field equations are highly nonlinear, we also assume the relation between metric coefficients $Q = x^n R^n$.

Using this relation, the field equations (9-13) admit the exact solution.

$$R = M(c_3 t + c_4)^{\frac{1}{n+2}} \tag{14}$$

$$Q = N x^n (c_3 t + c_4)^{\frac{n}{n+2}} \tag{15}$$

where $M = (n+2)^{\frac{1}{n+2}}$ and $N = m^n$

$$\lambda = \rho = c_5 \frac{1}{(c_3 t + c_4)^{\frac{2}{n+2}}} \text{ where } c_5 = \frac{k_1}{M^2} = \frac{k_1}{(n+2)^{\frac{2}{n+2}}} \tag{16}$$

$$\phi = \left[\left(\frac{n+2}{2}\right)\int \frac{c_2}{x^n N M^2} \frac{1}{(c_3 t + c_4)} dt\right] \tag{17}$$

Using equations (14) and (15) the Metric (4) becomes

$$ds^2 = dt^2 - N^2 x^{2n} (c_3 t + c_4)^{\frac{2n}{n+2}} dx^2 - M^2 (c_3 t + c_4)^{\frac{2}{n+2}} (dy^2 + h^2 dz^2) \tag{18}$$

This metric can be transformed through a proper choice of coordinates to the form

$$ds^2 = dT^2 - N^2 x^{2n} T^{\frac{2n}{n+2}} dx^2 - M^2 T^{\frac{2}{n+2}} (dy^2 + h^2 dz^2) \tag{19}$$

Where $T = c_3 t + c_4$

4. THE PHYSICAL AND KINEMATICAL PROPERTIES

The energy density ρ, the string tension density λ for the mode (19) are given by $\lambda = \rho = c_5 \frac{1}{(T)^{\frac{2}{n+2}}}$ (2oa)

$$\varphi = \left[\left(\frac{n+2}{2}\right)\int \frac{c_2}{c_3 x^n N M^2} \frac{1}{T^2} dT\right] \tag{2ob}$$

Also,

The spatial volume is given by $V = x^n N M^2 T$ (21)

The scalar expansion, $\theta = \frac{\dot{Q}}{Q} + 2\frac{\dot{R}}{R} = \frac{c_1}{T}$ (22)

shear scalar, $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{c_3}{T^2}$ (23)

Hubble parameter, $H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\theta}{3} = \frac{c_1}{3T}$ (24)

The model (19) has no initial singularities while the energy density, tension density of the string given by (20a) and scalar field ϕ given by (20b) possess initial singularities. However as T increases these singularities vanish. Spatial volume of the model given by (21) shows anisotropic expansion of the universe (19) with time. For the model the expansion Scalar θ and shear scalar σ tend to zero as $T \to \infty$.

© 2015, IJMA. All Rights Reserved

5. CONCLUSION

In this paper, we have studied cosmological model generated by a cloud of string in Ruban's space time in the frame work of scalar tensor theory of gravitation proposed by [5]. For finding the exact solution the relation as the equation of state of Nambu [11] is used. The model is free from initial singularities and also expanding, shearing and non rotating in the standard way.

ACKNOWLEDGEMENT

One of authors (**A. M. Pund**) is thankful to U.G.C., New Delhi for financial assistance under Minor Research Project

REFERENCES

1. Bran, C.H., Dicke, R.H.:Phys.Rev., 124,925(1961).
2. Norvedt, K.Jr.: astrophys. J.161, 1069, (1970).
3. Lyra,G.,Math.Z.54,52(1951).
4. Sen, D.K., Dunn, K.A.:J.Math.Phys.12, 578(1971).
5. Saez D., Ballester, V. J.: Phys. Lett.A113, 467(1985).
6. Singh, T. and Agrawal, A. K.; Astrophys. Space Sci., 182, 28 (1991).
7. Shri Ram, Tiwari, S.K.: Astrophys Space Sci. 259, 91(1998).
8. Reddy, D.R.K., Murthy, C.S.V.V.R., Venkateswarlu, R.: Astrophys. Space Sci. 301, 79-82 (2006).
9. Adhav, K.S., Nimkar, A.S., Naidu, R.L.: Astrophys. Space Sci. 312, 165-169 (2007).
10. Adhav, K.S., Mete, V.G.,Nimkar, A.S. and Pund, A.M.: Int. J. Phys. 47, 2314 (2008).
11. Letelier, P.S.:Phys.Rev., D.28,2414(1983).
12. Kibble T. W. B.: Phys, A Matgs. Gen. 9 1387 (1976).
13. Zel'dovich Ya B, Kobzarev, I Yu, and Okun, L. B.: Zh, Eksp. Teor. Fiz, 67 3(1975).
14. Zel'dovich Ya B, Kobzarev, I Yu, and Okun, L. B.: Sov. Phys,-JETP 40 1(1975).
15. Kibble T. W. B.: Phys. Rep, 67 183(1980).
16. Everett A. E.: Phys, Rev, 24 858(1981).
17. Vilenkin A .: Phys, Rev, D 24 2082(1981).
18. Banerjee A, Sanyal A. K. and Chakraborty S.: Pramana.-J. Phys, 34 1(1990).
19. Chakraborty S.: Ind. J. Pure Appl. Phys. 29 31(1991).
20. Tikekar R and Patel L. K. Gen. Rel. Grav. 24 397(1992).
21. Tikekar R and Patel L. K. Pramana.- J. Phys. 42 48(1994_.
22. Patel L K and Maharaj S. D.: Pramana.- J. Phys. 47 3(1996).
23. Ram, S and Singh, T. K.: Gen. Rel. Grav. 27 1207(1995).
24. Lima J. A. S., Nobre M. A. S.: Class. Quantum Grav.7 399-409 (1990).

Source of support: U.G.C., New Delhi, India. Conflict of interest: None Declared

[Copy right © 2015. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]