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INTERVAL VALUED FUZZY SUBSEMIRINGS OF A SEMIRING UNDER HOMOMORPHISM

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ABSTRACT

In this paper, we study some of the properties of interval valued fuzzy subsemiring of a semiring under homomorphism and anti-homomorphism and prove some results on these.

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Key Words: *Interval valued fuzzy subset, interval valued fuzzy subsemiring, pseudo interval valued fuzzy coset.*

INTRODUCTION

Interval valued fuzzy sets were introduced independently by Zadeh [11], Grattan-Guiness [4], Jahn [6], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval valued membership function. Jun.Y.B and Kin.K.H [7] defined an interval valued fuzzy R-subgroups of nearsemirings. Solairaju.A and Nagarajan.R [10] defined the charactarization of interval valued anti fuzzy Left h-ideals over hemisemirings. Azriel Rosenfeld [2] defined a fuzzy group. K.Murugalingam & K.Arjunan [8] defined an interval valued fuzzy subsemiring of a semiring. We introduce the concept of interval valued fuzzy subsemiring of a semiring under homomorphism and anti-homomorphism and established some results.

1. PRELIRMINARIES

1.1 Definition [8]: Let X be any nonempty set. A mapping $[M] : X \to D[0, 1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X. Thus $M^-(x)$ is an interval (a closed subset of [0, 1]) and not a number from the interval [0, 1] as in the case of fuzzy subset. Note that [0] = [0, 0] and [1] = [1, 1].

1.2 Remark [8]: Let D^X be the set of all interval valued fuzzy subset of X, where D means D[0, 1].

1.3 Definition: Let [A] be an interval valued fuzzy subset of X. Then the following operations are defined as

(i) $?([A]) = \{\langle x, \text{rmin}\{[½, ½], [A](x)\} / \text{for all } x \in X\}$.
(ii) $!([A]) = \{\langle x, \text{rmax}\{[½,½], [A](x)\} / \text{for all } x \in X\}$.
(iii) $Q_\alpha([A]) = \{\langle x, \text{rmin}\{\alpha, [A](x)\} / \text{for all } x \in X \text{ and } \alpha \text{ in } D[0, 1]\}$.
(iv) $P_\alpha([A]) = \{\langle x, \text{rmax}\{\alpha, [A](x)\} / \text{for all } x \in X \text{ and } \alpha \text{ in } D[0, 1]\}$.
(v) $G_\alpha([A]) = \{\langle x, \alpha [A](x)\}\rangle / \text{for all } x \in X \text{ and } \alpha \text{ in } [0, 1]\}$.

1.4 Definition [8]: Let (R, +, ·) be a semiring. An interval valued fuzzy subset [M] of R is said to be an **interval valued fuzzy subsemiring** of R if the following conditions are satisfied:

(i) $[M](x+y) \geq \text{rmin}\{[M](x), [M](y)\}$
(ii) $[M](xy) \geq \text{rmin}\{[M](x), [M](y)\}$ for all x and y in R.

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1.5 Definition: Let (R, +, ·) and (R^{I}, +, ·) be any two semirings. Let f : R→ R^{I} be any function and [M] be an interval valued fuzzy subsemiring in R, [V] be an interval valued fuzzy subsemiring in f(R) = R^{I}, defined by $[V](y) = \sup_{x \in f^{-1}(y)} [M](x)$, for all x in R and y in R^{I}. Then [M] is called a pre-image of [V] under f and is denoted by f^{-1}([V]).

1.6 Definition: Let [M] be an interval valued fuzzy subsemiring of a semiring (R, +, ·) and a in R. Then the **pseudo interval valued fuzzy coset** $(a[M])^p$ is defined by $((a[M])^p)(x)$ = p(a)[M](x) for every x in R and for some p in P.

2. SOME PROPERTIES

2.1 Theorem: Let (R, +, ·) and (R^{I}, +, ·) be any two semirings. The homomorphic image of an interval valued fuzzy subsemiring of R is an interval valued fuzzy subsemiring of R^{I}.

Proof: Let f: R→R^{I} be a homomorphism. Let [M] be an interval valued fuzzy subsemiring of R. Let [V] be the homomorphic image of [M] under f. We have to prove that [V] is an interval valued fuzzy subsemiring of f(R) = R^{I}. Let f(x) and f(y) in R^{I}. Then [V](f(x) +f(y)) = [V](f(x+y)) ≥ [M](x+y) ≥ rmin{[M](x), [M](y)} which implies that [V](f(x)+f(y)) ≥ rmin{[V](f(x)), [V](f(y))}. And [V](f(x)f(y)) = [V](f(xy)) ≥ [M](xy) ≥ rmin {[M](x), [M](y)} which implies that [V](f(x)f(y)) ≥ rmin{[V](f(x)), [V](f(y))}. Hence [V] is an interval valued fuzzy subsemiring of a semiring R^{I}.

2.2 Theorem: Let (R, +, ·) and (R^{I}, +, ·) be any two semirings. The homomorphic pre-image of an interval valued fuzzy subsemiring of R^{I} is an interval valued fuzzy subsemiring of R.

Proof: Let f: R→R^{I} be a homomorphism. Let [V] be an interval valued fuzzy subsemiring of f(R) = R^{I}. Let [M] be the pre-image of [V] under f. We have to prove that [M] is an interval valued fuzzy subsemiring of R. Let x and y in R. Then [M](x+y) = [V](f(x+y)) = [V](f(x)+f(y)) ≥ rmin {[V](f(x)), [V](f(y))}= rmin{[M](x), [M](y)} which implies that [M](x+y) ≥ rmin{[M](x), [M](y)} for x and y in R. And [M](xy) = [V](f(xy)) = [V](f(x)f(y)) ≥ rmin{[V](f(x)), [V](f(y))} = rmin{[M](x), [M](y)} which implies that [M](xy) ≥ rmin{[M](x), [M](y)} for x and y in R. Hence [M] is an interval valued fuzzy subsemiring of the semiring R.

2.3 Theorem: Let (R, +, ·) and (R^{I}, +, ·) be any two semirings. The anti-homomorphic image of an interval valued fuzzy subsemiring of R is an interval valued fuzzy subsemiring of R^{I}.

Proof: Let f: R→ R^{I} be a anti-homomorphism. Let [M] be an interval valued fuzzy subsemiring of R. Let [V] be the homomorphic image of [M] under f. We have to prove that [V] is an interval valued fuzzy subsemiring of f(R) = R^{I}. Let f(x) and f(y) in R^{I}. Then [V](f(x)+f(y)) = [V](f(y+x)) ≥ [M](y+x) ≥ rmin{[M](x), [M](y)} which implies that [V](f(x)+f(y)) ≥ rmin{[V](f(x)), [V](f(y))}. And [V](f(x)f(y)) = V(f(yx)) ≥ [M](yx) ≥ rmin{[M](x), [M](y)} which implies that [V](f(x)f(y)) ≥ rmin {[V](f(x)), [V](f(y))}. Hence [V] is an interval valued fuzzy subsemiring of R'.

2.4 Theorem: Let (R, +, ·) and (R^{I}, +, ·) be any two semirings. The anti-homomorphic pre-image of an interval valued fuzzy subsemiring of R^{I} is an interval valued fuzzy subsemiring of R.

Proof: Let f: R → R^{I} be a anti-homomorphism. Let [V] be an interval valued fuzzy subsemiring of f(R) = R^{I}. Let [M] be the pre-image of [V] under f. We have to prove that [M] is an interval valued fuzzy subsemiring of R. Let x and y in R. Then [M](x+y) = [V](f(x+y)) = [V](f(y)+f(x)) ≥ rmin{[V](f(x)), [V](f(y))} = rmin{[M](x), [M](y)} which implies that [M](x+y) ≥ rmin{[M](x), [M](y)} for all x and y in R. And [M](xy) = [V](f(xy)) = [V](f(y)f(x)) ≥ rmin{[V](f(x)), [V](f(y))} = rmin{[M](x), [M](y)} which implies that [M](xy) ≥ rmin{[M](x), [M](y)} for all x and y in R. Hence [M] is an interval valued fuzzy subsemiring of the semiring R.

In the following Theorem ◦ is the composition operation of functions:

2.5 Theorem: Let [M] be an interval valued fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then [M]◦f is an interval valued fuzzy subsemiring of R.

Proof: Let x and y in R and [M] be an interval valued fuzzy subsemiring of the semiring H. Then ([M]◦f)(x+y) = [M](f(x+y)) = [M](f(x)+f(y)) ≥ rmin{[M](f(x)), [M](f(y)) ≥ rmin{([M]◦f)(x), ([M]◦f)(y)}which implies that ([M]◦f)(x+y) ≥ rmin{([M]◦f)(x), ([M]◦f)(y). And ([M]◦f)(xy) = [M](f(xy)) = [M](f(x)f(y)) ≥ rmin{[M](f(x)), [M](f(y))} ≥ rmin{([M]◦f)(x), ([M]◦f)(y)} which implies that ([M]◦f)(xy) ≥ rmin{([M]◦f)(x), ([M]◦f)(y)}. Therefore ([M]◦f) is an interval valued fuzzy subsemiring of a semiring R.

2.6 Theorem: Let [M] be an interval valued fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then [M]◦f is an interval valued fuzzy subsemiring of R.

Proof: Let x and y in R and [M] be an interval valued fuzzy subsemiring of the semiring H. Then ([M]◦f)(x+y) = [M](f(x+y)) = [M](f(y)+f(x)) ≥ rmin{[M](f(x)), [M](f(y))}≥ rmin{([M]◦f)(x), ([M]◦f)(y)} which implies that ([M]◦f)(x+y)≥rmin{([M]◦f)(x), ([M]◦f)(y)}. And ([M]◦f)(xy)= [M](f(xy)) = [M](f(y)f(x)) ≥ rmin{[M](f(x)), [M](f(y))} ≥ rmin{([M]◦f)(x), ([M]◦f)(y)} which implies that ([M]◦f)(xy) ≥ rmin{([M]◦f)(x), ([M]◦f)(y)}. Therefore ([M]◦f) is an interval valued fuzzy subsemiring of R.

2.7 Theorem: Let [M] be an interval valued fuzzy subsemiring of a semiring R, then the pseudo interval valued fuzzy coset $(a[M])^p$ is an interval valued fuzzy subsemiring of the semiring R, for every a in R.

Proof: Let [M] be an interval valued fuzzy subsemiring of the semiring R. For every x and y in R, we have $((a[M])^p)(x+y)$ = p(a)[M](x+y) ≥ p(a) rmin{[M](x), [M](y)}= rmin {p(a)[M](x), p(a)[M](y)} = rmin{$((a[M])^p)(x)$, $((a[M])^p)(y)$}. Therefore $((a[M])^p)(x+y)$ ≥ rmin{$((a[M])^p)(x)$, $((a[M])^p)(y)$} for x and y in R. And $((a[M])^p)(xy)$ = p(a)[M](xy) ≥ p(a) rmin{[M](x), [M](y)} = rmin{p(a)[M](x), p(a)[M](y)}=rmin{$((a[M])^p)(x)$, $((a[M])^p)(y)$}. Therefore $((a[M])^p)(xy)$ ≥ rmin{$((a[M])^p)(x)$, $((a[M])^p)(y)$} for x and y in R. Hence $(a[M])^p$ is an interval valued fuzzy subsemiring of R.

2.8 Theorem [8]: If [M] and [N] are two interval valued fuzzy subsemirings of a semiring R, then their intersection [M]∩[N] is an interval valued fuzzy subsemiring of R.

2.9 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then ?([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, we have ?([M])(x+y) = rmin{[½,½], [M](x+y)} ≥ rmin{[½,½], rmin {[M](x), [M](y)}} = rmin{rmin {[½,½], [M](x)}, rmin {[½,½], [M](y)}} = rmin {?([M])(x), ?([M])(y)}. Therefore ?([M])(x+y) ≥ rmin {?A^+(x), ?A^+(y)} for all x and y in R. Also ?([M])(xy) = rmin {[½,½], [M](xy)} ≥ rmin {[½,½], rmin {[M](x), [M](y)}} = rmin {rmin{[½,½], [M](x)}, rmin {[½,½], [M](y)}} = rmin {?([M])(x), ?([M])(y)}. Therefore ?([M])(xy) ≥ rmin {?A^+(x), ?A^+(y) } for all x and y in R. Hence ?([M]) is an interval valued fuzzy subsemiring of R.

2.10 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then !([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, we have !([M])(x+y) = rmax{[½,½], [M](x+y)} ≥ rmax {[½,½], rmin{[M](x), [M](y)}} = rmin {rmax{[½,½], [M](x)}, rmax {[½,½], [M](y)}} = rmin {!([M])(x), !([M])(y)}. Therefore !([M])(x+y) ≥ rmin{!([M])(x), !([M])(y)} for all x and y in R. Also !([M])(xy) = rmax{[½,½], [M](xy)}≥ rmax {[½,½], rmin {[M](x), [M](y)}}= rmin{rmax {[½,½], [M](x)}, rmax {[½,½], [M](y)}}= rmin{!([M])(x), !([M])(y)}. Therefore !([M])(xy)≥ rmin{!([M])(x), !([M])(y)} for all x and y in R. Hence !([M]) is an interval valued fuzzy subsemiring of R.

2.11 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then Q_α([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in D[0, 1], we have Q_α([M])(x+y) = rmin {α, [M](x+y)}≥ rmin{α, rmin{[M](x), [M](y)}} = rmin {rmin {α, [M](x)}, rmin {α, [M](y)}} = rmin{Q_α([M])(x), Q_α([M])(y)}. Therefore Q_α([M])(x+y) ≥ rmin {Q_α([M])(x), Q_α([M])(y)} for all x and y in R. Also Q_α([M])(xy) = rmin {α, [M](xy)} ≥ rmin {α, rmin{[M](x), [M](y)}} = rmin {rmin {α, [M](x)}, rmin{α, [M](y)}} = rmin {Q_α([M])(x), Q_α([M])(y)}. Therefore Q_α([M])(xy) ≥ rmin {Q_α([M])(x), Q_α([M])(y)} for all x and y in R. Hence Q_α([M]) is an interval valued fuzzy subsemiring of R.

2.12 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then P_α([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in D[0, 1], we have P_α([M])(x+y) = rmax{α, [M](x+y)} ≥ rmax {α, rmin{[M](x), [M](y)}} = rmin{rmax {α, [M](x)}, rmax{α, [M](y)}} = rmin{P_α([M])(x), P_α([M])(y)}. Therefore P_α([M])(x+y) ≥ rmin {P_α([M])(x), P_α([M])(y)} for all x and y in R. Also P_α([M])(xy) = rmax{α, [M](xy)} ≥ rmax {α, rmin{[M](x), [M](y)}} = rmin {rmax {α, [M](x)}, rmax{α, [M](y)}} = rmin {P_α([M])(x), P_α([M])(y)}. Therefore P_α([M])(xy) ≥ rmin{ P_α([M])(x), P_α([M])(y)} for all x and y in R. Hence P_α ([M]) is an interval valued fuzzy subsemiring of R.

2.13 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then G_α([M]) is an interval valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in [0, 1], we have $G_\alpha([M])(x+y) = \alpha\ [M](x+y) \geq \alpha\ (rmin\{[M](x), [M](y)\}) = rmin\{\alpha\ [M](x), \alpha\ [M](y)\} = rmin\ \{G_\alpha([M])(x), G_\alpha([M])(y)\}$. Therefore $G_\alpha([M])(x+y) \geq rmin\ \{G_\alpha([M])(x), G_\alpha([M])(y)\}$ for all x and y in R. Also $G_\alpha([M])(xy) = \alpha\ [M](xy) \geq \alpha(rmin\{[M](x), [M](y)\}) = rmin\ \{\alpha\ [M](x), \alpha\ [M](y)\} = rmin\ \{G_\alpha([M])(x), G_\alpha([M])(y)\}$. Therefore $G_\alpha([M])(xy) \geq rmin\ \{G_\alpha([M])(x), G_\alpha([M])(y)\}$ for all x and y in R. Hence $G_\alpha([M])$ is an interval valued fuzzy subsemiring of R.

2.14 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
?([M]∩[N]) = ?([M]) ∩ ?([N]) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.9, the statement of the Theorem is true.

2.15 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
!([M]∩[N]) = !([M]) ∩!([N]) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.10, the statement of the Theorem is true.

2.16 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then !(?([M])) = ?(!([M])) is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.9, 2.10, the statement of the Theorem is true.

2.17 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
$Q_\alpha([M]\cap[N] = Q_\alpha([M]) \cap Q_\alpha([N])$ is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.11, the statement of the Theorem is true.

2.18 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
$P_\alpha([M]\cap[N]) = P_\alpha([M]) \cap P_\alpha([N])$ is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.12, the statement of the Theorem is true.

2.19 Theorem: If [M] is an interval valued fuzzy subsemiring of a semiring R, then $P_\alpha(Q_\alpha([M])) = Q_\alpha(P_\alpha([M]))$ is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.11, 2.12, the statement of the Theorem is true.

2.20 Theorem: If [M] and [N] are interval valued fuzzy subsemirings of a semiring R, then
$G_\alpha([M]\cap[N]) = G_\alpha([M]) \cap G_\alpha([N])$ is also an interval valued fuzzy subsemiring of R.

Proof: By Theorem 2.8, 2.13, the statement of the Theorem is true.

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