

## Pairwise $bg$ closed and Pairwise $*bg$ closed set in Bitopological Spaces

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### ABSTRACT

*In this paper, we introduce some new classes of sets namely Pairwise  $bg$  closed set, Pairwise  $*bg$  closed set. We obtain the basic properties and their relationships with other classes of sets in bitopological spaces. We devote the concept of Pairwise  $bg$  and Pairwise  $*bg$  continuous functions. The relationship between Pairwise  $bg$  continuous and Pairwise  $*bg$  continuous and other defined continuous functions are being deliberated.*

**Keywords and Phrases:** Pairwise  $bg$  closed set, Pairwise  $*bg$  closed set, Pairwise  $bg$  continuous function, Pairwise  $*bg$  continuous function, Pairwise  $bg$  irresolute function, Pairwise  $*bg$  irresolute function.

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## I. INTRODUCTION AND PRELIMINARIES

### INTRODUCTION

In 1963, J.C.Kelly [11] initiated the study of bitopological spaces. Following him several authors [3, 9, 11, 13, 15] have extended the concepts of topological spaces to bitopological spaces. Mean while Fukutake[8] introduced generalized closed sets and pairwise generalized closure operator in bitopological spaces in 1986. Abo khadra and Nasef [1] discussed  $b$ -open sets in bitopological spaces. In this paper we generalize the notions of Pairwise  $bg$  closed set, Pairwise  $*bg$  closed set in bitopological spaces and their characterizations are discussed. In 1991, K.Balachandran, P.Sundaram and H.Maki [17] defined a new class of mappings called generalized continuous mappings which contains the class of continuous mappings. We use the class of Pairwise  $bg$  closed and Pairwise  $*bg$  closed sets to develop the concept of continuity in bitopological spaces.

### PRELIMINARIES

**Definition: 1.1 [15]** Let  $(X, \tau)$  be a topological space. A set  $A$  is called semi-open set if  $A \subseteq C_1(\text{Int}(A))$ . The complement of semi - open set is semi - closed set.

**Definition: 1.2 [16]** Let  $(X, \tau)$  be a topological space. A set  $A$  is called pre - open set if  $A \subseteq \text{Int}(C_1(A))$ . The complement of pre - open set is pre - closed set.

**Definition: 1.3 [19]** Let  $(X, \tau)$  be a topological space. A set  $A$  is called  $\alpha$ - open set if  $A \subseteq \text{int}(C_1(\text{int}(A)))$ . The complement of  $\alpha$  - open set is  $\alpha$  - closed set.

**Definition: 1.4 [2]** Let  $(X, \tau)$  be a topological space. A set  $A$  is called  $b$  - open set if  $A \subseteq C_1(\text{Int}(A)) \cup \text{Int}(C_1(A))$  The complement of  $b$  - open set is called  $b$  - closed set.

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**Definition: 1.5 [9]** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called  $\tau_1\tau_2 - sg$  closed if  $\tau_2 - scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -semi open in  $X$ .

**Definition: 1.6 [12]** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called  $\tau_1\tau_2 - \omega$  closed if  $\tau_2 - cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -semi open in  $X$ .

**2. Pairwise *bg* closed and Pairwise \* *bg* closed set**

**Definition: 2.1** A set  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called Pairwise *bg* closed if  $\tau_2 - bcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -semi open in  $X$ .

**Definition: 2.2** A set  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called Pairwise \* *bg* closed if  $\tau_2 - bcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1 - \alpha$  open in  $X$ .

**Theorem: 2.3**

- (a) Every Pairwise *bg* closed set is Pairwise \* *bg* closed set.
- (b) Every  $\tau_1\tau_2 - \omega$  closed set is Pairwise *bg* closed set.
- (c) Every  $\tau_1\tau_2 - \omega$  closed set is Pairwise \* *bg* closed set.
- (d) Every  $\tau_1\tau_2 - sg$  closed set is Pairwise *bg* closed set.
- (e) Every  $\tau_1\tau_2 - sg$  closed set is Pairwise \* *bg* closed set.

**Proof:** a) Let  $A$  be Pairwise *bg* closed set. We have to prove  $A$  is Pairwise \* *bg* closed set. Let  $A \subseteq U$  and  $U$  is  $\tau_1 - \alpha$  open in  $X$ . Since every  $\alpha$  open set is semi open set then  $U$  is  $\tau_1$ -semi open in  $X$ . Also since  $A \subseteq U$  and  $U$  is  $\tau_1$ -semi open in  $X$  and  $A$  is Pairwise *bg* closed set, then  $\tau_2 - bcl(A) \subseteq U$ . Therefore  $A$  is Pairwise \* *bg* closed set. The other results follows from the definitions.

**Remark: 2.4** The converse of the above theorems are not true and it is shown by the following examples.

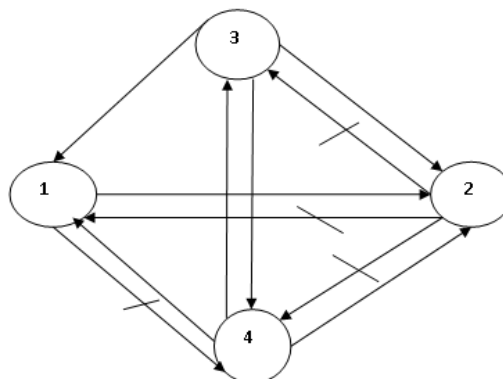
**Example: 2.5** Let  $X = \{a, b, c\}$ ;  $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ ;  $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ . Here  $\{a, b\}$  is Pairwise \* *bg* closed but not Pairwise *bg* closed set.

**Example: 2.6** Let  $X = \{a, b, c\}$ ;  $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ ;  $\tau_2 = \{\emptyset, X, \{b\}, \{b, c\}\}$ . Here  $\{c\}$  is Pairwise *bg* closed but not  $\tau_1\tau_2 - \omega$  closed set.

**Example: 2.7** Let  $X = \{a, b, c\}$ ;  $\tau_1 = \{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$ ;  $\tau_2 = \{\emptyset, X, \{a\}, \{a, c\}\}$ . Here  $\{c\}$  is Pairwise \* *bg* closed but not  $\tau_1\tau_2 - \omega$  closed set.

**Example: 2.8** Let  $X = \{a, b, c\}$ ;  $\tau_1 = \{\emptyset, X, \{c\}, \{a, b\}\}$ ;  $\tau_2 = \{\emptyset, X, \{a\}, \{a, c\}\}$ . Here  $\{a, c\}$  is Pairwise \* *bg* closed but not  $\tau_1\tau_2 - sg$  closed set.

**Remark: 2.9** From the above theorems and examples we have the following diagrammatic representation.



Here the numbers 1 to 4 represent the following:

1. Pairwise *bg* closed set
2. Pairwise \* *bg* closed
2.  $\tau_1\tau_2 - \text{sg}$  closed set
4.  $\tau_1\tau_2 - \omega$  closed set

**Proposition: 2.10** The finite union of Pairwise *bg* closed (Pairwise \* *bg* closed) set is Pairwise *bg* closed (Pairwise \* *bg* closed).

**Proof:** Let A and B be Pairwise *bg* closed (Pairwise \* *bg* closed) subsets of X and let U be  $\tau_1$ - semi open ( $\alpha$  open) in X such that  $A \cup B \subseteq U$ . Then  $\tau_2 - bcl(A) \subseteq U, \tau_2 - bcl(B) \subseteq U$ .

Therefore  $\tau_2 - bcl(A \cup B) = \tau_2 - bcl(A) \cup \tau_2 - bcl(B) \subseteq U$ . This implies  $\tau_2 - bcl(A \cup B) \subseteq U$ . Hence  $A \cup B$  is Pairwise *bg* closed (Pairwise \* *bg* closed) set.

**Theorem: 2.11** If A is an Pairwise *bg* closed (Pairwise \* *bg* closed) set of  $(X, \tau_1, \tau_2)$  such that  $A \subseteq B \subseteq \tau_2 - bcl(A)$  then B is also an Pairwise *bg* closed (Pairwise \* *bg* closed) set of X.

**Proof:** Let  $B \subseteq U$  where U is  $\tau_1$ - semi open ( $\alpha$  open) in X. Then  $A \subseteq B$  implies  $A \subseteq U$ . Since A is pairwise *bg* closed (Pairwise \* *bg* closed) then  $\tau_2 - bcl(A) \subseteq U$ .

Given  $B \subseteq \tau_2 - bcl(A)$  then  $\tau_2 - bcl(B) \subseteq \tau_2 - bcl(\tau_2 - bcl(A)) \subseteq \tau_2 - bcl(A) \subseteq U$ . Therefore B is Pairwise *bg* closed (Pairwise \* *bg* closed) set.

**Proposition: 2.12** If A is Pairwise *bg* closed (Pairwise \* *bg* closed) subset of  $(X, \tau_1, \tau_2)$  then  $[\tau_2 - bcl(A)] - A$  does not contain any non empty  $\tau_1 -$  semi closed ( $\alpha$  closed) sets.

**Proof:** Let A be Pairwise *bg* closed (Pairwise \* *bg* closed) set. Suppose  $F \neq \phi$  is  $\tau_1 -$  semi closed ( $\alpha$  closed) set of  $[\tau_2 - bcl(A)] - A$  then  $F \subseteq \tau_2 - bcl(A) - A$ . This implies  $F \subseteq \tau_2 - bcl(A)$  and  $F \subseteq X - A$ . Consider  $A \subseteq X - F$  then  $F \subseteq [\tau_2 - bcl(A)]^c$ . Therefore,  $F \subseteq [\tau_2 - bcl(A)] \cap [\tau_2 - bcl(A)]^c = \phi$ . Hence  $F = \phi$ .

**Corollary: 2.13** Let A be Pairwise *bg* closed (Pairwise \* *bg* closed) set in  $(X, \tau_1, \tau_2)$  then A is  $\tau_2 - b -$  closed iff  $[\tau_2 - bcl(A)] - A$  is  $\tau_1 -$  semi closed ( $\alpha$  closed) set.

**Proof:** Let A be Pairwise *bg* closed (Pairwise \* *bg* closed) set. If A is  $\tau_2 - b -$  closed we have  $\tau_2 - bcl(A) = A$  then  $[\tau_2 - bcl(A)] - A = \phi$  which is  $\tau_1 -$  semi closed ( $\alpha$  closed) set.

Conversely, let  $[\tau_2 - bcl(A)] - A$  is  $\tau_1 -$  semi closed ( $\alpha$  closed) set. Then by proposition 3.3,  $[\tau_2 - bcl(A)] - A$  is  $\tau_1 -$  semi closed ( $\alpha$  closed) subset of itself then  $[\tau_2 - bcl(A)] - A = \phi$ . This implies that  $\tau_2 - bcl(A) = A$ . Therefore A is  $\tau_2 - b -$  closed.

**Definition: 2.14** A subset  $A \subseteq X$  is called Pairwise *bg* open (Pairwise \* *bg* open) set iff its complement is Pairwise *bg* closed (Pairwise \* *bg* closed) set.

**Theorem: 2.15** A subset  $A \subseteq X$  is Pairwise *bg* open (Pairwise \* *bg* open) set iff  $F \subseteq \tau_2 - bint(A)$  whenever F is semi closed ( $\alpha$  closed) in  $\tau_1$  such that  $F \subseteq A$ .

**Proof:** Necessity: Let A be Pairwise *bg* open (Pairwise \* *bg* open) set and F be semi closed ( $\alpha$  closed) in  $\tau_1$  such that  $F \subseteq A$ . Then  $X - A$  is contained in  $X - F$  where  $X - F$  is semi open ( $\alpha$  open) in  $\tau_1$ . Since A is Pairwise *bg* open (Pairwise \* *bg* open),

$$\tau_2 - bcl(X - A) \subseteq X - F. \text{ This implies } X - [\tau_2 - bint(A)] \subseteq X - F. \text{ Thus } F \subseteq \tau_2 - bint(A).$$

Sufficiency: Suppose F is semi closed ( $\alpha$  closed) in  $\tau_1$  and  $F \subseteq A$ . This implies  $F \subseteq \tau_2 - bint(A)$ . Let  $X - A \subseteq U$ ,

where U is semi open ( $\alpha$  open) set in  $\tau_1$ . Then  $X - U \subseteq A$ , where  $X - U$  is semi closed ( $\alpha$  closed) in  $\tau_1$ . By hypothesis,  $X - U \subseteq \tau_2 - bint(A)$  (i.e)  $X - [\tau_2 - bint(A)] \subseteq U$ . Then  $\tau_2 - bcl(X - A) \subseteq U$  implies  $X - A$  is Pairwise *bg* closed (Pairwise \* *bg* closed) set. Therefore A is Pairwise *bg* open (Pairwise \* *bg* open) set.

**Theorem: 2.16** If  $A \subseteq X$  is Pairwise *bg* closed (Pairwise \* *bg* closed) set then  $[\tau_2 - bcl(A)] - A$  is Pairwise *bg* open (Pairwise \* *bg* open).

**Proof:** Let A be Pairwise *bg* closed(Pairwise \* *bg* closed). Let F be semi closed ( $\alpha$  closed) set in  $\tau_1$  such that  $F \subseteq [\tau_2 - bcl(A)] - A$ . Then by proposition 2.13,  $F = \phi$ . So  $F \subseteq [\tau_2 - bint([\tau_2 - bcl(A)] - A)]$ . This implies  $[\tau_2 - bcl(A)] - A$  is Pairwise *bg* open (Pairwise \* *bg* open) set.

### 3. Pairwise *bg* and Pairwise \* *bg* continuous functions

#### Definition: 3.1

(i) A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  where  $(X, \tau_1, \tau_2)$  and  $(Y, \sigma_1, \sigma_2)$  are bitopological space is pairwise *bg* continuous if  $f^{-1}(U)$  is Pairwise *bg* closed in X for each  $\sigma_j$  closed U in Y,  $i \neq j$  and  $i, j = 1, 2$

(ii) A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  where  $(X, \tau_1, \tau_2)$  and  $(Y, \sigma_1, \sigma_2)$  are bitopological space is pairwise \* *bg* continuous if  $f^{-1}(U)$  is pairwise \* *bg* closed in X for each  $\sigma_j$  closed U in Y,  $i \neq j$  and  $i, j = 1, 2$

#### Theorem: 3.2

- (a) Every pairwise *bg* continuous function is pairwise \* *bg* continuous function.
- (b) Every  $\tau_1 \tau_2 - \omega$  continuous function is pairwise *bg* continuous function.
- (c) Every  $\tau_1 \tau_2 - \omega$  continuous function is pairwise \* *bg* continuous function.
- (d) Every  $\tau_1 \tau_2 - sg$  continuous function is pairwise *bg* continuous function.
- (e) Every  $\tau_1 \tau_2 - sg$  continuous function is pairwise \* *bg* continuous function.

**Proof:** a) Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be pairwise *bg* continuous. Let U be  $\sigma_j$  closed set in Y. Then  $f^{-1}(U)$  is Pairwise *bg* closed set in X. Since every Pairwise *bg* closed set is pairwise \* *bg* closed set in X then  $f^{-1}(U)$  is pairwise \* *bg* closed set in X. Hence  $f$  is pairwise \* *bg* continuous function. The proof is obvious for others.

**Remark: 3.3** The converse of the above theorems are not true as shown by the following examples.

**Example: 3.4** Let  $X = Y = \{a, b, c\}$ ;  $\tau_1 = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ ;  $\tau_2 = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ ;  $\sigma_1 = \{\phi, Y, \{a, c\}\}$ ;  $\sigma_2 = \{\phi, Y, \{a\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function defined by  $f(a) = c$ ,  $f(b) = b$ ,  $f(c) = a$ .

Here  $f^{-1}(b, c) = \{a, b\}$  is pairwise \* *bg* closed but not pairwise *bg* closed set. Therefore  $f$  is pairwise \* *bg* continuous but not pairwise *bg* continuous function.

**Example: 3.5** Let  $X = Y = \{a, b, c\}$ ;  $\tau_1 = \{\phi, X, \{a\}, \{a, c\}, \{c\}, \{a, b\}\}$ ;  $\tau_2 = \{\phi, X, \{b\}, \{b, c\}\}$ ;  $\sigma_1 = \{\phi, Y, \{c\}, \{a, c\}\}$ ;  $\sigma_2 = \{\phi, Y, \{a, b\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be an identity function.

Here  $f^{-1}(c) = \{c\}$  is pairwise *bg* closed but not  $\tau_1 \tau_2 - \omega$  closed set. Therefore  $f$  is pairwise *bg* continuous but not  $\tau_1 \tau_2 - \omega$  continuous function.

**Example: 3.6** Let  $X = Y = \{a, b, c\}$ ;  $\tau_1 = \{\phi, X, \{c\}, \{b, c\}, \{a, c\}\}$ ;  $\tau_2 = \{\phi, X, \{a\}, \{a, c\}\}$ ;  $\sigma_1 = \{\phi, Y, \{b\}, \{b, c\}\}$ ;  $\sigma_2 = \{\phi, Y, \{a, b\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be an identity function. Here  $f^{-1}(c) = \{c\}$  is pairwise \* *bg* closed but not  $\tau_1 \tau_2 - \omega$  closed set. Therefore  $f$  is pairwise \* *bg* continuous but not  $\tau_1 \tau_2 - \omega$  continuous function.

Here  $f^{-1}(c) = \{c\}$  is pairwise \* *bg* closed but not  $\tau_1 \tau_2 - \omega$  closed set. Therefore  $f$  is pairwise \* *bg* continuous but not  $\tau_1 \tau_2 - \omega$  continuous function.

**Example: 3.7** Let  $X = Y = \{a, b, c\}$ ;  $\tau_1 = \{\phi, X, \{c\}, \{a, b\}\}$ ;  $\tau_2 = \{\phi, X, \{a\}, \{a, c\}\}$ ;  $\sigma_1 = \{\phi, Y, \{a\}, \{a, b\}\}$ ;  $\sigma_2 = \{\phi, Y, \{a\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function defined by  $f(a) = b$ ,  $f(b) = a$ ,  $f(c) = c$ . Here  $f^{-1}(b, c) = \{a, c\}$  is pairwise \* *bg* closed but not  $\tau_1 \tau_2 - sg$  closed set. Therefore  $f$  is but not  $\tau_1 \tau_2 - sg$  continuous function.

Here  $f^{-1}(b, c) = \{a, c\}$  is pairwise \* *bg* closed but not  $\tau_1 \tau_2 - sg$  closed set. Therefore  $f$  is but not  $\tau_1 \tau_2 - sg$  continuous function.

**Theorem: 3.8** The following are equivalent for a function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$

- (a) f is pairwise *bg* continuous (pairwise \* *bg* continuous).
- (b)  $f^{-1}(U)$  is pairwise *bg* open (pairwise \* *bg* open) in X for each  $\sigma_j -$  open set U in Y,  $i \neq j$  and  $i, j = 1, 2$ .

**Proof:** (a)  $\Rightarrow$  (b) Suppose that f is pairwise *bg* continuous (pairwise \* *bg* continuous). Let A be  $\sigma_j -$  open set in Y. Then  $Y - A$  is  $\sigma_j -$  closed set in Y. Since f is pairwise *bg* continuous (pairwise \* *bg* continuous),  $f^{-1}(Y - A)$  is pairwise *bg* closed (pairwise \* *bg* closed) in X,  $i \neq j$  and  $i, j = 1, 2$ . Consequently,  $f^{-1}(A)$  is pairwise *bg* open (pairwise \* *bg* open) in X.

(b)  $\Rightarrow$  (a) Suppose that  $f^{-1}(A)$  is pairwise *bg* open (pairwise \* *bg* open) in  $X$  for each  $\sigma_j$  – open set  $U$  in  $Y$ ,  $i \neq j$  and  $i, j = 1, 2$ . Let  $V$  be  $\sigma_j$  – closed set in  $Y$ . Then  $X - V$  is  $\sigma_j$  – open in  $Y$ . Then by our assumption,  $f^{-1}(X - V)$  is pairwise *bg* open (pairwise \* *bg* open) in  $X$ ,  $i \neq j$  and  $i, j = 1, 2$ . Then  $f^{-1}(V)$  is pairwise *bg* closed (pairwise \* *bg* closed) in  $X$ . Hence  $f$  is pairwise *bg* continuous (pairwise \* *bg* continuous).

**Remark: 3.9** The composition of two pairwise *bg* continuous (pairwise \* *bg* continuous) functions is not pairwise *bg* continuous (pairwise \* *bg* continuous) functions as shown by the following example.

**Example: 3.10** Let  $X = Y = Z = \{a, b, c\}$ ;  $\tau_1 = \{\phi, X, \{a, b\}, \{b, c\}, \{b\}\}$ ;  $\tau_2 = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}\}$ ;  $\sigma_1 = \{\phi, Y, \{b, c\}\}$ ;  $\sigma_2 = \{\phi, Y, \{c\}, \{a, c\}\}$ ;  $\gamma_1 = \{\phi, Z, \{b\}, \{a, c\}\}$ ;  $\gamma_2 = \{\phi, Z, \{b\}, \{b, c\}\}$ .

Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function defined by  $f(a) = a, f(b) = c, f(c) = b$ . And  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \gamma_1, \gamma_2)$  be an identity function. Then  $f$  and  $g$  are pairwise *bg* continuous function. But  $f^{-1}(g^{-1}(a)) = \{a\}$  is not pairwise *bg* closed in  $(X, \tau_1, \tau_2)$ . Hence  $g \circ f$  is not pairwise *bg* continuous function.

**Definition: 3.11** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is

- (a) pairwise *bg* irresolute if  $f^{-1}(U)$  is  $\tau_i \tau_j$  – pairwise *bg* closed for each  $\sigma_i \sigma_j$  – pairwise *bg* closed  $U$  in  $Y$ ,  $i \neq j$  and  $i, j = 1, 2$ .
- (b) pairwise \* *bg* irresolute if  $f^{-1}(U)$  is  $\tau_i \tau_j$  – pairwise \* *bg* closed for each  $\sigma_i \sigma_j$  – pairwise \* *bg* closed  $U$  in  $Y$ ,  $i \neq j$  and  $i, j = 1, 2$ .

**Proposition: 3.12** If  $f$  is pairwise *bg* irresolute (pairwise \* *bg* irresolute) then  $f$  is pairwise *bg* continuous (pairwise \* *bg* continuous) function.

**Proof:** Let  $V$  be  $\sigma_j$  – closed set in  $Y$ . Then  $V$  is  $\sigma_i \sigma_j$  – pairwise *bg* closed (pairwise \* *bg* closed) in  $Y$ . By assumption,  $f^{-1}(V)$  is pairwise *bg* closed (pairwise \* *bg* closed) in  $X$ . Hence  $f$  is pairwise *bg* continuous (pairwise \* *bg* continuous) function.

**Remark: 3.13** The converse of the above theorem is not true as shown by the following example.

**Example: 3.14** Let  $X = Y = \{a, b, c\}$ ;  $\tau_1 = \{\phi, X, \{a, c\}\}$ ;  $\tau_2 = \{\phi, X, \{c\}, \{a, c\}\}$ ;  $\sigma_1 = \{\phi, Y, \{b\}, \{a, c\}\}$ ;  $\sigma_2 = \{\phi, Y, \{a\}, \{a, b\}\}$ . Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function defined by  $f(a) = a, f(b) = c, f(c) = b$ .

Here  $f^{-1}(c) = \{b\}$  and  $f^{-1}(b, c) = \{b, c\}$  is pairwise *bg* closed in  $(X, \tau_1, \tau_2)$ . Hence  $f$  is pairwise *bg* continuous. But  $f^{-1}(a, b) = \{a, c\}$  is not pairwise *bg* closed in  $(X, \tau_1, \tau_2)$ . Hence it is not pairwise *bg* irresolute (pairwise \* *bg* irresolute) function.

**Theorem: 3.15** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$  be two functions. Then if  $f$  and  $g$  are pairwise *bg* irresolute (pairwise \* *bg* irresolute) then  $g \circ f$  is pairwise *bg* irresolute (pairwise \* *bg* irresolute) function.

**Proof:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$  be pairwise *bg* irresolute (pairwise \* *bg* irresolute). Let  $V$  be pairwise *bg* closed (pairwise \* *bg* closed) set in  $Z$ . Since  $g$  is pairwise *bg* irresolute (pairwise \* *bg* irresolute) function, then  $g^{-1}(v)$  is pairwise *bg* closed (pairwise \* *bg* closed) in  $Y$ . Since  $f$  is pairwise *bg* irresolute (pairwise \* *bg* irresolute) function,  $(g \circ f)^{-1}(v) = f^{-1}(g^{-1}(v))$  is pairwise *bg* closed (pairwise \* *bg* closed) in  $X$ . Therefore  $g \circ f$  is pairwise *bg* irresolute function (pairwise \* *bg* irresolute).

**Theorem: 3.16** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$  be two functions. Then if  $f$  is pairwise *bg* irresolute (pairwise \* *bg* irresolute) function and  $g$  is pairwise *bg* continuous (pairwise \* *bg* continuous) function. Then  $g \circ f$  is pairwise *bg* continuous (pairwise \* *bg* continuous) function.

**Proof:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be pairwise *bg* irresolute (pairwise \* *bg* irresolute) function and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$  be pairwise *bg* continuous (pairwise \* *bg* continuous) function. Let  $V$  be  $\sigma_j$  – closed set in  $Z$ . Since  $g$  is pairwise *bg* continuous (pairwise \* *bg* continuous) function, then  $g^{-1}(v)$  is pairwise *bg* closed (pairwise \* *bg* closed) in  $Y$ .

Since  $f$  is pairwise *bg* irresolute (pairwise \* *bg* irresolute) function,  $(g \circ f)^{-1}(v) = f^{-1}(g^{-1}(v))$  is pairwise *bg* closed (pairwise \* *bg* closed) in  $X$ . Therefore  $g \circ f$  is pairwise *bg* continuous (pairwise \* *bg* continuous) function.

**Theorem: 3.17** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$  be two functions. Then if  $f$  is pairwise  $bg$  continuous (pairwise  $*$   $bg$  continuous) function and  $g$  is pairwise continuous. Then  $g \circ f$  is pairwise  $bg$  continuous (pairwise  $*$   $bg$  continuous) function.

**Proof:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be pairwise  $bg$  continuous (pairwise  $*$   $bg$  continuous) function and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$  be pairwise continuous. Let  $V$  be  $\sigma_j$  - closed set in  $Z$ . Since  $g$  is pairwise continuous function, then  $g^{-1}(v)$  is  $\sigma_j$  closed in  $Y$ . Since  $f$  is pairwise  $bg$  continuous (pairwise  $*$   $bg$  continuous) function,  $(g \circ f)^{-1}(v) = f^{-1}(g^{-1}(v))$  is pairwise  $bg$  closed (pairwise  $*$   $bg$  closed) in  $X$ . Therefore  $g \circ f$  is pairwise  $bg$  continuous (pairwise  $*$   $bg$  continuous) function.

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