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A NOTE ON GENERALIZED L- CONTACT STRUCTURE

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ABSTRACT

In 1989, K. Matsumoto [1] introduced the notion of manifolds with Lorentzian paracontact metric structure. Also in 1988, K. Matsumoto and I. Mihai [2] discussed on a certain transformation in a Lorentzian Para-Sasakian manifold and in 2011, R. Nivas and A. Bajpai [5] studied on generalized Lorentzian Para-Sasakian manifolds. In 1994, the author with R.H.Ojha [6] studied on D-conformal transformation in LP-contact manifold. In 1980, R. S. Mishra and S. N. Pandey [3] discussed on quarter-symmetric metric F-connection and in 1970, K. Yano [8] studied on semi symmetric metric connections and their curvature tensors. Symmetric metric connections are also studied by K. Yano and T. Imai [9], Nirmala S. Agashe and Mangala R. Chafle [4], R. N. Singh and S. K. Pandey [7] and many others. The purpose of this paper is to study generalized induced connection in a generalized Lorentzian contact manifold. Generalized D-conformal transformation on Lorentzian contact structure has also been discussed.

Keywords: *Generalized Lorentzian structure, generalized induced connection, generalized D-conformal transformation.*

1. INTRODUCTION

Let V_n be an odd $(n = 2m + 1)$ dimensional differentiable manifold, on which there are defined a tensor field $F$of type (1, 1), contravariant vector fields T_i, covariant vector fields A_i, where $i = 3,4,5, \dots (n - 1)$, and a Lorentzian metric g, satisfying for arbitrary vector fields $X, Y, Z, \dots$

(1.1) $\overline{\overline{X}} = -X - \sum_{i=3}^{n-1} A_i(X)T_i, \quad \overline{T_i} = 0, \quad A_i(T_i) = -1, \quad \overline{X} \stackrel{\text{def}}{=} FX, \quad A_i(\overline{X}) = 0,$ rank $F = n - i$

(1.2) $g\,(\overline{X}, \overline{Y}) = g\,(X, Y) + \sum_{i=3}^{n-1} A_i(X)\,A_i(Y)$, where $A_i(X) = g(X, T_i)$, $`F(X, Y) \stackrel{\text{def}}{=} g(\overline{X},\ Y) = -g(\ X, \overline{Y}) = -`F(Y, X)$,

Then V_n will be called a generalized Lorentzian contact manifold and the structure $(F,\ T_i,\ A_i,\ g\ )$ will be known as generalized Lorentzian contact structure.

Let D be a Riemannian connection on V_n, then we have

(1.3) (a) $(D_X`F)(\overline{Y},\ Z) - (D_X`F)(Y, \overline{Z}) + \sum_{i=3}^{n-1} A_i(Y)(D_X A_i)(Z) + \sum_{i=3}^{n-1} A_i(Z)(D_X A_i)(Y) = 0$

(b) $(D_X`F)\left(\overline{Y},\ \overline{\overline{Z}}\right) = (D_X`F)\left(\overline{\overline{Y}}, \overline{Z}\right)$

(1.4) (a) $(D_X`F)(\overline{Y},\ \overline{Z}) + (D_X`F)(Y, Z) + \sum_{i=3}^{n-1} A_i(Y)(D_X A_i)(\overline{Z}) - \sum_{i=3}^{n-1} A_i(Z)(D_X A_i)(\overline{Y}) = 0$

(b) $(D_X`F)\left(\overline{\overline{Y}},\ \overline{\overline{Z}}\right) + (D_X`F)(\ \overline{Y},\ \overline{Z}\ ) = 0$

2. GENERALIZED CONNECTION IN A GENERALIZED LORENTZIAN CONTACT MANIFOLD

Let V_{2m-1} be submanifold of V_{2m+1} and let $c : V_{2m-1} \to V_{2m+1}$ be the inclusion map such that

$$d \in V_{2m-1} \to cd \in V_{2m+1},$$

Where c induces a linear transformation (Jacobian map) $J : T'_{2m-1} \to T'_{2m+1}$. T'_{2m-1} is a tangent space to V_{2m-1} at point d and T'_{2m+1} is a tangent space to V_{2m+1} at point cd such that $\hat{X}$ in V_{2m-1} at $d \to J\hat{X}$ in V_{2m+1} at cd

Let $\tilde{g}$ be the induced Lorentzian metric in V_{2m-1}. Then we have

(2.1) $\tilde{g}(\hat{X}, \hat{Y}) \stackrel{\text{def}}{=} g(J\hat{X}, J\hat{Y}\ )$

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Let us define a type of generalized semi-symmetric non-metric connection B in a generalized Lorentzian contact manifold is given by

(2.2) $iB_X Y = iD_X Y + \sum_{i=3}^{n-1} A_i(Y)X - \sum_{i=3}^{n-1} g(X,Y)T_i + \sum_{i=3}^{n-1} g(X,Y)U_i$,
Where U_i are vector fields associated with 1-form d_i defined by

(2.3) $d_i(X) \stackrel{\text{def}}{=} g(X, U_i\,),\ i = 3, 4, 5, \ldots\ldots(n-1)$. If
(2.4) (a) $T_i = Jt_i + \rho_i M + \sigma_i N$ and
(b) $U_i = Ju_i + \theta_i M + \Phi_i N\ ,\quad i = 3,4,5, \ldots\ldots(n-1)$.
Where t_i and $u_i,\ i = 3,4,5, \ldots\ldots(n-1)$ are C^∞ vector fields in V_{2m-1} and M, N are unit normal vectors to V_{2m-1}.

Denoting by $\grave{D}$ the connection induced on the submanifold from D, we have Gauss equation

(2.5) $D_{JX} J\hat{Y} = J(\grave{D}_X \hat{Y}\,) +\ p(\hat{X},\hat{Y})M + q(\hat{X},\hat{Y})N$ Where p and q are symmetric bilinear functions in V_{2m-1}.

Similarly we have

(2.6) $B_{JX} J\hat{Y} = J(\grave{B}_X \hat{Y}\,) +\ r(\hat{X},\hat{Y})M + s(\hat{X},\hat{Y})N$,
Where $\grave{B}$ is the connection induced on the submanifold from B and r and s are symmetric bilinear functions in V_{2m-1} Inconsequence of (2.2), we have

(2.7) $iB_{JX} J\hat{Y} = iD_{JX} J\hat{Y} + \sum_{i=3}^{n-1} A_i(J\hat{Y})J\hat{X} - \sum_{i=3}^{n-1} g(J\hat{X}, J\hat{Y})T_i + \sum_{i=3}^{n-1} g(J\hat{X}, J\hat{Y})U_i$

Using (2.5), (2.6) and (2.7), we get
(2.8) $iJ(\grave{B}_X \hat{Y}\,) +\ ir(\hat{X},\hat{Y})M +\ is(\hat{X},\hat{Y})N =\ iJ(\grave{D}_X \hat{Y}\,) + ip(\hat{X},\hat{Y})M +\ iq(\hat{X},\hat{Y})N + \sum_{i=3}^{n-1} A_i(J\hat{Y})J\hat{X}$
$-\ \sum_{i=3}^{n-1} g(J\hat{X}, J\hat{Y})T_i + \sum_{i=3}^{n-1} g(J\hat{X}, J\hat{Y})U_i$

Using (2.4) (a) and (2.4) (b), we obtain

(2.9) $iJ(\grave{B}_X \hat{Y}\,) +\ ir(\hat{X},\hat{Y})M +\ is(\hat{X},\hat{Y})N =\ iJ(\grave{D}_X \hat{Y}\,) +\ ip(\hat{X},\hat{Y})M +\ iq(\hat{X},\hat{Y})N + \sum_{i=3}^{n-1} a_i(\,\hat{Y})J\hat{X}$
$- \sum_{i=3}^{n-1} \tilde{g}(\,\hat{X},\hat{Y})(Jt_i + \rho_i M + \sigma_i N) + \sum_{i=3}^{n-1} \tilde{g}(\hat{X},\hat{Y})(Ju_i + \theta_i M + \Phi_i N)$
Where $\tilde{g}(\hat{Y}, t_i) \stackrel{\text{def}}{=} a_i(\,\hat{Y})$, where $i = 3,4,5, \ldots\ldots(n-1)$.

This gives

(2.10) $i\grave{B}_X \hat{Y} =\ i\grave{D}_X \hat{Y} + \sum_{i=3}^{n-1}\ a_i(\,\hat{Y})\hat{X} - \sum_{i=3}^{n-1} \tilde{g}(\hat{X},\hat{Y})t_i + \sum_{i=3}^{n-1} \tilde{g}(\hat{X},\hat{Y})u_i$
Iff
(2.11) (a) $ir(\hat{X},\hat{Y}) =\ ip(\hat{X},\hat{Y}) - \sum_{i=3}^{n-1} \rho_i \tilde{g}(\,\hat{X},\hat{Y}) + \sum_{i=3}^{n-1} \theta_i \tilde{g}(\,\hat{X},\hat{Y})$
(b) $is(\hat{X},\hat{Y}) =\ iq(\hat{X},\hat{Y}) - \sum_{i=3}^{n-1}\ \sigma_i \tilde{g}(\,\hat{X},\hat{Y}) + \sum_{i=3}^{n-1} \Phi_i \tilde{g}(\,\hat{X},\hat{Y})$

Thus we have

Theorem 2.1: The connection induced on a submanifold of a generalized Lorentzian contact manifold with a generalized semi-symmetric non-metric connection with respect to unit normal vectors M and N is also semi-symmetric non-metric connection iff (2.11) holds.

3. GENERALIZED D-CONFORMAL TRANSFORMATION

Let the corresponding Jacobian map J of the transformation b transforms the structure $(F,\ T_i,\ A_i,\ g)$ to the structure $(F, V_i,\ v_i, h\,)$ such that
(3.1) (a) $J\overline{Z} = \overline{JZ}$ (b) $h(\,JX, JY)ob = e^{\sigma}\, \mathrm{g}\,(\overline{X},\overline{Y}) - e^{2\sigma}\ \sum_{i=3}^{n-1} A_i(X)A_i(Y)$
(c) $V_i = e^{-\sigma}\, JT_i$ (d) $v_i(\,JX\,)ob = e^{\sigma}\, A_i(X)$
Where σ is a differentiable function on V_n, then the transformation is said to be generalized D-conformal transformation.

Theorem 3.1: The structure $(F, V_i,\ v_i, h\ )$ is generalized Lorentzian contact.

Proof: Inconsequence of (1.1), (1.2), (3.1) (b) and (3.1) (d), we get
$h(J\overline{X}, J\overline{Y}\,)ob = e^{\sigma}\, \mathrm{g}\,(\overline{X},\overline{Y}) =\ h(\,JX, JY)ob + \sum_{i=3}^{n-1} e^{2\sigma}\, A_i(X)A_i(Y)$
$= h(\,JX, JY)ob + \sum_{i=3}^{n-1}\{v_i(\,JX\,)ob\}\{v_i(\,JY\,)ob\}$

This gives

(3.2) $h(J\overline{X}, J\overline{Y}) = h(JX, JY) + \sum_{i=3}^{n-1} \upsilon_i(JX)\, \upsilon_i(JY)$

Using (1.1), (3.1) (a), (3.1) (c) and (3.1) (d), we get

(3.3) $\overline{\overline{JX}} = J\overline{\overline{X}} = -JX - \sum_{i=3}^{n-1} A_i(X) JT_i = -JX - \sum_{i=3}^{n-1}\{\upsilon_i(JX)ob\}V_i$

Also

(3.4) $\overline{V_i} = e^{-\sigma}\,\overline{JT_i} = 0$

Proof follows from equations (3.2), (3.3) and (3.4).

Theorem 3.2: Let E and D be the Riemannian connections with respect to h and g such that

(3.5) (a) $E_{JX}JY = JD_XY + JH(X,Y)$ and

(b) $`H(X,Y,Z) \overset{\text{def}}{=} g(H(X,Y),Z)$

Then

(3.6) $2E_{JX}JY = 2JD_XY - J[2e^{\sigma}\{\sum_{i=3}^{n-1}(X\sigma)A_i(Y)\,T_i + \sum_{i=3}^{n-1}(Y\sigma)A_i(X)\,T_i - \sum_{i=3}^{n-1}({}^{-1}G\nabla\sigma)\,A_i(X)\,A_i(Y)\}$
$+(e^{\sigma}-1)\sum_{i=3}^{n-1}\{(D_XA_i)(Y) + (D_YA_i)(X) - 2A_i(H(X,Y))\}T_i$
$+(e^{\sigma}-1)\sum_{i=3}^{n-1}\{A_i(X)(D_YT_i) + A_i(Y)(D_XT_i) - A_i(X)({}^{-1}G\nabla A_i)(Y) - A_i(Y)({}^{-1}G\nabla A_i)(X)\}]$

Proof: Inconsequence of (3.1) (b), we have

$JX(h(JY,JZ))ob = X\{e^{\sigma}\,g(\overline{Y},\overline{Z}) - \sum_{i=3}^{n-1} e^{2\sigma} A_i(Y)A_i(Z)\}$

Now

(3.7) $h(E_{JX}JY,JZ)ob + h(JY,E_{JX}JZ)ob = (X\sigma)e^{\sigma}\,g(\overline{Y},\overline{Z}) + e^{\sigma}\,g(D_X\overline{Y},\overline{Z}) + e^{\sigma}\,g(\overline{Y},D_X\overline{Z})$
$-\sum_{i=3}^{n-1}\{2(X\sigma)e^{2\sigma}A_i(Y)A_i(Z) + e^{2\sigma}(D_XA_i)(Y)A_i(Z) +$
$e^{2\sigma}(D_XA_i)(Z)A_i(Y) + e^{2\sigma}A_i(D_XY)A_i(Z) + e^{2\sigma}A_i(D_XZ)A_i(Y)\}$

Also

(3.8) $h(E_{JX}JY,JZ)ob + h(JY,E_{JX}JZ)ob = e^{\sigma}\,g(\overline{D_XY},\overline{Z}) + e^{\sigma}\,g(\overline{H(X,Y)},\overline{Z}) + e^{\sigma}\,g(\overline{Y},\overline{H(X,Z)}) + e^{\sigma}\,g(\overline{Y},\overline{D_XZ})$
$-\sum_{i=3}^{n-1}\begin{matrix}\{e^{2\sigma}A_i(D_XY)A_i(Z) + e^{2\sigma}A_i(Y)A_i(H(X,Z))\\ +e^{2\sigma}A_i(D_XZ)A_i(Y) + e^{2\sigma}A_i(H(X,Y))A_i(Z)\}\end{matrix}$

Inconsequence of (1.3) (a), (3.7) and (3.8), we have

(3.9) $(X\sigma)g(\overline{Y},\overline{Z}) - 2(X\sigma)e^{\sigma}\sum_{i=3}^{n-1}\{A_i(Y)A_i(Z)\} - (e^{\sigma}-1)\sum_{i=3}^{n-1}\{(D_XA_i)(Y)A_i(Z) + (D_XA_i)(Z)A_i(Y)\} +$
$(e^{\sigma}-1)\sum_{i=3}^{n-1}\{A_i(H(X,Y))A_i(Z) + A_i(H(X,Z))A_i(Y)\} = `H(X,Y,Z) + `H(X,Z,Y)$

Writing two other equations by cyclic permutation of X, Y, Z and subtracting the third equation from the sum of the first two. Also using symmetry of $`H$ in the first two slots, we get

(3.10) $2`H(X,Y,Z) = -2e^{\sigma}\sum_{i=3}^{n-1}\{(X\sigma)A_i(Y)A_i(Z) + (Y\sigma)A_i(Z)A_i(X) - (Z\sigma)A_i(X)A_i(Y)\}$
$-(e^{\sigma}-1)\sum_{i=3}^{n-1}\left[A_i(Z)\begin{Bmatrix}(D_XA_i)(Y) + (D_YA_i)(X)\\ -2A_i(H(X,Y))\end{Bmatrix} + A_i(X)\begin{Bmatrix}(D_YA_i)(Z)\\ -(D_ZA_i)(Y)\end{Bmatrix} + A_i(Y)\begin{Bmatrix}(D_XA_i)(Z)\\ -(D_ZA_i)(X)\end{Bmatrix}\right]$

This implies

(3.11) $2H(X,Y) = -2e^{\sigma}\sum_{i=3}^{n-1}[(X\sigma)A_i(Y)T_i + (Y\sigma)A_i(X)\,T_i - ({}^{-1}G\nabla\sigma)A_i(X)A_i(Y)] - (e^{\sigma}-1)\sum_{i=3}^{n-1}[\{(D_XA_i)(Y) + (D_YA_i)(X) - 2A_i(H(X,Y))\}\,T_i + A_i(X)(D_YT_i) + A_i(Y)(D_X\,T_i) - A_i(X)({}^{-1}G\nabla A_i)(Y) - A_i(Y)({}^{-1}G\nabla A_i)(X)]$ ..

(3.6) follows from (3.11) and (3.5) (a).

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