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$\zeta -$ CONNECTEDNESS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT

The focus of this paper is to explore the concepts of different connectedness in intuitionistic fuzzy topological spaces. Also we obtain their characterization and analyse their inter- relations.

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Keywords*: intuitionistic fuzzy $\zeta -$connected, intuitionistic fuzzy $\zeta C_5 -$connected, intuitionistic fuzzy $\zeta C_s -$ connected, intuitionistic fuzzy $\zeta C_M -$connected, intuitionistic fuzzy $\zeta -$super connected,*

1. INTRODUCTION

The concept of an intuitionistic fuzzy set (IFS), which is a generalization of the concept of a fuzzy set (FS), has been introduced by K. Atanassov [1]. Using the notion of intuitionistic fuzzy sets, Coker [2] introduced intuitionistic fuzzy topological space. Connectedness in intuitionistic fuzzy special topological spaces was introduced by Oscag and Coker [9]. Many researchers have extended their notions to study various forms of connectedness Sharmila.S and I.Arockiarani [6] discussed intuitionistic fuzzy $\zeta -$open sets and intuitionistic fuzzy $\zeta -$continuity.

In this paper we have introduced intuitionistic fuzzy $\zeta -$connected space and various forms of connectedness. Several properties concerning connectedness in these spaces are also explored.

2. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set (IFS, in short) A in X is an object having the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$ where the functions $\mu_A : X\rightarrow I$ and $\upsilon_A : X\rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\upsilon_A(x)$) of each element $x\in X$ to the set A on a nonempty set X and $0\le \mu_A(x)+\upsilon_A(x)\le 1$ for each $x\in X$. Obviously every fuzzy set A on a nonempty set X is an IFS's A and B be in the form $A=\{x,\mu_A(x),1-\mu_A(x)/x\in X\}$

Definition 2.2: [1] Let X be a nonempty set and the IFS's A and B be in the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$, $B=\{x,\mu_B(x),\upsilon_B(x)/x\in X\}$ and let $A=\{A_j : j\in J\}$ be an arbitrary family of IFS's in X. Then we define

(i) $A\subseteq B$ if and only if $\mu_A(x)\le\mu_B(x)$ and $\upsilon_A(x)\ge\upsilon_B(x)$ for all $x\in X$.
(ii) A=B if and only if $A\subseteq B$ and $B\subseteq A$.
(iii) $\overline{A}=\{x,\upsilon_A(x),\mu_A(x)/x\in X\}$.
(iv) $A\cap B=\{x,\mu_A(x)\cap\mu_B(x),\upsilon_A(x)\cup\upsilon_B(x)/x\in X\}$.
(v) $A\cup B=\{x,\mu_A(x)\cup\mu_B(x),\upsilon_A(x)\cap\upsilon_B(x)/x\in X\}$
(vi) $1_\sim=\{\langle x,1,0\rangle x\in X\}$ and $0_\sim=\{\langle x,0,1\rangle x\in X\}$.

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Definition 2.3: [2] An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of an intuitionistic fuzzy set (IFS, in short) in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$.

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.

(iii) $\bigcup A_j \in \tau$ for any $A_j : j \in J \subseteq \tau$.

In this paper we denote intuitionistic fuzzy topological space (IFTS, in short) by $(X,\tau),(Y,\kappa)$ or X,Y. Each IFS which belongs to τ is called *an intuitionistic fuzzy open set* (*IFOS,* in short) in X. The complement $\overline{A}$ of an IFOS A in X is called an *intuitionistic fuzzy closed set* (*IFCS*, in short). An IFS X is called *intuitionistic fuzzy clopen* (*IF clopen*) if and only if it is both intutionistic fuzzy open and intuitionistic fuzzy closed.

Definition 2.4: [2] Let (X,τ) be an IFTS and $A=\{x,\mu_A(x),\upsilon_A(x)\}$ be an IFS in X. Then the fuzzy interior and closure of A are denoted by

(i) $cl(A)=\bigcap$ {K: K is an IFCS in X and $A \subseteq K$ }.

(ii) $\text{int}(A)=\bigcup$ {G: G is an IFOS in X and $G \subseteq A$}.

Note that, for any IFS A in (X,τ), we have $cl(\overline{A})=\overline{\text{int}(A)}$ and $\text{int}(\overline{A})=\overline{cl(A)}$.

Definition 2.5: [5] Let X and Y be two non-empty sets and $f : X \to Y$ be a function.

If $B=\{<y,\mu_B(y),\upsilon_B(y)>/\,y\in Y\}$ is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B)=\{<x, f^{-1}(\mu_B(x)), f^{-1}(\upsilon_B(x))>/\,x\in X\}$. Since $\mu_B(x),\upsilon_B(x)$ are fuzzy sets, we explain that $f^{-1}(\mu_B(x))=\mu_B(x)(f(x)), f^{-1}(\upsilon_B(x))=\upsilon_B(x)(f(x))$.

Definition 2.6[5]: An IFS $p(\alpha,\beta)=\langle x, C_\alpha, C_{1-\beta}\rangle$ where $\alpha\in(0,1], \beta\in[0,1)$ and $\alpha+\beta\le 1$ is called an *intuitionistic fuzzy point* (IFP) in X.

Note that an IFP $p(\alpha,\beta)$ is said to belong to an IFS $A=\langle X,\mu_A,\upsilon_A\rangle$ of X denoted by $p(\alpha,\beta)\in A$ if $\alpha\le\mu_A$ and $\beta\ge\upsilon_A$.

Definition 2.7[5]: Let $p(\alpha,\beta)$ be an IFP of an IFTS (X,τ). An IFS A of X is called an *intuitionistic fuzzy neighbourhood* (IFN) of $p(\alpha,\beta)$ if there exists an IFOS B in X such that $p(\alpha,\beta)\in B\subseteq A$.

Definition 2.8: [3] Two intuitionistic fuzzy sets A and B are said to be q-coincident (AqB) if and only if there exists an element $x\in X$ such that $\mu_A(x)\supset \nu_B(x)$ or $\nu_A(x)\subset\mu_B(x)$.

Definition 2.9: [3] Two intuitionistic fuzzy sets A and B are said to be not q-coincident $A\overline{q}B$ if and only if $A\subseteq\overline{B}$.

Definition 2.10: [8] An IFTS (X,τ) is called intuitionistic fuzzy C_5 connected between two intuitionistic fuzzy sets A and B if there is no IFOS E in (X,τ) such that $A\subseteq E$ and $E\overline{q}B$.

Definition 2.11: [6] Let A be an IFTS (X,τ). Then A is called an *intuitionistic fuzzy* ζ *open set*($IF\zeta OS$, in short) in X if $A\subseteq bcl(\text{int}(A))$.

Definition 2.12: [6] Let A be an IFTS (X,τ). Then A is called an *intuitionistic fuzzy* ζ *closed set* (IFζ CS, in short) in X if $b\,\text{int}(cl(A))\subseteq$ A.

Definition 2.13:[6] Let (X,τ) be an IFTS and $A=\{<x,\mu_A(x),\upsilon_A(x)>/\,x\in X\}$ be an IFS in X. Then the intuitionistic fuzzy ζ *–closure* and ζ *-interior* of A are defined by

(i) $\zeta cl(A)=\bigcap$ {U:U is an $IF\zeta CS$ in X and $U\supseteq A$};

(ii) $\zeta\,\text{int}(A)=\bigcup$ {V:V is an $IF\zeta OS$ in X and $V\subseteq A$};

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Definition 2.14: Let $f : X \to Y$ from an IFTS X into an IFTS Y. Then f is said to be an

(i) *Intuitionistic fuzzy* ζ *-continuous* ($IF\zeta -$ cont, in short) [6] if $f^{-1}(B) \in IF\zeta OS(\text{X})$ for every $B \in \kappa$.

(ii) *Intuitionistic fuzzy continuous* [4] if $f^{-1}(B) \in IFO(X)$ for every $B \in \kappa$.

Definition 2.15: [7] Let f be a mapping from IFTS (X,τ) into an IFTS (Y,κ). Then f is said to be *intuitionistic fuzzy* $\zeta -$ *irresolute* ($IF\zeta -$ *irresolute*, in short) if $f^{-1}(B) \in IF\zeta O(X)$ for every $IF\zeta OS$ B in Y.

3. INTUITIONISTIC FUZZY $\zeta -$ CONNECTED SPACES

Definition 3.1: An IFTS (X,τ) is $IF\zeta -$ *disconnected* if there exists $IF\zeta OS$ s U, V in X, $U \neq 0_{\sim}, V \neq 0_{\sim}$ such that $U \cup V = 1_{\sim}$ and $U \cap V = 0_{\sim}$. If X is not $IF\zeta -$ *disconnected* then it is said to be $IF\zeta -$ *connected.*

Example 3.2: Let $X = \{a,b\}$, $\tau = \{0_{\sim}, 1_{\sim}, G_1\}$ where

$G_1 = \{< x,(0.2.0.1),(0.7,0.5) >, x \in X\}$, $G_2 = \{< x,(0.3.0.2),(0.6,0.4) >, x \in X\}$

G_1 and G_2 are $IF\zeta OS$ in X, $G_1 \neq 0_{\sim}, G_2 \neq 0_{\sim}$ and $G_1 \cup G_2 = G_2 \neq 1$, $G_1 \cap G_2 = G_1 \neq 0$. Hence X is $IF\zeta -$ *connected* .

Example 3.3: Let $X = \{a,b\}$, $\tau = \{0_{\sim}, 1_{\sim}, G_1\}$ where

$G_1 = \{< x,(0.2.0.1),(0.7,0.5) >, x \in X\}$,

$G_2 = \{< x,(1,0),(0,1) >, x \in X\}$,

$G_3 = \{< x,(0,1),(1,0) >, x \in X\}$.

G_2 and G_3 are $IF\zeta OS$ in X, $G_2 \neq 0_{\sim}, G_3 \neq 0_{\sim}$ and $G_2 \cup G_3 = 1_{\sim}$, $G_1 \cap G_2 = 0_{\sim}$.

Hence X is $IF\zeta -$ *disconnected* .

Definition 3.4: Let N be an IFS in IFTS (X,τ)

(a) If there exists $IF\zeta OS$ s U and V in X satisfying the following properties, then N is called $IF\zeta_i -$ *disconnected* (i=1,2,3,4) :

C_1: $N \subseteq U \cup V$, $U \cap V \subseteq \overline{N}$, $N \cap U \neq 0_{\sim}$, $N \cap V \neq 0_{\sim}$.

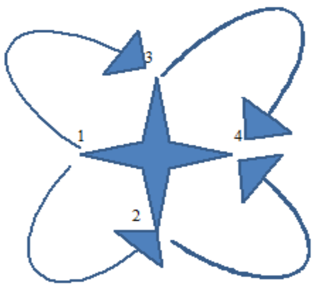
C_2: $N \subseteq U \cup V$, $N \cap U \cap V = 0_{\sim}$, $N \cap U \neq 0_{\sim}$, $N \cap V \neq 0_{\sim}$.

C_3: $N \subseteq U \cup V$, $U \cap V \subseteq \overline{N}$, $U \not\subset \overline{N}$, $V \not\subset \overline{N}$.

C_4: $N \subseteq U \cup V$, $N \cap U \cap V = 0_{\sim}$, $U \not\subset \overline{N}$, $V \not\subset \overline{N}$.

(b) N is said to be $IF\zeta_i -$ *connected* (i=1,2,3,4) if N is not $IF\zeta_i -$ *disconnected* (i=1,2,3,4).

Obviously, we can obtain the following implications between several types of $IF\zeta_i -$ *connected* (i=1,2,3,4..).



1. $IF\zeta_1 -$ *connectedness*
2. $IF\zeta_2 -$ *connectedness*
3. $IF\zeta_3 -$ *connectedness*
4. $IF\zeta_4 -$ *connectedness*

Example 3.5: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1,G_2\}$ where $G_1=\{<x,(0.4.0.1),(0.6,0.9)>,x\in X\}$, $G_2=\{<x,(0.5,0.3),(0.5,0.7)>,x\in X\}$. Consider the IFS $G_3=\{<x,(0.3,0.1),(0.7,0.9)>,x\in X\}$; G_3 is *IF* ζ_2 *– connected, IF* ζ_3 *– connected, IF* ζ_4 *– connected,* but *IF* ζ_1 *– disconnected.*

Example 3.6: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1,G_2,G_1\cup G_2,G_1\cap G_2\}$ where $G_1=\{<x,(0.2.0.8),(0.6,0.2)>,x\in X\}$, $G_2=\{<x,(0.8,0.6),(0.2,0.2)>,x\in X\}$. Consider the IFS $G_3=\{<x,(0.1,0.1),(0.7,0.7)>,x\in X\}$; G_3 is *IF* ζ_4 *– connected,* but *IF* ζ_3 *– disconnected.*

Example 3.7: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1,G_2,G_1\cup G_2\}$ where $G_1=\{<x,(0.,0.2),(1,0.8)>,x\in X\}$, $G_2=\{<x,(0.2,0),(0.8,1)>,x\in X\}$. Consider the IFS $G_3=\{<x,(0.1,0.1),(0.9,0.9)>,x\in X\}$; G_1 and G_2 are $IF\zeta OS$. G_3 is *IF* ζ_4 *– connected,* but *IF* ζ_2 *– disconnected.*

Definition 3.8: An IFTS (X,τ) is $IF\zeta$ *C_5 –disconnected,* if there exists IFS U and V in X, which is both $IF\zeta OS$ and $IF\zeta CS$ $U\neq 0_\sim, U\neq 1_\sim$. If X is not $IF\zeta$ *C_5 –disconnected,* then it is said to be $IF\zeta$ *C_5 –connected.*

Example 3.9: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1\}$ where $G_1=\{<(x,0.2,0.1),(0.7,0.5)>;x\in X\}$
G_1 is an $IF\zeta OS$ in X, but not an $IF\zeta CS$ and $G_1\neq 0_\sim, G_1\neq 1_\sim$. Thus X is $IF\zeta$ *C_5 –connected.*

Example 3.10: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1\}$ where $G_1=\{<(x,0.2,0.1),(0.7,0.5)>;x\in X\}$
$G_2=\{<(x,1,0),(0,1)>;x\in X\}$.
G_2 is an $IF\zeta OS$ in X. Also G_2 is an $IF\zeta CS$ since $b\,\mathrm{int}(cl(G_2))\subseteq G_2$. Hence there exists an IFS G_2 in X such that $G_2\neq 0_\sim, G_2\neq 1_\sim$ which is both $IF\zeta OS$ and $IF\zeta CS$ in X. Thus X is $IF\zeta$ *C_5 –disconnected.*

Theorem 3.11: *$IF\zeta$ C_5 –disconnectedness implies IF ζ – connectedness.*

Proof: Suppose that there exists nonempty $IF\zeta OS$ s U and V such that $U\cup V=1_\sim$ and $U\cap V=0_\sim$ (*IF* ζ *– disconnected*) then $\mu_A\cup\mu_B=1$, $\upsilon_A\cap\upsilon_B=0$, and $\mu_A\cup\mu_B=0$, $\upsilon_A\cap\upsilon_B=1$. In other words $\overline{V}=U$. Hence U is $IF\zeta$ *-clopen* which implies X is $IF\zeta$ *C_5 –disconnected.*
But the converse may not be true as shown by the following example.

Example 3.12: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1\}$ where $G_1=\{<(x,0.6,0.6),(0.6,0.6)>;x\in X\}$
$G_2=\{<(x,0.3,0.1),(0.2,0.6)>;x\in X\}$
.Then G_1 and G_2 are $IF\zeta OS$ in X.
$G_1\cup G_2=\{<x,(0.6,0.6),(0.2,0.6)>;x\in X\}\neq 1_\sim$.
$G_1\cap G_2=\{<x,(0.2,0.6),(0.6,0.6)>;x\in X\}\neq 0_\sim$. Hence X is *IF* ζ *– connected.* Since IFS G_1 is both $IF\zeta OS$ and $IF\zeta CS$ in X, X is $IF\zeta$ *C_5 –disconnected.*

Theorem 3.13: Let $f:(X,\tau)\to(Y,\kappa)$ be an $IF\zeta$ *-irresolute surjection,* (X,τ) is an *IF* ζ *– connected, then* (Y,κ) *is IF* ζ *– connected.*

Proof: Assume that (Y,κ) is not *IF* ζ *– connected,* then there exists nonempty $IF\zeta OS$ s U and V in (Y,κ) such that $U\cup V=1_\sim$ and $U\cap V=0_\sim$. Since f is $IF\zeta$ *-irresolute mapping,* $A=f^{-1}(U)\neq 0_\sim$, $B=f^{-1}(V)\neq 0_\sim$, which are $IF\zeta OS$ in X and $f^{-1}(U)\cup f^{-1}(V)=f^{-1}(1)=1_\sim$, which implies $A\cup B=1_\sim$. $f^{-1}(U)\cap f^{-1}(V)=f^{-1}(0)=0_\sim$, which implies $A\cap B=0_\sim$. Thus X is *IF* ζ *– disconnected,* which is a contradiction to our hypothesis. Hence Y is *IF* ζ *– connected.*

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Theorem 3.14: (X,τ) is $IF\zeta$ C_5 *–connected* if and only if there exists no nonempty $IF\zeta OS$ U and V in X such that $U=\overline{V}$.

Proof: Suppose that U and V are $IF\zeta OS$ s in X such that $U\neq 0_\sim, V\neq 0_\sim$ and $U=\overline{V}$. Since $U=\overline{V}$, $\overline{V}$ is an $IF\zeta OS$ and V is $IF\zeta CS$ and $U\neq 0_\sim$ implies $V\neq 1_\sim$. But this is a contradiction to the fact that X is $IF\zeta$ C_5 *–connected* **.**

Conversely , let U be both $IF\zeta OS$ and $IF\zeta CS$ in X such that $U\neq 0_\sim, U\neq 1_\sim$. Now take $\overline{U}=V$. V is an $IF\zeta OS$ and $U\neq 1_\sim$ which implies $\overline{U}=V\neq 0$ which is a contradiction.

Theorem 3.15: *An IFTS* (X,τ) *is* $IF\zeta$ *–connected space if and only if there exists no non-zero* $IF\zeta OS$ U and V in (X,τ), such that $U=\overline{V}$.

Proof: Necessity: Let U and V be two $IF\zeta OS$ in (X,τ) such that $U\neq 0_\sim, V\neq 0_\sim$ and $U=\overline{V}$. Therefore $\overline{V}$ is an $IF\zeta CS$. *Since* $U\neq 0_\sim, V\neq 1_\sim$. This implies V is a proper IFS which is both $IF\zeta OS$ and $IF\zeta CS$ in (X,τ). Hence (X,τ) is not an $IF\zeta$ *–connected space.* But this is a contradiction to our hypothesis. Thus there exist no non-zero $IF\zeta OS$ U and V in (X,τ), such that $U=\overline{V}$.

Sufficiency: Let U be both $IF\zeta OS$ and $IF\zeta CS$ in (X,τ) such that $U\neq 0_\sim, U\neq 1_\sim$. Now let $V=\overline{U}$. Then V is an $IF\zeta OS$ and $V\neq 1_\sim$. This implies $V=\overline{U}\neq 0_\sim$, which is a contradiction to our hypothesis. Therefore is an (X,τ) *is* $IF\zeta$ *–connected space.*

Theorem 3.16: *An IFTS* (X,τ) *is* $IF\zeta$ *–connected space if and only if there exists no non-zero* $IF\zeta OS$ U and V in (X,τ), such that $U=\overline{V}$, $V=\overline{\zeta cl(U)}$ and $U=\overline{\zeta cl(V)}$.

Proof: Necessity : Assume that there exists IFSs U and V such that $U\neq 0_\sim, V\neq 0_\sim$, $V=\overline{U}$, $V=\overline{\zeta cl(U)}$ and $U=\overline{\zeta cl(V)}$. Since $\overline{\zeta cl(U)}$ and $\overline{\zeta cl(V)}$ are $IF\zeta OS$ in (X,τ), U and V are $IF\zeta OS$ in (X,τ). This implies (X,τ) is not an $IF\zeta$ *–connected space*, which is a contradiction. Therefore *there exists no non-zero* $IF\zeta OS$ U and V in (X,τ), such that $U=\overline{V}$, $V=\overline{\zeta cl(U)}$ and $U=\overline{\zeta cl(V)}$.

Sufficiency: Let U be both $IF\zeta OS$ and $IF\zeta CS$ in (X,τ) such that $U\neq 0_\sim, U\neq 1_\sim$. Now by taking $V=\overline{U}$ we obtain a contradiction to our hypothesis. Hence (X,τ) *is* $IF\zeta$ *–connected space.*

Definition 3.17: An IFTS (X,τ) is $IF\zeta$ –strongly *connected,* if there exists no nonempty $IF\zeta CS$ U and V in X such that $\mu_A+\mu_B\supseteq 1, \upsilon_A+\upsilon_B\subseteq 1$.

In other words, an IFTS (X,τ) is $IF\zeta$ –strongly *connected,* if there exists no nonempty $IF\zeta CS$ U and V in X such that $U\cap V=0_\sim$.

Theorem 3.18: An IFTS (X,τ) is $IF\zeta$ –strongly *connected,* if there exists no nonempty $IF\zeta CS$ U and V in X, $\overline{U}=V\neq 1$ such that $\mu_A+\mu_B\supseteq 1, \upsilon_A+\upsilon_B\subseteq 1$.

Example 3.19: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1\}$ where $G_1=\{<(x,0.6,0.6),(0.6,0.6)>; x\in X\}$
$G_2=\{<(x,0.3,0.1),(0.2,0.6)>; x\in X\}$
.Then G_1 and G_2 are $IF\zeta OS$ in X, also $\mu_A+\mu_B\supseteq 1, \upsilon_A+\upsilon_B\subseteq 1$. Hence X is $IF\zeta$ –strongly *connected.*

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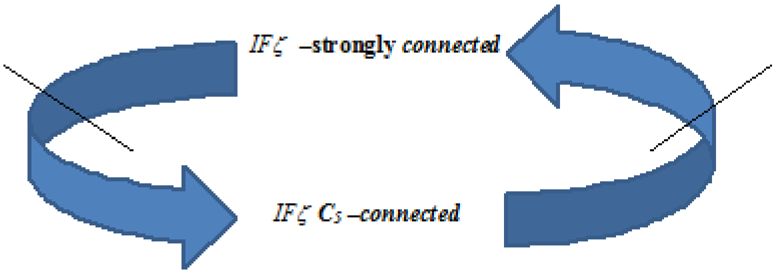
Theorem 3.20: Let $f:(X,\tau)\to(Y,\kappa)$ be an $IF\zeta$ *-irresolute surjection*. If X is an $IF\zeta$ –strongly *connected*, then so is Y.

Proof: Suppose that Y is not $IF\zeta$ –strongly *connected*, then there exists $IF\zeta CS$ U and V in Y such that $U\neq 0_\sim, V\neq 0_\sim$, $U\cap V=0_\sim$. Since f is $IF\zeta$ *-irresolute*, f^{1} *(U)*, f^{1} *(V)* are $IF\zeta CS$ in X and $f^{-1}(U)\cap f^{-1}(V)=0_\sim$, $f^{-1}(U)\neq 0_\sim, f^{-1}(V)\neq 0_\sim$. [If $f^{-1}(U)=0_\sim$ then $f(f^{1}(U))=U$ which implies $f(0_\sim)=U$. So $U=0_\sim$ a contradiction]. Hence X is $IF\zeta$ –strongly *connected*, a contradiction. Thus (Y,κ) is $IF\zeta$ –strongly *connected.*

Remark 3.21: $IF\zeta$ –strongly *connected* and $IF\zeta$ C_5 *–connected* are independent.



Example 3.22: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1\}$ where $G_1=\{<(x,0.6,0.6),(0.6,0.6)>;x\in X\}$ $G_2=\{<(x,0.3,0.1),(0.2,0.6)>;x\in X\}$.
Then G_1 and G_2 are $IF\zeta OS$ in X. Also $\mu_A+\mu_B\supseteq 1, \upsilon_A+\upsilon_B\subseteq 1$. Hence X is $IF\zeta$ –strongly *connected*. But X is not $IF\zeta$ C_5 *–connected*, since G_1 is both $IF\zeta OS$ and $IF\zeta CS$ in X

Example 3.23: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1,G_2,G_1\cup G_2,G_1\cap G_2\}$ where
$G_1=\{<(x,0.6,0.5),(0.3,0.4)>;x\in X\}$
$G_2=\{<(x,0.5,0.4),(0.2,0.4)>;x\in X\}$.
X is $IF\zeta$ C_5 *–connected*, but X is not $IF\zeta$ –strongly *connected* since G_1 and G_2 are $IF\zeta OS$ in X such that $\mu_A+\mu_B\supseteq 1, \upsilon_A+\upsilon_B\subseteq 1$.

Lemma 3.24: [8] (i) $A\cap B=0_\sim \Rightarrow A\subseteq \overline{B}$. (ii) $A\not\subset \overline{B}\Rightarrow A\cap B\neq 0_\sim$

Definition 3.25: A and B are non-zero intuitionistic fuzzy sets in (X,τ). Then A and B are said to be

(i) $IF\zeta$ *–weakly separated* if $\zeta cl(A)\subseteq \overline{B}$ and $\zeta cl(B)\subseteq \overline{A}$.

(ii) $IF\zeta$ *–q- separated* if $\zeta cl(A)\cap B=0_\sim, A\cap(\zeta cl(B)=0_\sim$.

Definition 3.26: An IFTS (X,τ) is $IF\zeta$ C_S *–disconnected* if there exists $IF\zeta$ *–weakly separated* non-zero intuitionistic sets U and V in (X,τ) such that $U\cup V=1_\sim$.

Example 3.27: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1\}$ where $G_1=\{<(x,0.2,0.1),(0.7,0.5)>;x\in X\}$ $G_2=\{<(x,1,0),(0,1)>;x\in X\}$, $G_3=\{<(x,0,1),(1,0)>;x\in X\}$
G_2 and G_3 are $IF\zeta OS$ in X. Hence G_2 and G_3 are $IF\zeta$ *–weakly separated* and $G_2\cup G_3=1_\sim$. Hence X is $IF\zeta$ C_S *–disconnected.*

Definition 3.28: An IFTS (X,τ) is $IF\zeta$ C_M *–disconnected* if there exists $IF\zeta$ *–q- separated* non-zero IFS's U and V in (X,τ) such that $U\cup V=1_\sim$.

Example 3.29: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1\}$ where $G_1=\{<(x,0.2,0.1),(0.7,0.5)>;x\in X\}$ $G_2=\{<(x,1,0),(0,1)>;x\in X\}$, $G_3=\{<(x,0,1),(1,0)>;x\in X\}$

G_2 and G_3 are $IF\zeta OS$ in X. $\zeta cl(G_2)\cap G_3=0_\sim, G_2\cap(\zeta cl(G_3)=0_\sim$ which implies G_2 and G_3 are $IF\zeta$ *–q-separated* and $G_2\cup G_3=1_\sim$. Hence X is $IF\zeta$ C_M *–disconnected.*

Remark 3.30: An IFTS (X,τ) is $IF\zeta$ C_S *–disconnected* if and only if (X,τ) is $IF\zeta$ C_M *–connected.*

Definition 3.31: An IFTS (X,τ) is $IF\zeta$ *–regular open* set if $\zeta\operatorname{int}(\zeta cl(A))=A$ and $IF\zeta$ *–regular closed* if $\zeta cl(\zeta\operatorname{int}(A))=A$.

Definition 3.32: An IFTS (X,τ) is $IF\zeta$ *–super disconnected* if there exists an $IF\zeta$ *–regular open set* A in X such that $A\neq 0_\sim$, $A\neq 1_\sim$, . X is called $IF\zeta$ *–super connected if X is not* $IF\zeta$ *–super disconnected.*

Example 3.33: Let $X=\{a,b\}$, $\tau=\{0_\sim,1_\sim,G_1\}$ where $G_1=\{<(x,0.2,0.1),(0.7,0.5)>;x\in X\}$ $G_2=\{<(x,1,0),(0,1)>;x\in X\}$, $G_3=\{<(x,0,1),(1,0)>;x\in X\}$

G_2 and G_3 are $IF\zeta OS$ in X and $\zeta\operatorname{int}(\zeta cl(G_2))=G_2$. This implies G_2 is an $IF\zeta$ *–regular open* in X. Hence X is an $IF\zeta$ *–super disconnected.*

Theorem 3.34: Let (X,τ) be an IFTS. Then the following are equivalent:

(a) X is $IF\zeta$ *–super connected.*

(b) For each $IF\zeta OS$ $U\neq 0_\sim$ in X, we have $\zeta cl(U)=1_\sim$.

(c) For each $IF\zeta CS$ $U\neq 1_\sim$ in X, we have $\zeta\operatorname{int}(U)=0_\sim$.

(d) There exists no $IF\zeta OS$ s U and V in X such that $U\neq 0_\sim$, $V\neq 0_\sim$ and $U\subseteq\overline{V}$.

(e) There exists no $IF\zeta OS$ s U and V in X such that $U\neq 0_\sim$, $V\neq 0_\sim$, $V=\overline{\zeta cl(U)}$ and $U=\overline{\zeta cl(V)}$.

(f) There exists no $IF\zeta CS$ s U and V in X such that $U\neq 1_\sim$, $V\neq 1_\sim$, $V=\overline{\zeta\operatorname{int}(U)}$ and $U=\overline{\zeta\operatorname{int}(V)}$.

Proof:

$(a)\Rightarrow(b)$ Assume that there exists an $U\neq 0_\sim$ such that $\zeta cl(U)\neq 1_\sim$. Take $U=\zeta\operatorname{int}(\zeta cl(A))$. Then A is proper ζ –regular open set in X which contradicts that X is $IF\zeta$ –super connectedness.

$(c)\Rightarrow(d)$ Let U and V be $IF\zeta OS$ in X such that $U\neq 0_\sim$, $V\neq 0_\sim$ and $U\subseteq\overline{V}$. Since $\overline{V}$ is an $IF\zeta CS$ in X, $\overline{V}\neq 1_\sim$ by (c) $\zeta\operatorname{int}(\overline{V})=0_\sim$. But $U\subseteq\overline{V}$ implies $0_\sim\neq U=\zeta\operatorname{int}(U)\subseteq\zeta\operatorname{int}(\overline{V})=0_\sim$ which is a contradiction.

$(d)\Rightarrow(a)$ Let $U\neq 0_\sim$, $U\neq 1_\sim$ be an $IF\zeta ROS$ in X. If we take $V=\overline{\zeta cl(U)}$, we get $V\neq 0_\sim$. (If not $V\neq 0_\sim$ implies $\overline{\zeta cl(U)}=0\Rightarrow\zeta cl(U)=1_\sim\Rightarrow U=\zeta\operatorname{int}(\zeta cl(A))=\zeta\operatorname{int}(1_\sim)=1_\sim\Rightarrow U=1_\sim$). We also have $U\subseteq\overline{V}$ which is a contradiction. Therefore X is $IF\zeta$ –super connected.

$(a)\Rightarrow(e)$ Let U and V be $IF\zeta OS$ in X such that $U\neq 0_\sim$, $V\neq 0_\sim$, $V=\overline{\zeta cl(U)}$ and $U=\overline{\zeta cl(V)}$. Now we have $\zeta\operatorname{int}(\zeta cl(U))=\zeta\operatorname{int}(\overline{U})=\overline{\zeta cl(U)}=U$, $U\neq 0_\sim$, $U\neq 1_\sim$, since if $U=1_\sim$, then $1_\sim=\overline{\zeta cl(V)}\Rightarrow\zeta cl(V)=0\Rightarrow V=0$. But $V\neq 0_\sim$. Therefore $U\neq 1_\sim$, implies U is proper $IF\zeta ROS$ in X which is a contradiction to (a). Hence (e) is true.

$(e)\Rightarrow(a)$ Let U be $IF\zeta OS$ in X such that $U=\zeta\operatorname{int}(\zeta cl(A))$, $U\neq 0_\sim$, $U\neq 1_\sim$. Now take $V=\overline{\zeta cl(U)}$. In this case, $V\neq 0_\sim$ and V is an $IF\zeta OS$ in X and $V=\overline{\zeta cl(U)}$ and $\overline{\zeta cl(U)}=\overline{\zeta cl(\overline{\zeta cl(U)}}=\overline{\overline{\zeta\operatorname{int}(\zeta cl(U)}}=\zeta\operatorname{int}(\zeta cl(U))=U$. But this is a contradiction to (e). Therefore X is $IF\zeta$ –super connected space.

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$(e)\Rightarrow(f)$ Let U and V be $IF\zeta CS$ in X such that $U\neq 1_{\sim}$, $V\neq 1_{\sim}$, $V=\overline{\zeta\,\text{int}(U)}$ and $U=\overline{\zeta\,\text{int}(V)}$. Taking $C=\overline{U}$ and $D=\overline{V}$, C and D become $IF\zeta OS$ in X and $C\neq 0_{\sim}, D\neq 0_{\sim}$, $\overline{\zeta cl(C)}=\overline{\zeta cl(\overline{U})}=\overline{(\overline{\zeta\,\text{int}(U)})}=\zeta\,\text{int}(U)=\overline{V}=D$ and similarly $\overline{\zeta cl(D)}=C$. But this is a contradiction to (e). Hence (f) is true.

$(f)\Rightarrow(e)$ We can prove this by the similar way as in $(e)\Rightarrow(f)$.

Theorem 3.34: Let $f:(X,\tau)\to(Y,\kappa)$ be an $IF\zeta$ *-irresolute surjection*. If X is an $IF\zeta$ *–super connected,* then so is Y.

Proof: Suppose that Y is not $IF\zeta$ *–super connected,* then there exists $IF\zeta OS$ U and V in Y such that $U\neq 0_{\sim}, V\neq 0_{\sim}$, $U\subseteq\overline{V}$. Since f is $IF\zeta$ *-irresolute, f^{1} (U), f^{1} (V)* are $IF\zeta OS$ in X and $U\subseteq\overline{V}\Rightarrow f^{-1}(U)\subseteq f^{-1}(\overline{V})=\overline{f^{-1}(V)}$. Hence $f^{-1}(U)\neq 0_{\sim}$, $f^{-1}(\overline{V})\neq 0_{\sim}$ which means that X is $IF\zeta$ *– super connected,* which is a contradiction.

Definition 3.35: An IFTS (X,τ) is called $IF\zeta$ *–connected* between two intuitionistic fuzzy sets A and B if there is no $IF\zeta OS$ E in (X,τ) such that $A\subseteq E$ and $E\overline{q}B$.

Example 3.36: Let $X=\{a,b\}$, $\tau=\{0_{\sim},1_{\sim},G_1\}$ where $G_1=\{<(x,0.6,0.6),(0.3,0.3)>;x\in X\}$ be IFTS. Consider the IFSs

$G_2=\{<(x,0.2,0.4),(0.7,0.6)>;x\in X\}$, $G_3=\{<(x,0.6,0.6),(0.6,0.3)>;x\in X\}$

G_1 is $IF\zeta OS$ in X. Then X is $IF\zeta$ *–connected* between G_2 and G_3.

Theorem 3.37: If an IFTS (X,τ) is an $IF\zeta$ *–connected* between two intuitionistic fuzzy sets A and B, then it is IFC_5 *–connected* between two intuitionistic fuzzy sets U and V.

Proof: Suppose (X,τ) is not IFC_5 *–connected* between two intuitionistic fuzzy sets U and V then there exists an IFOS E in (X,τ) such that $U\subseteq E$ and $E\overline{q}V$ which implies (X,τ) is not $IF\zeta$ *–connected* between U and V, a contradiction to our hypothesis. Therefore (X,τ) is IFC_5 *–connected* between U and V.

However, the converse of the above theorem may not be true, as shown by the following example,

Example 3.38: Let $X=\{a,b\}$, $\tau=\{0_{\sim},1_{\sim},G_1\}$ where $G_1=\{<(x,0.6,0.6),(0.3,0.3)>;x\in X\}$ be IFTS. Consider the IFSs

$G_2=\{<(x,0.2,0.4),(0.7,0.6)>;x\in X\}$, $G_3=\{<(x,0.6,0.6),(0.6,0.3)>;x\in X\}$

G_1 is $IF\zeta OS$ in X. Then X is $IF\zeta_5-$*connected* between G_2 and G_3. Consider IFS $G_4=\{<(x,0.5,0.4),(0.5,0.6)>;x\in X\}$. G_4 is an $IF\zeta OS$ such that $G_2\subseteq G_3$ and which implies X is $IF\zeta$ *– disconnected* between G_2 and G_3.

Theorem 3.39: Let (X,τ) be an IFTS and U and V be IFS in (X,τ). If UqV then (X,τ) is $IF\zeta$ *–connected* between U and V.

Proof: Suppose (X,τ) is not $IF\zeta$ *–connected* between U and V. Then there exists an $IF\zeta OS$ E in (X,τ) such that $U\subseteq E$ and $E\overline{q}V$. This implies that $U\subseteq\overline{V}$. That is $U\overline{q}B$ which is a contradiction to our hypothesis. Therefore (X,τ) is $IF\zeta$ *–connected* between U and V.

However, the converse of the above theorem need not be true, as shown by the following example.

Example 3.40: In the above example 3.38, X is $IF\zeta$ *–connected* between G_2 and G_3. But G_2 is not q-coincident with G_3, since $\mu_{G_2}(x)\subset\upsilon_{G_3}(x)$.

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