International Journal of Mathematical Archive-6(8), 2015, 128-134
IJMA Available online through www.ijma.info ISSN 2229 – 5046

INTERVAL VALUED INTUITIONISTIC FUZZY GENERALIZED SEMIPRECLOSED MAPPINGS

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(Received On: 05-08-15; Revised & Accepted On: 29-08-15)

ABSTRACT

In this paper, we introduce interval valued intuitionistic fuzzy generalized semi-preclosed mappings and interval valued intuitionistic fuzzy generalized semi-preopen mappings. Also we investigate some of their properties.

Keywords: *interval valued intuitionistic fuzzy subset, interval valued intuitionistic fuzzy topological space, interval valued intuitionistic fuzzy interior, interval valued intuitionistic fuzzy closure, interval valued intuitionistic fuzzy continuous mapping, interval valued intuitionistic fuzzy generalized semi-preclosed set, interval valued intuitionistic fuzzy generalized semi-preopen set, interval valued intuitionistic fuzzy generalized semi-preclosed mapping, interval valued intuitionistic fuzzy generalized semi-preopen mapping.*

2000 Ams Subject Classification: *54A40, 08A72.*

INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [16] in the year 1965, the subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper C.L.Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces many researchers like, and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] has introduced semipreclosed sets and Dontchev [4] has introduced generalized semipreclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by saraf and khanna [14]. Tapas kumar mondal and S.K.Samantha [9] have introduced the topology of interval valued fuzzy sets. Now we have generalized the set to interval valued intuitionistic fuzzy topological spaces. In this paper, we introduce interval valued intuitionistic fuzzy generalized semi-preclosed mappings and interval valued intuitionistic fuzzy generalized semi-preopen mappings and some properties are investigated.

1. PRELIMINARIES

1.1 Definition:[9] Let X be any nonempty set. A mapping $\bar{A}: X \to D[0,1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X, where $D[0,1]$ denotes the family of all closed subintervals of [0, 1] and $\bar{A}(x) = [A^-(x), A^+(x)]$ for all $x \in X$, where A^- and A^+ are fuzzy subsets of X such that $A^-(x) \le A^+(x)$ for all $x \in X$. Thus $\bar{A}(x)$ is an interval (a closed subset of [0, 1]) and not a number from the interval [0, 1] as in the case of fuzzy subset. Note that $\bar{0} = [0, 0]$ and $\bar{1} = [1, 1]$.

1.2 Definition: An **interval valued intuitionistic fuzzy subset** (**IVIFS**) $\bar{A}$ of a set X is defined as an object of the form $\bar{A} = \{\langle x, \mu_{\bar{A}}(x), \vartheta_{\bar{A}}(x)\rangle / x \in X\}$, where $\mu_{\bar{A}} : X \to D[0, 1]$ and $\vartheta_{\bar{A}} : X \to D[0, 1]$ define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying $0 \le \sup \mu_{\bar{A}}(x) + \sup \vartheta_{\bar{A}}(x) \le 1$.

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1.3 Definition: Let $\bar{A}$ and $\bar{B}$ be any two interval valued intuitionistic fuzzy subsets of a set X. We define the following relations and operations:

(i) $\bar{A} \subseteq \bar{B}$ if and only if $\mu_{\bar{A}}(x) \leq \mu_{\bar{B}}(x)$ and $\vartheta_{\bar{A}}(x) \geq \vartheta_{\bar{B}}(x)$ for all x in X.
(ii) $\bar{A} = \bar{B}$ if and only if $\mu_{\bar{A}}(x) = \mu_{\bar{B}}(x)$ and $\vartheta_{\bar{A}}(x) = \vartheta_{\bar{B}}(x)$ for all x in X.
(iii) $(\bar{A})^c = \{\langle x, \vartheta_{\bar{A}}(x), \mu_{\bar{A}}(x) \rangle / x \in X\}$.
(iv) $\bar{A} \cap \bar{B} = \{\langle x, \text{rmin}\{\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)\}, \text{rmax}\{\vartheta_{\bar{A}}(x), \vartheta_{\bar{B}}(x)\}\rangle / x \in X\}$.
(v) $\bar{A} \cup \bar{B} = \{\langle x, \text{rmax}\{\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)\}, \text{rmin}\{\vartheta_{\bar{A}}(x), \vartheta_{\bar{B}}(x)\}\rangle / x \in X\}$.

1.4 Remark: $\bar{0} = \{< x, [0, 0], [1,1] > : x \in X\}$ and $\bar{1} = \{< x, [1, 1], [0, 0] > : x \in X\}$.

1.5 Definition [9]: Let X be a set and $\Im$ be a family of interval valued intuitionistic fuzzy subsets of X. The family $\Im$ is called an interval valued intuitionistic fuzzy topology (IVIFT) on X if $\Im$ satisfies the following axioms

(i) $\bar{0}, \bar{1} \in \Im$ (ii) If $\{ \bar{A}_i ; i \in I \} \subseteq \Im$, then $\bigcup_{i \in I} \bar{A}_i \in \Im$

(iii) If $\bar{A}_1, \bar{A}_2, \bar{A}_3, \ldots, \bar{A}_n \in \Im$, then $\bigcap_{i=1}^{i=n} \bar{A}_i \in \Im$.

The pair (X, $\Im$) is called an interval valued intuitionistic fuzzy topological space (IVIFTS). The members of $\Im$ are called interval valued intuitionistic fuzzy open sets (IVIFOS) in X. An interval valued intuitionistic fuzzy subset $\bar{A}$ in X is said to be interval valued intuitionistic fuzzy closed set (IVIFCS) in X if and only if $(\bar{A})^c$ is an IVIFOS in X.

1.6 Definition: Let (X, $\Im$) be an IVIFTS and $\bar{A}$ be an IVIFS in X. Then the interval valued intuitionistic fuzzy interior and interval valued intuitionistic fuzzy closure are defined by $ivifint(\bar{A}) = \cup \{\bar{G} : \bar{G}$ is an IVIFOS in X and $\bar{G} \subseteq \bar{A}\}$, $ivifcl(\bar{A}) = \cap \{\bar{K} : \bar{K}$ is an IVIFCS in X and $\bar{A} \subseteq \bar{K}\}$. For any IVIFS $\bar{A}$ in (X, $\Im$), we have $ivifcl(A^c) = (ivifint(A))^c$ and $ivifint(A^c) = (ivifcl(A))^c$.

1.7 Definition: An IVIFS $\bar{A}$ of an IVIFTS (X, $\Im$) is said to be an

(i) interval valued intuitionistic fuzzy regular closed set (IVIFRCS for short) if $\bar{A} = ivifcl(ivifint(A))$
(ii) interval valued intuitionistic fuzzy semiclosed set (IVIFSCS for short) if $ivifint(ivifcl(\bar{A})) \subseteq \bar{A}$
(iii) interval valued intuitionistic fuzzy preclosed set (IVIFPCS for short) if $ivifcl(ivifint(\bar{A})) \subseteq \bar{A}$
(iv) interval valued intuitionistic fuzzy α closed set (IVIF**α**CS for short) if $ivifcl\left(ivifint(ivifcl(\bar{A}))\right) \subseteq \bar{A}$
(v) interval valued intuitionistic fuzzy β closed set (IVIF**β**CS for short) if $ivifint\left(ivifcl(ivifint(\bar{A}))\right) \subseteq \bar{A}$.

1.8 Definition: An IVIFS $\bar{A}$ of an IVIFTS (X, $\Im$) is said to be an

(i) interval valued intuitionistic fuzzy generalized closed set (IVIFGCS for short) if $ivifcl(\bar{A}) = \bar{U}$, whenever $\bar{A} \subseteq \bar{U}$ and $\bar{U}$ is an IVIFOS
(ii) interval valued intuitionistic fuzzy regular generalized closed set (IVIFRGCS for short) if $ivifcl(\bar{A}) \subseteq \bar{U}$, whenever $\bar{A} \subseteq \bar{U}$ and $\bar{U}$ is an IVIFROS.

1.9 Definition: An IVIFS $\bar{A}$ of an IVIFTS (X, $\Im$) is said to be an

(i) interval valued intuitionistic fuzzy semipreclosed set (IVFSPCS for short) if there exists an IVIFPCS $\bar{B}$ such that $ivifint(\bar{B}) \subseteq \bar{A} \subseteq \bar{B}$
(ii) interval valued intuitionistic fuzzy semipreopen set (IVIFSPOS for short) if there exists an IVIFPOS $\bar{B}$ such that $\bar{B} \subseteq \bar{A} \subseteq ivifcl(\bar{B})$.

1.10 Definition: Let $\bar{A}$ be an IVIFS in an IVIFTS (X, $\Im$). Then the interval valued intuitionistic fuzzy semipre interior of $\bar{A}$ ($ivifspint(\bar{A})$ for short) and the interval valued intuitionistic fuzzy semipre closure of $\bar{A}$ ($ivifspcl(\bar{A})$ for short) are defined by $ivifspint(\bar{A}) = \cup \{\bar{G} : \bar{G}$ is an IVIFSPOS in X and $\bar{G} \subseteq \bar{A}\}$, $ivifspcl(\bar{A}) = \cap \{\bar{K} : \bar{K}$ is an IVIFSPCS in X and $\bar{A} \subseteq \bar{K}\}$. For any IVIFS $\bar{A}$ in $(X, \Im)$, we have $ivifspcl(\bar{A}^c) = (ivifspint(\bar{A}))^c$ and $ivifspint(\bar{A}^c) = (ivifspcl(\bar{A}))^c$.

1.11 Definition: An IVIFS $\bar{A}$ in IVIFTS (X, $\Im$) is said to be an interval valued intuitionistic fuzzy generalized semipreclosed set (IVIFGSPCS for short) if $ivifspcl(\bar{A}) \subseteq \bar{U}$ whenever $\bar{A} \subseteq \bar{U}$ and $\bar{U}$ is an IVIFOS in (X,$\Im$).

1.12 Example: Let $X = \{a, b\}$ and $\bar{G} = \{\langle a, [0.5, 0.5], [0.5, 0.5] \rangle, \langle b, [0.4, 0.4], [0.6, 0.6] \rangle$ Then $\Im = \{\bar{0}, \bar{G}, \bar{1}\}$ is an IVIFT on X and the IVIFS $\bar{A} = \{\langle a, [0.4, 0.4], [0.6, 0.6] \rangle, \langle b, [0.2, 0.2], [0.7, 0.7] \rangle\}$ is an IVIFGSPCS in (X,$\Im$).

1.13 Definition: The complement $\overline{A}^c$ of an IVIFGSPCS $\overline{A}$ in an IVIFTS (X, ℑ) is called an interval valued intuitionistic fuzzy generalized semi-preopen set (IVIFGSPOS) in X.

1.14 Definition: An IVIFTS (X, ℑ) is called an interval valued intuitionistic fuzzy semi-pre $T_{1/2}$ space (IVIFSPT$_{1/2}$), if every IVIFGSPCS is an IVIFSPCS in X .

1.15 Definition: Let (X, ℑ) and (Y, σ) be IVIFTSs. Then a map g: X→Y is called an

(i) interval valued intuitionistic fuzzy continuous (IVIF continuous) mapping if $g^{-1}(\overline{B})$ is an IVIFOS in X for all IVIFOS $\overline{B}$ in Y.
(ii) an interval valued intuitionistic fuzzy closed mapping (IVIFC mapping) if $g(\overline{A})$ is an IVIFCS in Y for each IVIFCS $\overline{A}$ in X.
(iii) interval valued intuitionistic fuzzy semi-closed mapping (IVIFSC mapping) if $g(\overline{A})$ is an IVIFSCS in Y for each IVIFCS $\overline{A}$ in X.
(iv) interval valued intuitionistic fuzzy preclosed mapping (IVIFPC mapping) if $g(\overline{A})$ is an IVIFPCS in Y for each IVIFCS $\overline{A}$ in X.
(v) interval valued intuitionistic fuzzy semi-open mapping (IVIFSO mapping) if $g(\overline{A})$ is an IVIFSOS in Y for each IVIFOS $\overline{A}$ in X.
(vi) interval valued intuitionistic fuzzy generalized semi-preopen mapping (IVIFGSPO mapping) if $g(\overline{A})$ is an IVIFGSPOS in Y for each IVIFOS $\overline{A}$ in X.
(vii) interval valued intuitionistic fuzzy generalized semi-preclosed mapping (IVIFGSPC mapping) if $g(\overline{A})$ is an IVIFGSPCS in Y for each IVIFCS $\overline{A}$ in X.

2. SOME PROPERTIES

2.1 Theorem: [15] Every IVIFCS in (X,ℑ) is an IVIFGSPCS in (X,ℑ).

2.2 Theorem: Every IVIFC mapping is an IVIFGSPC mapping.

Proof. Let X and Y be two IVIFTSs. Assume that g : X→Y is an IVIFC mapping. Let $\overline{A}$ be an IVIFCS in X. Then $g(\overline{A})$ is an IVIFCS in Y. Since every IVIFCS is an IVIFGSPCS (by Theorem 2.1), $g(\overline{A})$ is an IVIFGSPCS in Y and hence g is an IVIFGSPC mapping.

2.3 Remark: The converse of the above theorem 2.2 need not be true from the following example: Let X={a, b}, Y= {u, v} and $\overline{K}_1 = \{< a,[0.6,0.6],[0.4,0.4] >,< b,[0.7,0.7],[0.2,0.2] >\}$, $\overline{L}_1 = \{< u,[0.5,0.5],[0.5,0.5] >,< v,[0.4,0.4],[0.6,0.6] >\}$. Then $\Im = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are IVIFT on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an IVIFGSPC mapping but not an IVIFC mapping, since $\overline{K}_1^{\,c}$ is an IVIFCS in X but $g(\overline{K}_1^{\,c}) = \{< u,[0.4,0.4],[0.6,0.6] >,< v,[0.2,0.2],[0.7,0.7] >\}$ is not an IVIFCS in Y, because ivifcl $(g(\overline{K}_1^{\,c})) = \overline{L}_1^{\,c} \neq g(\overline{K}_1^{\,c})$.

2.4 Theorem: [15] Every IVIFαCS in (X,ℑ) is an IVIFGSPCS in (X,ℑ).

2.5 Theorem: Every IVIFαC mapping is an IVIFGSPC mapping.

Proof: Let X and Y be two IVIFTSs. Assume that g: X→Y is an IVIFαC mapping. Let $\overline{A}$ be an IVIFCS in X. Then $f(\overline{A})$ is an IVIFαCS in Y. Since every IVIFαCS is an IVIFGSPCS(by Theorem 2.4), $g(\overline{A})$ is an IVIFGSPCS in Y and hence g is an IVIFGSPC mapping.

2.6 Remark: The converse of the above theorem 2.5 need not be true from the following example:

Let X= {a, b}, Y= {u, v} and $\overline{K_1} = \{< a,[0.6,0.6],[0.4,0.4]>,< b,[0.7,0.7],[0.2,0.2]>\}$, $\overline{L_1} = \{< u,[0.5,0.5],[0.5,0.5]>,< v,[0.4,0.4],[0.6,0.6]>\}$. Then $\Im = \{\overline{0}_X, \overline{K_1}, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L_1}, \overline{1}_Y\}$ are IVIFT on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an IVIFGSPC mapping but not an IVIFαC mapping, since $\overline{K_1}^c$ is an IVIFCS in X but $g(\overline{K_1}^c) = \{< u,[0.4,0.4],[0.6,0.6]>,< v,[0.2,0.2],[0.7,0.7]>\}$ is not an IVIFαCS in Y, because ivifcl (ivifint(ivifcl $(g(\overline{K_1}^c)))) = ivifcl(ivifint(\overline{L_1}^c)) = ivifcl(\overline{K_1}) = \overline{L_1^c} \not\subset g(\overline{K_1}^c)$.

2.7 Theorem: [15] Every IVIFSCS in (X,ℑ) is an IVIFGSPCS in (X,ℑ).

2.8 Theorem: Every IVIFSC mapping is an IVIFGSPC mapping.

Proof: Assume that g: X→Y is an IVIFSC mapping, where X and Y be two IVIFTSs. Let $\overline{A}$ be an IVIFCS in X. Then $g(\overline{A})$ is an IVIFSCS in Y. Since every IVIFSCS is an IVIFGSPCS (by Theorem 2.7), $g(\overline{A})$ is an IVIFGSPCS in Y and hence g is an IVIFGSPC mapping.

2.9 Remark: The converse of the above theorem 2.8 need not be true from the following example:

Let X= {a, b}, Y= {u, v} and $\overline{K_1} = \{< a,[0.6,0.6],[0.4,0.4]>,< b,[0.7,0.7],[0.2,0.2]>\}$, $\overline{L_1} = \{< u,[0.5,0.5],[0.5,0.5]>,< v,[0.4,0.4],[0.6,0.6]>\}$. Then $\Im = \{\overline{0}_X, \overline{K_1}, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L_1}, \overline{1}_Y\}$ are IVIFT on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an IVIFGSPC mapping but not an IVIFSC mapping, since $\overline{K_1}^c$ is an IVIFCS in X but $g(\overline{K_1}^c) = \{< u,[0.4,0.4],[0.6,0.6]>,< v,[0.2,0.2],[0.7,0.7]>\}$ is not an IVIFSCS in Y, because ivifint (ivifcl $(g(\overline{K_1}^c))) = ivifint(\overline{L_1}^c) = \overline{L_1} \not\subset g(\overline{K_1}^c)$.

2.10 Theorem:[15] Every IVIFPCS in (X,ℑ) is an IVIFGSPCS in (X,ℑ).

2.11 Theorem: Every IVIFPC mapping is an IVIFGSPC mapping.

Proof: Assume that g: X→Y is an IVIFPC mapping, where X and Y be two IVIFTSs.

Let $\overline{A}$ be an IVIFCS in X. Then $g(\overline{A})$ is an IVIFPCS in Y. Since every IVIFPCS is an IVIFGSPCS(by Theorem 2.10), $g(\overline{A})$ is an IVIFGSPCS in Y and hence g is an IVIFGSPC mapping.

2.12 Remark: The converse of the above theorem 2.11 need not be true from the following example:
Let X= {a, b}, Y= {u, v} and $\overline{K_1} = \{< a,[0.5,0.5],[0.5,0.5]>,< b,[0.3,0.3],[0.7,0.7]>\}$, $\overline{L_1} = \{< u,[0.5,0.5],[0.5,0.5]>,< v,[0.6,0.6],[0.4,0.4]>\}$. Then $\Im = \{\overline{0}_X, \overline{K_1}, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L_1}, \overline{1}_Y\}$ are IVIFT on X and Y respectively. Define a mapping $g:(X,\Im)\to(Y,\sigma)$ by $g(a)=u$ and $g(b)=v$. Then g is an IVIFGSPC mapping but not an IVIFPC mapping, since $\overline{K_1}^c$ is an IVIFCS in X but $g(\overline{K_1}^c) = \{< u,[0.5,0.5],[0.5,0.5]>,< v,[0.7,0.7],[0.3,0.3]>\}$ is not an IVIFPCS in Y, because ivifcl (ivifint $(g(\overline{K_1}^c))) = ivifcl(\overline{L_1}) = \overline{1}_Y \not\subset g(\overline{K_1}^c)$.

2.13 Theorem: Let g: X→Y be an IVIFGSPC mapping between two IVIFTSs X and Y. Then for every IVIFS $\overline{A}$ of X, $g(ivifcl(\overline{A}))$ is an IVIFGSPCS in Y.

Proof: Let $\overline{A}$ be any IVIFS in X. Then $ivifcl(\overline{A})$ is an IVIFCS in X. By hypothesis $g(ivifcl(\overline{A}))$ is an IVIFGSPCS in Y.

2.14 Theorem: Let $\overline{A}$ be an IVIFGCS in X. If a mapping g: X→Y from an IVIFTS X onto an IVIFTS Y is both IVIF continuous and an IVIFGSPC, then $g(\overline{A})$ is an IVIFGSPCS in Y.

Proof: Let $g(\overline{A}) \subseteq \overline{U}$ where $\overline{U}$ is an IVIFOS in Y. Then $\overline{A} \subseteq g^{-1}(g(\overline{A})) \subseteq g^{-1}(\overline{U})$, where $g^{-1}(\overline{U})$ is an IVIFOS in X, by hypothesis. Since $\overline{A}$ is an IVIFGCS, $ivifcl(\overline{A}) \subseteq g^{-1}(\overline{U})$ in X. This implies $g(ivifcl(\overline{A})) \subseteq g(g^{-1}(\overline{U}) = \overline{U}$. But $g(ivifcl(\overline{A}))$ is an IVIFGSPCS in Y, since $ivifcl(\overline{A})$ is an IVIFCS in X and by hypothesis. Therefore $ivifspcl\ (g(ivifcl(\overline{A}))) \subseteq \overline{U}$. Now $ivifspcl(g(\overline{A})) \subseteq ivifspcl\ (g(ivifcl(\overline{A}))) \subseteq \overline{U}$ .Hence $g(\overline{A})$ is an IVIFGSPCS in Y.

2.15 Theorem: A bijective mapping g: X→Y from an IVIFTS X into an IVIFTS Y is an IVIFGSPC mapping if and only if for every IVIFS $\overline{B}$ of Y and for every IVIFOS $\overline{U}$ containing $g^{-1}(\overline{B})$, there is an IVIFGSPOS $\overline{A}$ of Y such that $\overline{B} \subseteq \overline{A}$ and $g^{-1}(\overline{A}) \subseteq \overline{U}$.

Proof:
Necessity: Let $\overline{B}$ be any IVIFS in Y. Let $\overline{U}$ be an IVIFOS in X such that $g^{-1}(\overline{B}) \subseteq \overline{U}$. Then $\overline{U}^c$ is an IVIFCS in X. By hypothesis $g(\overline{U}^c)$ is an IVIFGSPCS in Y. Let $\overline{A} = (g(\overline{U}^c))^c$. Then $\overline{A}$ is an IVIFGSPOS in Y and $\overline{B} \subseteq \overline{A}$. Now $g^{-1}(\overline{A}) = g^{-1}(g(\overline{U}^c))^c = (g^{-1}(g(\overline{U}^c)))^c \subseteq \overline{U}$.

Sufficiency: Let $\overline{A}$ be any IVIFCS in X. Then $\overline{A}^c$ is an IVIFOS in X and $g^{-1}(g(\overline{A}^c)) \subseteq \overline{A}^c$ where $g(\overline{A})$ is an IVIFS in Y. By hypothesis, there exists an IVIFGSPOS $\overline{B}$ in Y such that $g(\overline{A}^c) \subseteq \overline{B}$ and $g^{-1}\overline{B} \subseteq \overline{A}^c$. Therefore $\overline{A} \subseteq (g^{-1}(\overline{B}))^c$. Hence $\overline{B}^c \subseteq g(\overline{A}) \subseteq g(g^{-1}(\overline{B}))^c \subseteq \overline{B}^c$. This implies that $g(\overline{A}) = \overline{B}^c$. Since $\overline{B}^c$ is an IVIFGSPCS in Y, $g(\overline{A})$ is an IVIFGSPCS in Y. Hence g is an IVIFGPC mapping.

2.16 Theorem: Let X, Y and Z be IVIFTSs. If g: X→Y is an IVIFC mapping and h: Y→Z is an IVIFGSPC mapping, then h•g is an IVIFGSPC mapping.

Proof: Let $\overline{A}$ be an IVIFCS in X. Then $g(\overline{A})$ is an IVIFCS in Y, by hypothesis. Since h is an IVIFGSPC mapping, $h(g(\overline{A}))$ is an IVIFGSPCS in Z. Therefore h•g is an IVIFGSPC mapping.

2.17 Theorem: Let g: X→Y be a bijection from an IVIFTS X to an IVIFSPT$_{1/2}$ space Y. Then the following statements are equivalent:

(i) g is an IVIFGSPC mapping,

(ii) ivifspcl $(g(\overline{A})) \subseteq g(ivifcl(\overline{A}))$ for each IVIFS $\overline{A}$ of X,

(iii) $g^{-1}(ivifspcl(\overline{B})) \subseteq ivifcl\ \ (g^{-1}(\overline{B}))$ for every IVIFS $\overline{B}$ of Y.

Proof:
$(i) \Rightarrow (ii)$: Let $\overline{A}$ be an IVIFS in X. Then $ivifcl(\overline{A})$ is an IVIFCS in X. (i) implies that $g(ivifcl(\overline{A}))$ is an IVIFGSPC in Y. Since Y is an IVIFSPT$_{1/2}$ space, $g(ivifcl(\overline{A}))$ is an IVIFSPCS in Y. Therefore $ivifspcl(g(ivifcl(\overline{A}))) = g(ivifcl(\overline{A}))$.

Now $ivifspcl(g(\overline{A})) \subseteq ivifspcl(g(ivifcl(\overline{A}))) = g(ivifcl(\overline{A}))$. Hence $ivifspcl(g(\overline{A})) \subseteq g(ivifcl(\overline{A}))$ for each IVIFS $\overline{A}$ of X.

$(ii) \Rightarrow (i)$: Let $\overline{A}$ be any IVIFCS in X. Then ivifcl $(\overline{A}) = \overline{A}$. (ii) implies that $ivifspcl\ (g(\overline{A})) \subseteq g(ivifcl(\overline{A}))$ $= g(\overline{A})$. But $g(\overline{A}) \subseteq ivifspcl(g(\overline{A}))$. Therefore $ivifspcl(g(\overline{A})) = g(\overline{A})$. This implies $g(\overline{A})$ is an IVIFSPC in Y. Since every IVIFSPCS is an IVIFGSPCS, $g(\overline{A})$ is an IVIFGSPCS in Y. Hence g is an IVIFGSPC mapping.

$(ii) \Rightarrow (iii)$: Let $\overline{B}$ be an IVIFS in Y. Then $g^{-1}(\overline{B})$ is an IVIFS in X. Since g is onto, $ivifspcl(\overline{B}) = ivifspcl(g(g^{-1}(\overline{B})))$ and (ii) implies $ivifspcl\ (g(g^{-1}(\overline{B}))) \subseteq g(ivifcl(g^{-1}(\overline{B})))$. Therefore we have $ivifspcl\ (\overline{B}) \subseteq g(ivifcl\ (g^{-1}(\overline{B})))$. Now $g^{-1}(ivifspcl(\overline{B})) \subseteq g^{-1}(g(ivifcl(g^{-1}(\overline{B})))) = ivifcl(g^{-1}(\overline{B}))$, since g is one to one. Hence $g^{-1}(ivifspcl(\overline{B})) \subseteq ivifcl(g^{-1}(\overline{B}))$.

$(iii) \Rightarrow (ii)$: Let $\overline{A}$ be any IVIFS in X. Then $g(\overline{A})$ is an IVIFS in Y. Since g is one to one, (iii) implies that $g^{-1}(ivifspcl\ (g(\overline{A})) \subseteq ivifcl(g^{-1}(g(\overline{A}))) = ivifcl(\overline{A})$. Therefore $g(g^{-1}(ivifspcl(g(\overline{A})))) \subseteq g(ivifcl(\overline{A}))$. Since g is onto $ivifspcl(g(\overline{A})) = g(g^{-1}(ivifspcl(g(\overline{A})))) \subseteq g(ivifcl(\overline{A}))$.

2.18 Theorem: Let g: X→Y be bijective mapping, where X is an IVIFTS and Y is an IVIFSPT$_{1/2}$ space. Then the following statements are equivalent:

(i) g is an IVIFGSPC mapping,

(ii) g is an IVIFGSPO mapping,

(iii) $g(ivifint(\overline{B})) \subseteq ivifcl(ivifint(ivifcl(g(\overline{B}))))$ for every IVIFS $\overline{B}$ in X.

Proof:

$(i) \Leftrightarrow (ii)$: is obvious.

$(ii) \Rightarrow (iii)$: Let $\overline{B}$ be an IVIFS in X. Then $ivifint(\overline{B})$ is an IVIFOS in X. By hypothesis $g(ivifint(\overline{B}))$ is an IVIFGSPOS in Y. Since Y is an IVIFSPT$_{1/2}$ space, $g(ivifint(\overline{B}))$ is an IVIFSPOS in Y. Therefore $g(ivifint\ (\overline{B})) \subseteq ivifcl(ivifint(ivifcl(g(ivifint(\overline{B}))))) \subseteq ivifcl(ivifint(ivifcl(g(\overline{B}))))$.

$(iii) \Rightarrow (i)$: Let $\overline{A}$ be an IVIFCS in X. Then $\overline{A}^c$ is an IVIFOS in X. By hypothesis, $g(ivifint(\overline{A}^c)) = g(\overline{A}^c) \subseteq ivifcl(ivifint(ivifcl(g(\overline{A}^c))))$.

That is $ivifint(ivifcl(ivifint(g(\overline{A})))) \subseteq g(\overline{A})$. This implies $g(\overline{A})$ is an IVIFβCS in Y and hence an IVIFGSPCS in Y. Therefore g is an IVIFGSPC mapping.

2.19 Theorem: Let g: X→Y be bijective mapping, where X is an IVIFTS and Y is an IVIFSPT$_{1/2}$ space. Then the following statements are equivalent:

(i) g is an IVIFGSPC mapping,

(ii) $g(\overline{B})$ is an IVIFGSPCS in Y for every IVIFCS $\overline{B}$ in X,

(iii) $ivifint(ivifcl(ivifint(g(\overline{B})))) \subseteq g(ivifcl(\overline{B}))$ for every IVIFS $\overline{B}$ in X.

Proof:

$(i) \Leftrightarrow (ii)$: is obvious.

$(ii) \Rightarrow (iii)$: Let $\overline{B}$ be an IVIFS in X. Then $ivifcl(\overline{B})$ is an IVIFCS in X. By hypothesis $g(ivifcl(\overline{B}))$ is an IVIFGSPCS in Y. Since Y is an IVIFSPT$_{1/2}$ space, $g(ivifcl(\overline{B}))$ is an IVIFSPCS in Y. Therefore $g(ivifcl\ (\overline{B})) \supseteq ivifint(ivifcl(ivifint(g(ivifcl(\overline{B}))))) \supseteq ivifint(ivifcl(ivifint(g(\overline{B}))))$.

$(iii) \Rightarrow (i)$: Let $\overline{A}$ be an IVIFCS in X. By hypothesis, $g(ivifcl(\overline{A})) = g(\overline{A}) \supseteq ivifint(ivifcl(ivifint(g(\overline{A}))))$. This implies $g(\overline{A})$ is an IVIFβCS in Y and hence an IVIFGSPCS in Y. Therefore g is an IVIFGSPC mapping.

2.20 Theorem: Let X and Y be IVIFTSs. A mapping g: X→Y is an IVIFGSPC mapping if $g(ivifspint\ (\overline{A})) \subseteq ivifspint(g(\overline{A}))$ for every $\overline{A}$ in X.

Proof: Let $\overline{A}$ be an IVIFOS in X. Then $ivifint(\overline{A}) = \overline{A}$. Now $g(\overline{A}) = g(ivifint(\overline{A})) \subseteq g(ivifspint(\overline{A})) \subseteq ivifspint\ (g(\overline{A}))$, by hypothesis. But $ivifspint\ (g(\overline{A})) \subseteq g(\overline{A})$. Therfore $g(\overline{A})$ is an IVIFSPOS in Y. That is $g(\overline{A})$ is an IVIFGSPOS in Y. Hence g is an IVIFGSPC mapping, by theorem 2.18.

2.21 Theorem: Let X be an IVIFTS and Y be an IVIFSPT$_{1/2}$ space. Let g: X→Y be bijection. Then the following statements are equivalent:

(i) g is an IVIFGSPC mapping,

(ii) $g(ivifint(\overline{A}) \subseteq ivifspint(g(\overline{A}))$ for each IVIFS $\overline{A}$ of X,

(iii) $ivifint(g^{-1}(\overline{B})) \subseteq g^{-1}(ivifspint(\overline{B}))$ for every IVIFS $\overline{B}$ of Y.

Proof:

$(i) \Rightarrow (ii)$: Let g be an IVIFGSPC mapping. Let $\overline{A}$ be any IVIFS in X. Then $ivifint(\overline{A})$ is an IVIFOS in X. Now $g(ivifint(\overline{A}))$ is an IVIFGSPOS in Y, by theorem 2.18. Since Y is an IVIFSPT$_{1/2}$ space, $g(ivifint(\overline{A}))$ is an IVIFSPOS in Y. Therefore $ivifspint(g(ivifint(\overline{A}))) = g(ivfint(\overline{A}))$. Now $g(ivifint(\overline{A})) = ivifspint(g(ivifint(\overline{A}))) \subseteq ivifspint(g(\overline{A}))$.

$(ii) \Rightarrow (iii)$: Let $\overline{B}$ be an IVIFS in Y. Then $g^{-1}(\overline{B})$ is an IVIFS in X. By (ii), $g(ivifint(g^{-1}(\overline{B}))) \subseteq ivifspint(g(g^{-1}(\overline{B}))) \subseteq ivifspint(\overline{B})$. Now $ivifint(g^{-1}(\overline{B})) \subseteq g^{-1}(g(ivifint(g^{-1}(\overline{B})))) \subseteq g^{-1}(ivifspint(\overline{B}))$.

$(iii) \Rightarrow (i)$: Let $\overline{A}$ be an IVIFOS in X. Then $ivifint(\overline{A}) = \overline{A}$ and $g(\overline{A})$ is an IVIFS in Y. By (iii), $ivifint(g^{-1}(g(\overline{A}))) \subseteq g^{-1}(ivifspint(g(\overline{A})))$. Now $\overline{A} = ivifint(\overline{A}) \subseteq ivifint(g^{-1}(g(\overline{A}))) \subseteq g^{-1}(ivifspint(g(\overline{A})))$. Therefore $g(\overline{A}) \subseteq g(g^{-1}(ivifspint(g(\overline{A})))) \subseteq ivifspint(g(\overline{A})) \subseteq g(\overline{A})$. Therefore $ivifspint(g(\overline{A})) = g(\overline{A})$ is an IVIFSPOS in Y and hence an IVIFGSPOS in Y. Thus g is an IVIFGSPC mapping, by theorem 2.18**.**

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Source of support: Nil, Conflict of interest: None Declared