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INTUITIONISTIC FUZZY SUBBIGROUPS OF A BIGROUP

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of intuitionistic fuzzy subbigroup of a bigroup.

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INTRODUCTION

In 1965, the fuzzy subset was introduced by L.A.Zadeh [10], after that several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov [2, 3], as a generalization of the notion of fuzzy set. The notion of fuzzy subgroups was introduced by Azriel Rosenfeld [4]. Palaniappan.N & K.Arjunan [7] defined the intuitionistic fuzzy subgroups of a group. In this paper, we introduce the some theorems in intuitionistic fuzzy subbigroup of a bigroup.

1. PRELIMINARIES

1.1 Definition: A set $(G, +, \bullet)$ with two binary operations + and $\bullet$ is called a bigroup if there exist two proper subsets G_1 and G_2 of G such that (i) $G = G_1 \cup G_2$ (ii) $(G_1, +)$ is a group (iii) $(G_2, \bullet)$ is a group.

1.2 Definition: Let X be a non-empty set. A **fuzzy subset A** of X is a function $A: X \to [0, 1]$.

1.3 Definition: Let X be a non-empty set. A **intuitionistic fuzzy subset** A in X is defined as an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \text{ in } X\}$, where $\mu_{A:}\ X \to [0, 1]$ and $\nu_{A:}\ X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $\mu_A(x) + \nu_A(x) \leq 1$.

1.4 Definition: Let (G, +) be a group. A fuzzy subset A of G is said to be a fuzzy subgroup of G if $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all x and y in G.

1.5 Definition: Let (G, +) be a group. An intuitionistic fuzzy subset A of G is said to be an intuitionistic fuzzy subgroup of G if it satisfies the following axioms:
- (i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$
- (ii) $\nu_A(x-y) \leq \max\{\nu_A(x), \nu_A(y)\}$ for all x and y in G.

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1.6 Definition: Let G = $(G_1 \cup G_2, +, \bullet)$ be a bigroup. Then a fuzzy set A of G is said to be a fuzzy subbigroup of G if there exist two fuzzy subsets A_1 of G_1 and A_2 of G_2 such that (i) $A = A_1 \cup A_2$ (ii) A_1 is a fuzzy subgroup of $(G_1, +)$ (iii) A_2 is a fuzzy subgroup of $(G_2, \bullet)$.

1.7 Definition: Let G = $(G_1 \cup G_2, +, \bullet)$ be a bigroup. Then an intuitionistic fuzzy set A of G is said to be an intuitionistic fuzzy subbigroup of G if there exist two intuitionistic fuzzy subsets A_1 of G_1 and A_2 of G_2 such that (i) $A = A_1 \cup A_2$ (ii) A_1 is an intuitionistic fuzzy subgroup of $(G_1, +)$ (iii) A_2 is an intuitionistic fuzzy subgroup of $(G_2, \bullet)$.

1.8 Definition: Let A = M∪N and B = O∪P be any two intuitionistic fuzzy subbigroups of bigroups G = E∪F and H= I∪J respectively. The product of A and B, denoted by A×B, is defined as A×B = (M×O) ∪ (N×P) where $\mu_{M\times O}(x, y) = \min\{\mu_M(x), \mu_O(y)\}$, $\nu_{M\times O}(x, y) = \max\{\nu_M(x), \nu_O(y)\}$, $\mu_{N\times P}(x, y) = \min\{\mu_N(x), \mu_P(y)\}$ and $\nu_{N\times P}(x, y) = \max\{\nu_N(x), \nu_P(y)\}$.

2. PROPERTIES

2.1 Theorem: If A = M∪N is an intuitionistic fuzzy subbigroup of a bigroup G = E∪F, then $\mu_M(-x) = \mu_M(x)$, $\mu_M(x) \leq \mu_M(e)$, $\nu_M(-x) = \nu_M(x)$, $\nu_M(x) \geq \nu_M(e)$ for all x, e in E, $\mu_N(x^{-1}) = \mu_N(x)$, $\mu_N(x) \leq \mu_N(e')$, $\nu_N(x^{-1}) = \nu_N(x)$, $\nu_N(x) \geq \nu_N(e')$ for all x, e' in F.

2.2 Theorem: If A= M∪N is an intuitionistic fuzzy subbigroup of a bigroup G = E∪F, then

(i) $\mu_M(x+y) = \mu_M(y+x)$ if and only if $\mu_M(x) = \mu_M(-y+x+y)$ for all x and y in E
(ii) $\mu_N(xy) = \mu_N(yx)$ if and only if $\mu_N(x) = \mu_N(y^{-1}xy)$ for all x and y in F
(iii) $\nu_M(x+y) = \nu_M(y+x)$ if and only if $\nu_M(x) = \nu_M(-y+x+y)$ for all x and y in E
(iv) $\nu_N(xy) = \nu_N(yx)$ if and only if $\nu_N(x) = \nu_N(y^{-1}xy)$ for all x and y in F.

Proof: (i) Let x and y belongs to E and e_1 be an identity element of E. Assume that $\mu_M(x+y) = \mu_M(y+x)$, then $\mu_M(-y+x+y) = \mu_M(-y+y+x) = \mu_M(e_1+x) = \mu_M(x)$. Therefore $\mu_M(x) = \mu_M(-y+x+y)$ for all x and y in E. Conversely, assume that $\mu_M(x) = \mu_M(-y+x+y)$, then $\mu_M(x+y) = \mu_M(x+y-x+x) = \mu_M(y+x)$. Therefore $\mu_M(x+y) = \mu_M(y+x)$ for all x and y in E. (ii) Let x and y belongs to F and e_2 be an identity element of F. Assume that $\mu_N(xy) = \mu_N(yx)$, then $\mu_N(y^{-1}xy) = \mu_N(y^{-1}yx) = \mu_N(e_2x) = \mu_N(x)$. Therefore $\mu_N(x) = \mu_N(y^{-1}xy)$ for all x and y in F. Conversely, assume that $\mu_N(x) = \mu_N(y^{-1}xy)$, then $\mu_N(xy) = \mu_N(xyxx^{-1}) = \mu_N(yx)$. Therefore $\mu_N(xy) = \mu_N(yx)$ for all x and y in F. (iii) Let x and y belongs to E and e_1 be an identity element of E. Assume that $\nu_M(x+y) = \nu_M(y+x)$, then $\nu_M(-y+x+y) = \nu_M(-y+y+x) = \nu_M(e_1+x) = \nu_M(x)$. Therefore $\nu_M(x) = \nu_M(-y+x+y)$ for all x and y in E.

Conversely, assume that $\nu_M(x) = \nu_M(-y+x+y)$, then $\nu_M(x+y) = \nu_M(x+y-x+x) = \nu_M(y+x)$.

Therefore $\nu_M(x+y) = \nu_M(y+x)$ for all x and y in E. (iv) Let x and y belongs to F and e_2 be an identity element of F. Assume that $\nu_N(xy) = \nu_N(yx)$, then $\nu_N(y^{-1}xy) = \nu_N(y^{-1}yx) = \nu_N(e_2x) = \nu_N(x)$. Therefore $\nu_N(x) = \nu_N(y^{-1}xy)$ for all x and y in F. Conversely, assume that $\nu_N(x) = \nu_N(y^{-1}xy)$, then $\nu_N(xy) = \nu_N(xyxx^{-1}) = \nu_N(yx)$. Therefore $\nu_N(xy) = \nu_N(yx)$ for all x and y in F.

2.3 Theorem: Let A= M∪N be an intuitionistic fuzzy subbigroup of a bigroup G = E∪F. If

(i) $\mu_M(x) < \mu_M(y)$, then $\mu_M(x+y) = \mu_M(x) = \mu_M(y+x)$ for all x and y in E
(ii) $\mu_N(x) < \mu_N(y)$, then $\mu_N(xy) = \mu_N(x) = \mu_N(yx)$ for all x and y in F
(iii) $\nu_M(x) < \nu_M(y)$, then $\nu_M(x+y) = \nu_M(y) = \nu_M(y+x)$ for all x and y in E
(iv) $\nu_N(x) < \nu_N(y)$, then $\nu_N(xy) = \nu_N(y) = \nu_N(yx)$ for all x and y in F.

Proof: (i) Let x and y belongs to E. Assume that $\mu_M(x) < \mu_M(y)$, then $\mu_M(x+y) \geq \min\{\mu_M(x), \mu_M(y)\} = \mu_M(x)$; and $\mu_M(x) = \mu_M(x+y-y) \geq \min\{\mu_M(x+y), \mu_M(y)\} = \mu_M(x+y)$. Therefore $\mu_M(x+y) = \mu_M(x)$ for all x and y in E. And $\mu_M(y+x) \geq \min\{\mu_M(y), \mu_M(x)\} = \mu_M(x)$; and $\mu_M(x) = \mu_M(-y+y+x) \geq \min\{\mu_M(y), \mu_M(y+x)\} = \mu_M(y+x)$. Therefore $\mu_M(y+x) = \mu_M(x)$ for all x and y in E. Hence $\mu_M(x+y) = \mu_M(x) = \mu_M(y+x)$ for all x and y in E. (ii) Let x and y belongs to F. Assume that $\mu_N(x) < \mu_N(y)$, then $\mu_N(xy) \geq \min\{\mu_N(x), \mu_N(y)\} = \mu_N(x)$; and $\mu_N(x) = \mu_N(xyy^{-1}) \geq \min\{\mu_N(xy), \mu_N(y)\} = \mu_N(xy)$. Therefore $\mu_N(xy) = \mu_N(x)$ for all x and y in F. And $\mu_N(yx) \geq \min\{\mu_N(y), \mu_N(x)\} = \mu_N(x)$; and $\mu_N(x) = \mu_N(y^{-1}yx) \geq \min\{\mu_N(y), \mu_N(yx)\} = \mu_N(yx)$. Therefore $\mu_N(yx) = \mu_N(x)$ for all x and y in F. Hence $\mu_N(xy) = \mu_N(x) = \mu_N(yx)$ for all x and y in F. (iii) Let x and y belongs to E. Assume that $\nu_M(x) < \nu_M(y)$, then $\nu_M(x+y) \leq \max\{\nu_M(x), \nu_M(y)\} = \nu_M(y)$; and $\nu_M(y) = \nu_M(-x+x+y) \leq \max\{\nu_M(x+y), \nu_M(x)\} = \nu_M(x+y)$. Therefore $\nu_M(x+y) = \nu_M(y)$ for all x and y in E. And $\nu_M(y+x) \leq \max\{\nu_M(y), \nu_M(x)\} = \nu_M(y)$; and $\nu_M(y) = \nu_M(y+x-x) \leq \max\{\nu_M(x), \nu_M(y+x)\} = \nu_M(y+x)$. Therefore $\nu_M(y+x) = \nu_M(y)$ for all x and y in E. Hence $\nu_M(x+y) = \nu_M(y) = \nu_M(y+x)$ for all x and y in E. (iv) Let x and y belongs to F. Assume that $\nu_N(x) < \nu_N(y)$, then $\nu_N(xy) \leq \max\{\nu_N(x), \nu_N(y)\} = \nu_N(y)$; and $\nu_N(y) = \nu_N(x^{-1}xy) \leq$

max{ν_N(xy), ν_N(x)} = ν_N(xy). Therefore ν_N(xy) = ν_N(y) for all x and y in F. And ν_N(yx) ≤ max{ ν_N(y), ν_N(x)} = ν_N(y); and ν_N(y) = ν_N(yxx^{-1}) ≤ max{ν_N(x), ν_N(yx)} = ν_N(yx). Therefore ν_N(yx) = ν_N(y) for all x and y in F. Hence ν_N(xy) = ν_N(y) = ν_N(yx) for all x and y in F.

2.4 Theorem: Let A = M∪N be an intuitionistic fuzzy subbigroup of a bigroup G = E∪F. If
(i) μ_M(x) > μ_M(y), then μ_M(x+y) = μ_M(y) = μ_M(y+x) for all x and y in E
(ii) μ_N(x) > μ_N(y), then μ_N(xy) = μ_N(y) = μ_N(yx) for all x and y in F
(iii) ν_M(x) > ν_M(y), then ν_M(x+y) = ν_M(x) = ν_M(y+x) for all x and y in E
(iv) ν_N(x) > ν_N(y), then ν_N(xy) = ν_N(x) = ν_N(yx) for all x and y in F.

Proof: It is trivial.

2.5 Theorem: Let A = M∪N be an intuitionistic fuzzy subbigroup of a bigroup G = E∪F. If (i) there is a sequence {x_n} in E such that $\lim_{n\to\alpha}$ min { $\mu_M(x_n)$, $\mu_M(x_n)$} = 1, then $\mu_M(e_1)$ = 1, where e_1 is the identity element in E, (ii) there is a sequence {x_n} in F such that $\lim_{n\to\alpha}$ min{ $\mu_N(x_n)$, $\mu_N(x_n)$} = 1, then $\mu_N(e_2)$ = 1, where e_2 is the identity in F, (iii) there is a sequence {x_n} in E such that $\lim_{n\to\alpha}$ max { $\nu_M(x_n)$, $\nu_M(x_n)$} = 0, then $\nu_M(e_1)$ = 0, where e_1 is the identity element in E, (iv) there is a sequence {x_n} in F such that $\lim_{n\to\alpha}$ max{$\nu_N(x_n)$, $\nu_N(x_n)$} = 0, then $\nu_N(e_2)$ = 0, where e_2 is the identity in F.

Proof: (i) Let e_1 be the identity element in E and x_n in E. Then $\mu_M(e_1)$ = $\mu_M(x_n - x_n)$ ≥ min{$\mu_M(x_n)$, $\mu_M(x_n)$} = $\mu_M(x_n)$. Therefore for each n, we have $\mu_M(e_1)$ ≥ $\mu_M(x_n)$. But $\mu_M(e_1)$ ≥ $\lim_{n\to\alpha}$ min{$\mu_M(x_n)$, $\mu_M(x_n)$} = 1. Therefore $\mu_M(e_1)$ = 1
(ii) Let e_2 be the identity element in F and x_n in F. Then $\mu_N(e_2)$ = $\mu_N(x_n x_n^{-1})$ ≥ min{ $\mu_N(x_n)$, $\mu_N(x_n)$} = $\mu_N(x_n)$. Therefore for each n, we have $\mu_N(e_2)$ ≥ $\mu_N(x_n)$. But $\mu_N(e_2)$ ≥ $\lim_{n\to\alpha}$ min{$\mu_N(x_n)$, $\mu_N(x_n)$}= 1. Therefore $\mu_N(e_2)$ = 1.
(iii) Let e_1 be the identity element in E and x_n in E. Then $\nu_M(e_1)$ = $\nu_M(x_n - x_n)$ ≤ max{$\nu_M(x_n)$, $\nu_M(x_n)$} = $\nu_M(x_n)$. Therefore for each n, we have $\nu_M(e_1)$ ≤ $\nu_M(x_n)$. But $\nu_M(e_1)$ ≤ $\lim_{n\to\alpha}$ max{$\nu_M(x_n)$, $\nu_M(x_n)$} = 0. Therefore $\nu_M(e_1)$ = 0. (iv) Let e_2 be the identity element in F and x_n in F. Then $\nu_N(e_2)$ = $\nu_N(x_n x_n^{-1})$ ≤ max{$\nu_N(x_n)$, $\nu_N(x_n)$} = $\nu_N(x_n)$. Therefore for each n, we have $\nu_N(e_2)$ ≤ $\nu_N(x_n)$. But $\nu_N(e_2)$ ≤ $\lim_{n\to\alpha}$ max{$\nu_N(x_n)$, $\nu_N(x_n)$} = 0. Therefore $\nu_N(e_2)$ = 0.

2.6 Theorem: If A = M∪N and B = O∪P are intuitionistic fuzzy subbigroups of the bigroups G = E∪F and H = I∪J, respectively, then A×B = (M×O) ∪ (N×P) is an intuitionistic fuzzy subbigroup of G×H = (E×I) ∪ (F×J).

Proof: Let x_1 and x_2 be in E, y_1 and y_2 be in I. Then (x_1, y_1) and (x_2, y_2) are in E×I. Now $\mu_{M\times O}$[(x_1, y_1) – (x_2, y_2)] = $\mu_{M\times O}$((x_1–x_2), (y_1– y_2)) = min{μ_M(x_1– x_2), μ_O(y_1–y_2)}≥ min{ min{$\mu_M(x_1)$, $\mu_M(x_2)$}, min{$\mu_O(y_1)$, $\mu_O(y_2)$}} = min{min{$\mu_M(x_1)$, $\mu_O(y_1)$}, min{ $\mu_M(x_2)$, $\mu_O(y_2)$}}= min{$\mu_{M\times O}$(x_1, y_1), $\mu_{M\times O}$(x_2, y_2)}. Therefore $\mu_{M\times O}$[(x_1, y_1) – (x_2, y_2)]≥ min{$\mu_{M\times O}$(x_1, y_1), $\mu_{M\times O}$(x_2, y_2)}. And $\nu_{M\times O}$[(x_1, y_1)–(x_2, y_2)] = $\nu_{M\times O}$((x_1–x_2), (y_1– y_2)) = max {ν_M(x_1– x_2), ν_O(y_1–y_2)}≤ max{max{$\nu_M(x_1)$, $\nu_M(x_2)$}, max{$\nu_O(y_1)$, $\nu_O(y_2)$}} = max {max{$\nu_M(x_1)$, $\nu_O(y_1)$}, max {$\nu_M(x_2)$, $\nu_O(y_2)$}} = max{$\nu_{M\times O}$ (x_1, y_1), $\nu_{M\times O}$(x_2, y_2)}. Therefore $\nu_{M\times O}$[(x_1, y_1) – (x_2, y_2)] ≤ max{$\nu_{M\times O}$(x_1, y_1), $\nu_{M\times O}$(x_2, y_2)}. Hence M×O is an intuitionistic fuzzy subgroup of E×I. Let x_1 and x_2 be in F, y_1 and y_2 be in J. Then (x_1, y_1) and (x_2, y_2) are in F×J. Also $\mu_{N\times P}$[(x_1, y_1)(x_2, y_2)$^{-1}$] = $\mu_{N\times P}$($x_1x_2^{-1}$, $y_1y_2^{-1}$) = min {$\mu_N(x_1x_2^{-1})$, $\mu_P(y_1y_2^{-1})$} ≥ min {min{ $\mu_N(x_1)$, $\mu_N(x_2)$ }, min{$\mu_P(y_1)$, $\mu_P(y_2)$}}= min{min{$\mu_N(x_1)$, $\mu_P(y_1)$}, min{$\mu_N(x_2)$, $\mu_P(y_2)$}}=min{$\mu_{N\times P}$(x_1, y_1), $\mu_{N\times P}$(x_2, y_2)}. Therefore $\mu_{N\times P}$[(x_1, y_1) (x_2, y_2)$^{-1}$] ≥ min{$\mu_{N\times P}$(x_1, y_1), $\mu_{N\times P}$(x_2, y_2)}. And $\nu_{N\times P}$[(x_1, y_1)(x_2, y_2)$^{-1}$] = $\nu_{N\times P}$($x_1x_2^{-1}$, $y_1y_2^{-1}$) = max {$\nu_N(x_1x_2^{-1})$, $\nu_P(y_1y_2^{-1})$} ≤ max{max{$\nu_N(x_1)$, $\nu_N(x_2)$ }, max{$\nu_P(y_1)$, $\nu_P(y_2)$}} = max {max{$\nu_N(x_1)$, $\nu_P(y_1)$}, max{$\nu_N(x_2)$, $\nu_P(y_2)$}}= max {$\nu_{N\times P}$(x_1, y_1), $\nu_{N\times P}$(x_2, y_2)}. Therefore $\nu_{N\times P}$[(x_1, y_1)(x_2, y_2)$^{-1}$] ≤ max {$\nu_{N\times P}$(x_1, y_1), $\nu_{N\times P}$(x_2, y_2)}. Therefore N×P is an intuitionistic fuzzy subgroup of F×J. Hence A×B is an intuitionistic fuzzy subbigroup of G×H.

2.7 Theorem: Let an intuitionistic fuzzy subbigroup A= M∪N of a bigroup G = E∪F be conjugate to an intuitionistic fuzzy subbigroup K = Q∪R of G = E∪F and an intuitionistic fuzzy subbigroup B = O∪P of a bigroup H = I∪J be conjugate to an intuitionistic fuzzy subbigroup L= S∪T of H= I∪J. Then an intuitionistic fuzzy subbigroup A×B = (M×O) ∪ (N×P) of a bigroup G×H = (E×I) ∪ (F×J) is conjugate to an intuitionistic fuzzy subbigroup K×L = (Q×S) ∪ (R×T) of G×H = (E×I) ∪ (F×J).

Proof: Let x, −x and f be in E and y, −y and g be in I. Then (x, y), (−x, −y) and (f, g) are in E×I. Now $\mu_{M\times O}(f, g) = \min\{\mu_M(f), \mu_O(g)\} = \min\{\mu_Q(x + f–x), \mu_S(y + g– y)\} = \mu_{Q\times S}(x+f–x, y+g–y) = \mu_{Q\times S}[(x, y) +(f, g)+(–x, –y)] = \mu_{Q\times S}[(x, y)+(f, g) – (x, y)]$. Therefore $\mu_{M\times O}(f, g) = \mu_{Q\times S}[(x, y) + (f, g) –(x, y)]$. And $\nu_{M\times O}(f, g) = \max\{\nu_M(f), \nu_O(g)\} = \max\{\nu_Q(x+f–x), \nu_S(y+g–y)\} = \nu_{Q\times S}(x+f–x, y+g–y) = \nu_{Q\times S}[(x, y) +(f, g)+(–x, –y)] = \nu_{Q\times S}[(x, y)+(f, g) – (x, y)]$. Therefore $\nu_{M\times O}(f, g) = \nu_{Q\times S}[(x, y) + (f, g) – (x, y)]$. Hence an intuitionistic fuzzy subgroup M×O of a group E×I is conjugate to an intuitionistic fuzzy subgroup Q×S of E×I. Let x, x^{-1} and f be in F and y, y^{-1} and g be in J. Then (x, y), (x^{-1}, y^{-1}) and (f, g) are in F×J. Now $\mu_{N\times P}(f, g) = \min\{\mu_N(f), \mu_P(g)\} = \min\{\mu_R(xfx^{-1}), \mu_T(ygy^{-1})\} = \mu_{R\times T}(xfx^{-1}, ygy^{-1}) = \mu_{R\times T}[(x, y)(f, g) \quad (x^{-1}, y^{-1})] = \mu_{R\times T}[(x, y)(f, g)(x, y)^{-1}]$. Therefore $\mu_{N\times P}(f, g) = \mu_{R\times T}[(x, y)(f, g)(x, y)^{-1}]$. And $\nu_{N\times P}(f, g) = \max\{\nu_N(f), \nu_P(g)\} = \max\{\nu_R(xfx^{-1}), \nu_T(ygy^{-1})\} = \nu_{R\times T}(xfx^{-1}, ygy^{-1}) = \nu_{R\times T}[(x, y)(f, g)(x^{-1}, y^{-1})] = \nu_{R\times T}[(x, y)(f, g)(x, y)^{-1}]$. Therefore $\nu_{N\times P}(f, g) = \nu_{R\times T}[(x, y)(f, g)(x, y)^{-1}]$. Therefore an intuitionistic fuzzy subgroup N×P of a group F×J is conjugate to an intuitionistic fuzzy subgroup R×T of F×J. Hence an intuitionistic fuzzy subbigroup A×B = (M×O) ∪ (N×P) of a bigroup G×H = (E×I) ∪ (F×J) is conjugate to an intuitionistic fuzzy subbigroup K×L = (Q×S) ∪ (R×T) of G×H = (E×I) ∪ (F×J).

2.8 Theorem: Let A = M∪N and B = O∪P be intuitionistic fuzzy subsets of the bigroups G = E∪F and H = I∪J, respectively and A×B = (M×O) ∪ (N×P) be an intuitionistic fuzzy subbigroup of G×H = (E×I) ∪ (F×J). Then the followings are true:

(i) if $\mu_M(x) \le \mu_O(e_2)$, $\nu_M(x) \ge \nu_O(e_2)$, then M is an intuitionistic fuzzy subgroup of E
(ii) if $\mu_N(x) \le \mu_P(e_2^{I})$, $\mu_N(x) \ge \mu_P(e_2^{I})$, then N is an intuitionistic fuzzy subgroup of F
(iii) A is an intuitionistic fuzzy subbigroup of G
(iv) if $\mu_O(x) \le \mu_M(e_1)$, $\nu_O(x) \ge \nu_M(e_1)$, then O is an intuitionistic fuzzy subgroup of I
(v) if $\mu_P(x) \le \mu_N(e_1^{I})$, $\mu_P(x) \ge \mu_N(e_1^{I})$, then P is an intuitionistic fuzzy subgroup of J
(vi) B is an intuitionistic fuzzy subbigroup of H
(vii) either A is an intuitionistic fuzzy subbigroup of G or B is an intuitionistic fuzzy subbigroup of H.

Proof: Let A×B = (M×O) ∪ (N×P) be an intuitionistic fuzzy subbigroup of G×H = (E×I) ∪ (F×J). (i) Let x and y be in E and e_2 be in I. Then (x, e_2) and (y, e_2) are in E×I. Using the property $\mu_M(x) \le \mu_O(e_2)$, $\nu_M(x) \ge \nu_O(e_2)$, we get $\mu_M(x–y) = \min\{\mu_M(x–y), \mu_O(e_2+e_2)\}=\mu_{M\times O}((x–y), (e_2+e_2))= \mu_{M\times O}[(x, e_2)+(–y, e_2)] \ge \min\{\mu_{M\times O}(x, e_2), \mu_{M\times O}(–y, e_2)\}= \min\{\min\{\mu_M(x), \mu_O(e_2)\}, \min\{\mu_M(y), \mu_O(e_2)\}\}= \min\{\mu_M(x), \mu_M(y)\}$. Therefore $\mu_M(x–y) \ge \min\{\mu_M(x), \mu_M(y)\}$for all x and y in E. And $\nu_M(x–y) = \max\{\nu_M(x–y), \nu_O(e_2+e_2)\} = \nu_{M\times O}((x–y), (e_2+e_2)) = \nu_{M\times O}[(x, e_2)+(–y, e_2)] \le \max\{\nu_{M\times O}(x, e_2), \nu_{M\times O}(–y, e_2)\} = \max\{\max\{\nu_M(x), \nu_O(e_2)\}, \max\{\nu_M(y), \nu_O(e_2)\}\} = \max\{\nu_M(x), \nu_M(y)\}$. Therefore $\nu_M(x–y) \le \max\{\nu_M(x), \nu_M(y)\}$for all x and y in E. Hence M is an intuitionistic fuzzy subgroup of E. (ii) Let x and y be in F and e_2^{I} be in J. Then (x, e_2^{I}) and (y, e_2^{I}) are in F×J. Using the property $\mu_N(x) \le \mu_P(e_2^{I})$, $\nu_N(x) \ge \nu_P(e_2^{I})$, we get $\mu_N(xy^{-1}) = \min\{\mu_N(xy^{-1}), \mu_P(e_2^{I} e_2^{I})\} = \mu_{N\times P}((xy^{-1}), (e_2^{I}e_2^{I})) = \mu_{N\times P}[(x, e_2^{I})(y^{-1}, e_2^{I})] \ge \min\{\mu_{N\times P}(x, e_2^{I}), \mu_{N\times P}(y^{-1}, e_2^{I})\} = \min\{\min\{\mu_N(x), \mu_P(e_2^{I})\}, \min\{\mu_N(y), \mu_P(e_2^{I})\}\} = \min\{\mu_N(x), \mu_N(y)\}$. Therefore $\mu_N(xy^{-1}) \ge \min\{\mu_N(x), \mu_N(y)\}$for all x and y in F. And $\nu_N(xy^{-1}) = \max\{\nu_N(xy^{-1}), \nu_P(e_2^{I}e_2^{I})\} = \nu_{N\times P}((xy^{-1}), (e_2^{I}e_2^{I})) = \nu_{N\times P}[(x, e_2^{I})(y^{-1}, e_2^{I})] \le \max\{\nu_{N\times P}(x, e_2^{I}), \nu_{N\times P}(y^{-1}, e_2^{I})\} = \max\{\max\{\nu_N(x), \nu_P(e_2^{I})\}, \max\{\nu_N(y), \nu_P(e_2^{I})\}\} = \max\{\nu_N(x), \nu_N(y)\}$. Therefore $\nu_N(xy^{-1}) \le \max\{\nu_N(x), \nu_N(y)\}$ for all x and y in F. Hence N is an intuitionistic fuzzy subgroup of F. (iii) From (i) and (ii), A is an intuitionistic fuzzy subbigroup of G. (iv) Let x and y be in I and e_1 be in E. Then (e_1, x) and (e_1, y) are in E×I. Using the property $\mu_O(x) \le \mu_M(e_1)$, $\nu_O(x) \ge \nu_M(e_1)$, we get $\mu_O(x–y) = \min\{\mu_O(x–y), \mu_M(e_1+e_1)\} = \mu_{M\times O}((e_1+e_1), (x–y)) = \mu_{M\times O}[(e_1, x)+(e_1,–y)] \ge \min\{\mu_{M\times O}(e_1, x), \mu_{M\times O}(e_1,–y)\} = \min\{\min\{\mu_M(e_1), \mu_O(x)\}, \min\{\mu_M(e_1), \mu_O(y)\}\} = \min\{\mu_O(x), \mu_O(y)\}$. Therefore $\mu_O(x–y) \ge \min\{\mu_O(x), \mu_O(y)\}$ for all x and y in I. And $\nu_O(x–y) = \max\{\nu_O(x–y), \nu_M(e_1+e_1)\} = \nu_{M\times O}((e_1+e_1), (x–y)) = \nu_{M\times O}[(e_1, x)+(e_1,–y)] \le \max\{\nu_{M\times O}(e_1, x), \nu_{M\times O}(e_1,–y)\} = \max\{\max\{\nu_M(e_1), \nu_O(x)\}, \max\{\nu_M(e_1), \nu_O(y)\}\} = \max\{\nu_O(x), \nu_O(y)\}$. Therefore $\nu_O(x–y) \le \max\{\nu_O(x), \nu_O(y)\}$ for all x and y in I. Hence O is an intuitionistic fuzzy subgroup of I (v) Let x and y be in J and e_1^{I} be in F. Then (e_1^{I}, x) and (e_1^{I}, y) are in F×J. Using the property $\mu_P(x) \le \mu_N(e_1^{I})$, $\nu_P(x) \ge \nu_N(e_1^{I})$, we get $\mu_P(xy^{-1})= \min\{\mu_P(xy^{-1}), \mu_N(e_1^{I}e_1^{I})\} = \mu_{N\times P}((e_1^{I}e_1^{I}), (xy^{-1}))= \mu_{N\times P}[(e_1^{I}, x)(e_1^{I}, y^{-1})] \ge \min\{\mu_{N\times P}(e_1^{I}, x), \mu_{N\times P}(e_1^{I}, y^{-1})\} = \min\{\min\{\mu_N(e_1^{I}), \mu_P(x)\}, \min\{\mu_N(e_1^{I}), \mu_P(y)\}\} = \min\{\mu_P(x), \mu_P(y)\}$. Therefore $\mu_P(xy^{-1}) \ge \min\{\mu_P(x), \mu_P(y)\}$ for all x and y in J. And $\nu_P(xy^{-1}) = \max\{\nu_P(xy^{-1}), \nu_N(e_1^{I}e_1^{I})\} = \nu_{N\times P}((e_1^{I}e_1^{I}), (xy^{-1})) = \nu_{N\times P}[(e_1^{I}, x)(e_1^{I}, y^{-1})] \le \max\{\nu_{N\times P}(e_1^{I}, x), \nu_{N\times P}(e_1^{I}, y^{-1})\} = \max\{\max\{\nu_N(e_1^{I}), \nu_P(x)\}, \max\{\nu_N(e_1^{I}), \nu_P(y)\}\} = \max\{\nu_P(x), \nu_P(y)\}$. Therefore $\nu_P(xy^{-1}) \le \max\{\nu_P(x), \nu_P(y)\}$ for all x and y in J. Hence P is an intuitionistic fuzzy subgroup of J. (vi) From (iv) and (v), B is an intuitionistic fuzzy subbigroup of H. (vii) is clear.

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