International Journal of Mathematical Archive-6(8), 2015, 51-65
Available online through www.ijma.info ISSN 2229 – 5046

A NOTE ON TL-IDEALS OF NEAR-FIELD SPACES OVER REGULAR δ-NEAR-RINGS

Dr. N. V. NAGENDRAM*

**Professor, Department of Mathematics(S&H),
Kakinada Institute of Technology & Science (KITS),
Tirupathi (v), Divili 533 433, East Godavari District, Andhra Pradesh. India.**

(Received On: 08-08-15; Revised & Accepted On: 21-08-15)

ABSTRACT

*The aim of this paper is to introduce and study TL-sub near-fields, TL-ideals of near-fields over regular delta near-rings and to obtain their characterizations. We define T-sum, T-difference, T-product of L-sub near-fields of a near-field N over regular delta near-ring of a ring R and obtain their properties. The set of near-fields of TL-ideals of a near-field N over –field regular delta near-ring of a ring R equipped with L-sub near-fields inclusion relation ≤ constitutes a complete lattice with L- sub near-field intersection as its meet is proved. The quotient near-field (N/μ, +T, *) is obtained and proved that it is isomorphic to the quotient near-field (N/Nμ,+T,*). Also we obtain some characterizations of homomorphism of TL-ideals of near-fields over regular delta near-rings.*

Keywords: *Fuzzy set, TL-subgroup, TL-sub near-field, TL-ideal, Isomorphism, Near-field, TL-subgroup, TL-sub-near-ring, TL–ideal, Isomorphism, Near-ring.*

Subject Classification Code: MSC (2010):*16D25, 54G05, 54C40.*

SECTION 1: INTRODUCTION

Near-field over a regular delta Near-Ring is a generalized structure of a near-field. Dr. N.V. Nagendram introduced the notion of fuzzy N- sub-near-fields and fuzzy ideals of near-field spaces over a regular delta near-ring.

Dr. N.V. Nagendram also introduced fuzzy ideals and fuzzy cosets of fuzzy ideals of near-field spaces over regular delta near-rings and extended the concept to TL-Ideals of near-field spaces over regular delta near-ring of a near-field.

As in near-ring theory it is interesting to fuzzify some substructures of near-field spaces over regular delta near-rings of a near-ring. Hence our aim in this paper is to study TL-sub-near-field spaces and TL-ideals of near-field spaces.

The organization of this paper is as follows:

In section 2, some preliminary definitions are given.

In section 3, T-sum, T-difference and T-product of L-sub-near-field spaces of a near-field over regular delta near-ring are defined. Also TL- sub-near-field space, TL- sub-near-field space and TL-ideals of a near-field space over regular delta near-ring are defined and their properties are studied.

In section 4, the properties of homomorphism of TL-ideals of a near-field space over regular delta near-ring are studied.

Section 5 concludes the paper.

The theory of fuzzy sets was introduced by Zadeh [14]. Goguen [9] introduced the concept of L- fuzzy sets. Rosenfeld [13] first introduced the fuzzification of the algebraic structures and defined fuzzy sub-groups. Anthony and Sherwood [3], Asaad and Abou-zaid [4], Akgul [2], Das [6], Dixit, Bhambri and Kumar [7] contributed the theory of fuzzy subgroups. Fuzzy ideals of ring are first defined by Liu. [11]. Cheng, Mordeson and Yandong [5] discussed TL-sub-rings and TL-ideals of a ring.

Corresponding Author: Dr. N. V. Nagendram*, Professor, Department of Mathematics(S&H), Kakinada Institute of Technology & Science (KITS), Tirupathi (v), Divili 533 433, East Godavari District, Andhra Pradesh. India.

In depth study of various types of near-fields [10], Dr. N V Nagendram extended the TL ideal of near-fields over regular delta near-ring in a near-field space by applying the topological applications making it into algebraic topology in modern and abstract algebra for large scale scope of applications in algebra of mathematics. Algebraic topology is a branch of mathematics that uses tools from abstract algebra to study topological spaces. The basic goal is to find algebraic invariants that classify topological spaces up to homeomorphism, though usually most classify up to homotopy equivalence.Although algebraic topology primarily uses algebra to study topological problems, using topology to solve algebraic problems is sometimes also possible. Algebraic topology, for example, allows for a convenient proof that any subgroup of a free group is again a free group

SECTION 2: PRELIMINARIES

In this section we recall some fundamental definitions for the sake of completeness in the sense of topology:

Definition (2.1) [10, 12]: By a near-ring we mean a non-empty set R with two binary operations '+' and '·' satisfying the following axioms:

(i) (R, +) is a group,
(ii) (R, ·) is a semi-group,
(iii) $x\cdot (y + z) = x\cdot y + x\cdot z$ for all $x, y, z \in R$.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-ring" instead of "left near-ring". We denote xy instead of x· y. Note that $x0 = 0$ and $x(-y) = -xy$, but $0x \neq 0$ for $x, y \in R$.

Definition (2.2): A binary operation T on a lattice L is called a t-norm if it satisfies the following conditions:

(i) T (T (a, b), c) = T (a, T (b, c)),
(ii) T (a, b) = T (b, a),
(iii) $b \leq c \Rightarrow T(a, b) \leq T(a, c)$,
(iv) T (a, 1) = a for all a, b, c in L.

Definition (2.3) [1, 8]: An ideal I of a near-ring R is a subset of R such that

(i) (I, +) is a normal subgroup of (R, +),
(ii) $RI \subset I$,
(iii) $(r + i) s - rs \in I$ for all i I and r, s R.

Note that if I satisfies (i) and (ii) then it is called a left ideal of R.

If I satisfies (i) and (iii) then it is called a right ideal of R.

SECTION 3: TL-IDEALS OF NEAR-FIELD SPACES OVER REGULAR DELTA NEAR - RINGS

In this section, I discuss about the operations known as T-sum, T- difference and T- product of L-sub-near-field spaces of a near-field N over a regular delta near-ring. Now we shall define various operations on L- sub-near-field spaces of a near-field N over regular delta near-ring R.

Let N be a near-field space over a regular delta near-ring and L be a complete lattice.

3.1 Definition: A function $\mu: N \rightarrow L$ is called an L-sub-near-field space of N.

The set of all L-sub-near-field spaces of N is called the L-power near-field space of N and is denoted by L^N.

3.2 Definition: Let μ, L^N and T be a t-norm on L. Then

(i) $(\mu + T)(x) = \vee\{\mu(y)T(z) \mid y, z \in R, y + z = x\}$.
(ii) $(-\mu)(x) = (\mu)(-x)$.
(iii) $(\mu - T)(x) = \vee\{\mu(y)\, T(z) \mid y, z \in N, y - z = x\}$.
(iv) $(\mu \,.\, T)(x) = \vee\{\mu(y)T(z) \mid y, z \in N, y\,.\,z = x\}$, where $x \in N$.

3.3 Remarks:

(i) μ+ T, μ- T and μ**.** T are called T-sum, T-difference and T- product of μ and respectively. – μ is called the negative of μ.
(ii) If T= $\wedge$ then μ+T, μ- T and μ**.** T are known as the sum, difference and product of μ and T respectively and are denoted by μ+ , μ - and μ. respectively.
(iii) μ+ T = + T μ and μ- T = μ+ T (-) for all μ, LR.As an elementary result we prove:

3.4 Theorem: Let μ, ξ ∈ L^N. Then μ (+ξ) ≤ μ + μ ξ.

3.5 Definition: For μ, L^N define n ∈ *n*
(μ T)(x) = ∨{T **(** μ(y i) T (z i))| y i, z i ∈ N, 1≤ i ≤ n, n ∈ N, ∑y i z i = x},x ∈N. *i* ≤1

3.6 Theorem: Let μ, ξ ∈ L^N. Then (i) μ.T ≤ μ T (ii) ≤ξ ⇒ μ T ≤ μ T ξ.

3.7 Definition: An L-sub-near-field space μ of a near-field space N over regular delta near-ring is called a TL-sub-near-field space of N if it satisfies the following conditions for all x, y ∈N:
(i) μ (0) = 1,
(ii) μ (- x) ≥ μ(x),
(iii) μ (x + y) ≥ μ(x) T μ(y).

Now we shall define TL- sub-near-field space over a regular delta near-ring and TL-ideal of a near-field space over a regular delta near-ring R as follows:

3.8 Definition: An L-sub-near-field space μ of a near-field space over regular delta near-ring R is called a TL- sub-near-field space over regular delta near-ring of R if it satisfies the following conditions:
(i) μ (0) = 1,
(ii) μ (- x) ≥ μ(x),
(iii) μ (x + y) ≥ μ(x) T μ(y),
(iv) μ (x y) ≥ μ(x) T μ(y), ∀ x, y ∈ N.

3.9 Remarks:
(i) When T= ∧, a TL-sub-near-field space over a regular delta near-ring is called L-sub-near-field space over a regular delta near-ring.
(i) The set of all TL- sub-near-field space over regular delta near-rings of N and set of all L- sub-near-field spaces over regular delta near-rings of N are denoted by TL (N) and L (N) respectively.
(ii) If L = [0, 1], TL- sub-near-field space over regular delta near-ring and L-sub- near-field space over regular delta near-ring of N are known as T-fuzzy sub- near-field space over regular delta near-ring and fuzzy sub-near-field space over regular delta near-ring of N respectively.

3.10 Definition: An L-sub- near-field space μ over regular delta of a near-ring N is called a TL-ideal of N if
(i) μ (0) = 1,
(ii) μ (-x) ≥ μ(x),
(iii) μ (x + y) ≥ μ(x) T μ(y),
(iv) μ(y+x-y) ≥ μ (x),
(v) μ (xy) ≥ μ(y),
(vi) μ ((x+i)y -xy) ≥ μ(i) for all x, y, i ∈ N.

3.11Note:
(1) If μ satisfies (i), (ii), (iii), (iv) and (v) then it is TL-left ideal of near-field space N.
(2) If μ satisfies (i),(ii),(iii),(iv) and (vi) then it is called TL-right ideal of near-field space N.
(3) When T= ∧, a TL-left ideal and TL-right ideal are known as L-left ideal and L-right ideal respectively.
(4) The set of all TL- left ideals and TL-right ideals of N are denoted by TL_I l(N) and TLI r (N) respectively. The set of all L- left ideals and L-right ideals of N are denoted by LI l (N) and LI r(N) respectively.
(5) When L = [0, 1], TL-left ideals and TL-right ideals are known as T-fuzzy left ideals and T-fuzzy right ideals of N respectively and when T= ∧, they are known as fuzzy left ideals and fuzzy right ideals of N respectively.

3.12Example: Let R = {a, b, c, d} be a non-empty set with two binary operations '+' and '·' defined as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	d

Then (R, +, ·) is a near-ring. Define a fuzzy set μ: R L by μ (c) = μ (d) < μ (b) < μ (a) = 1. Now define a binary operation T on L by
T(x, y) = max (x+y-1, 0) for all x, y ∈ R.
Then T is a t-norm on L and μ defined above is a TL-ideal of R. From the definition immediately we can obtain the following theorem:

Theorem (3.13): Every TL-ideal of a near-field N over a regular delta near-ring R is a TL-sub near-field of N over a regular delta near-ring of R.

Remark (3.14): The converse of the above theorem is not true. For this consider the following example:

Example (3.15): Let N= {a, b, c, d} be a non-empty set with two binary operations and '·' defined as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	b	c	d

Then (N, +, ·) is a near-field space over a regular delta near-ring.

Define a fuzzy set μ: N → L by μ (c) = μ (d) < μ (b) < μ (a) = 1.

Define a binary operation T on L by T(x, y) = max (x+y-1, 0) for all x, y ∈N.

Then T is a t-norm on L and μ defined above is a TL- sub near-field of N over regular delta near-ring R but not a TL-ideal of a near-field of N over regular delta near-ring R as μ ((c + b)d – cd) = μ (d) < μ (b).

Now we prove various properties of TL-ideals and TL-sub near-field of N over regular delta near-rings R. At the outset we characterize TL-sub near-field of N over regular delta near-ring as:

3.16 Theorem: Let μ L^N. Then μ is a TL- sub near-field of N over regular delta near-ring R if and only if

(1) μ (0) = 1,
(2) μ (x-y) ≥ μ(x) T μ(y) ,
(3) μ (x y) ≥ μ(x) T μ(y) for all x, y ∈N.

3.17 Theorem: Let μ be a TL-ideal of a near-field of N over regular delta near-ring R. Then μ (x -y) = μ (0) implies μ(x) =μ(y) for all x, y ∈ N.

In the following theorem we give a necessary and sufficient condition for μ and L^N to be fuzzy ideal of near-field space N over regular delta near-ring R:

3.18 Theorem: Let L be a chain and μ LR .Then a necessary and sufficient condition for μLI r (R) (resp. μ LIl (R)) is that every μa (a L) is a right (or left) ideal of R.

Proof:⇒ Let μ LI r (N).

Let x, y ∈ μa. Clearly μa is a non-empty sub near-field of N over a set of regular delta near-rings R.

Then μ (x - y) = μ (x + (- y)) ≥ μ (x) μ (y) = a.
∴ x - y ∈ μa for all x, y ∈ μa.

Again as μ LI r (R), μ (x) = μ(y + x - y) for all x, y ∈ N.

Now x μa ⇒ μ (x) ≥ a ⇒μ (y+x-y) ≥ a ⇒ y+x-y μa.
∴ y + x - y ∈ μa ∀ x μa and y ∈ N.

Hence (μa, +) is a normal subgroup of (N, +).

Again since μ LI r (N), μ((x + i)y - xy) ≥ μ(i) for all x, y, i ∈ N .

Let i μa. Then μ (i) ≥ a.
∴ μ ((x + i)y - xy) ≥ μ (i) ⇒ μ ((x + i)y - xy) ≥ a.

Thus, (x + i) y - xy μa for all i μa and x, y ∈ N.

Hence μa (a L) is a right ideal of N.

Conversely let μa (a L) be a right ideal of ∈ N.

If possible suppose that μ LI r (N).

When μ does not satisfy the condition μ (0) = 1, then μ (0) < 1, which implies that Nμ is not a sub near-field space of N over a regular delta near-ring R, which is a contradiction.

Again when μ does not satisfy the condition μ (-x) ≥ μ(x) ∀ x ∈ N.

Then there exists x N such that μ (- x) μ(x).

Take μ (x) = a.

Then x μa but -x μa, which is a contradiction.

Further, when μ does not satisfy the condition μ(x+y) ≥ μ(x) μ(y) ∀ x, y ∈N.

Then there exist x, y R such that μ(x+y) μ(x) μ(y).

Take μ(x) μ(y) = a.

Then x, y μa, but x + y μa which implies that μa is not a sub near-field space of N over a regular delta near-ring R of (N, +) and hence a contradiction.

Next when μ does not satisfy the condition μ(y + x - y) ≥μ(x).

Then there exist x,y ∈N such that μ(y+x-y) μ(x). Take μ(x) = a.

Then x μa but y + x - y μa, which is a contradiction.

When μ does not satisfy the condition μ ((x+i)y-xy) ≥μ(i).

Then there exist x, y ∈N such that μ ((x + i)y - xy) μ(i).

Take μ(i) = a.

Then i μa but (x + i)y - xy μa, which is a contradiction.

Hence μ is a L- right ideal of near-field space of N over a regular delta near-ring R i.e. μ LI r (N).

⇐: Similarly μ LIl (N) if and only if μa (a L) is a left ideal of near-field space N over a regular delta near-ring R.

A necessary condition for μ L^N to be fuzzy ideal of near-field space N over a regular delta near-ring R when L is a chain is mentioned in the following theorem:

This completes the proof of the theorem.

3.19 Theorem: Let μ L^N and L be a chain. Then a necessary condition forμ LI r(N)(resp.μ L_I ∈ l(N)) is that μ[a] (a L\{1}) is a right(resp. left) ideal of a near-field space N over regular delta near-ring R.

Proof: Let μ ∈ L^N and L be a chain.
⇒: Suppose μ LI r(N) and choose a L\ {1}.

To prove that μ[a] is a right ideal of near-field space N over regular delta near-ring R.

(i) μ (0) = 1 implies that 0 μ[a].
(ii) Let x μ[a]. Then μ (x) > a and as μ (- x) ≥ μ (x), μ (- x) > a. Hence - x μ[a] .
(iii) Again let x, y μ[a] .
Then μ (x + y) ≥ μ(x) μ(y) > a, which implies that x+y μ[a].
(iv) Let x μ [a].
Then μ(y + x - y) ≥ μ(x) ⇒ y + x - y μ[a] for all x μ[a] and y N.
(v) Let i μ[a] and x, y N.
Then μ ((x + i)y - xy) ≥ μ (i)⇒ μ((x + i)y - xy) > a
∴ (x +)y - xy μ[a] for all i μ[a] and x, y ∈ N. y N.

Hence μ[a] is a right ideal of a near-field space N over a regular delta near-ring R.

⇐: Suppose μ LIl (N) and choose a L\{1}.

To prove that μ[a] is a left ideal of a near-field space N over a regular delta near-ring R.

As in part (I), (μa, +) is a normal sub near-field space N over a regular delta near-ring of (N, +).

Let y N μ[a]. Then y Nμ[a] implies y = r x where r N and x μ[a] .

Again since μ LI l (N), μ (rx) ≥ μ(x) for r N and x μ[a] .

Thus μ(y) = μ (rx) ≥ μ(x) implies μ(y) = μ(rx) > a.
∴ y μ[a] and hence Nμ[a] ⊆ μ[a] . Thus μ[a] (a L) is a left ideal of near-field space N over a regular delta near-ring R.

This completes the proof of the theorem.

A sufficient condition for μ ∈ L^N to be fuzzy ideal of near-field space N over a regular delta near-ring R when L is dense is proved in the following theorem:

3.20 Theorem: Let μ ∈ LN and L be dense near-field spaces. Then a sufficient condition for μ LI r (N) (resp. μ LI l (N)) is that μ[a] (a L\{1}) is a right (or left) ideal of near-field space N over a regular delta near-ring R.

Proof: Let μ ∈ L^N and L be dense near-field spaces of N over a regular delta near-ring R.

⇒: Suppose that every μ[a] (a L\{1}) is a right ideal of near-field space of N over a regular delta near-ring R.

(i) Since 0 μ[a] , μ(0) > a for all a L\{1}.Therefore μ (0) = 1.
(ii) let x ∈ μ [a].Then x ∈ μ [a] ⇒ - x μ [a] ⇒ μ (- x) > a ∀ a ∈ L\{1}. Takingμ (x) = a we get μ (- x) > μ (x) ∀ x ∈ N.
(iii) Now suppose x, y ∈ μ [a].

Let μ (x) ∧ μ (y) > a. Then μ (x) ≥ μ (x) ∧ μ (y)⇒ x ∈ μ [a].

Similarly, y ∈ μ [a].

Hence x+y ∈ μ [a] and so μ (x + y) > a.

Let a = μ (x) ∧ μ (y).

If a = 0 then μ (x + y) ≥ 0 = μ (x)∧μ (y).

If a >0 then for any b ∈ L, b< a we observe that μ [b] is a right ideal of near-field space over regular delta near-ring R and x, y ∈ μ [b] ⇒ x + y ∈ μ [b]. i.e. μ(x + y) > b.
∴ μ (x + y) ≥ ∨{b| b ∈ L, b < a}.

Since L is dense, ∨ {b| b ∈ L, b < a} = a.
∴ μ (x + y) ≥ a = μ (x) ∧ μ (y).

(iv) Now let x, y ∈ N and μ (x) = a.

Let a>0. Then for any b ∈ L, b < a we observe that (μ[b], +) is a normal sub near-field space of (N,+) and x μ ⇒ μ (y + x - y) > b.

Thus μ (y + x - y) ≥ ∨{b| b ∈ L, b < a}.Since L is dense,∨ {b| b L, b < a} = a.

Hence μ (y + x - y) ≥ μ (x) for all x, y ∈ N.

(v) Finally, let i μ [a] and x, y ∈N.

If a = 0 then clearly μ ((x + i) y - xy) ≥ μ (i).

If a > 0 then for any b ∈ L, b < a we observe that μ [b] is a right ideal of near-field space N over a regular delta near-ring R and x ∈ μ [b] implies that μ ((x + i)y - xy) > b.

Hence μ ((x + i)y - xy) ≥∨{b| b ∈ L, b < a}.

Since L is a dense, ∨{b| b ∈ L, b < a}= a.
∴ μ ((x + i)y - xy) ≥ a = μ (i).

Hence μ ((x + i)y - xy) ≥ μ (i) for all x, y, i ∈ N.

Thus, μ LI r (R).

⇐ : Similarly if μ[a] (a ∈ L\{1}) is a left ideal of near-field space N over a regular delta near-ring R then μ LI l(N).

This completes the proof of the theorem.

For μ TLIr (N), Nμ is always right ideal of near-field space N over a regular delta near-ring R, is proved in the following theorem:

3.21 Theorem: Let μ TLIr (N) (or μ TLI l(N)). Then Nμ is a right (or left) ideal of near-field space N over a regular delta near-ring R.

When T is regular, Supp (μ) is always an ideal of R is proved below:

3.22 Theorem: Let μ TLI r (N) (resp. μ TLI l (N)). Then Supp (μ) is a right (resp. left) ideal of near-field space N over a regular delta near-ring R when T is regular.

How to obtain a complete lattice from the set of TL-ideals of a near-field space N over a regular delta near-ring is established in the following theorem:

3.23 Theorem: The set of near-field spaces of N, TLI r (N) (resp.TLI l (N)) equipped with L-sub near-field space of N inclusion '≤' constitutes a complete lattice with L- sub near-field space of N intersection as its meet. Its maximal and minimal elements are 1N and 1{0} respectively.

Further it is closed under L- sub near-field space of N T-intersection.

Proof: If part: Let μλ TLI r (N), λ where is any non empty index set. Then we observe the following points.
(i) μλ (0) = 1 for all λ .Therefore { μλ (0)| λ }= 1.

(ii) Since μλ (-x) ≥ μλ (x) ∀ x ∈ N and ∀ λ.
∴ {μλ (-x)| λ}≥ {μλ (x)| λ}, ∀ x ∈ N.

(iii) As μλ (x + y) ≥ μλ (x) T μλ (y), ∀ x, y ∈ N and ∀ λ.
∴ {μλ (x+y)|λ}≥({μλ (x) |λ })T({μλ (y)|λ }),∀ x, y ∈N.

(iv) Also μλ (y + x - y) ≥ μλ (x) ∀ x, y ∈ N and ∀ λ.
∴ {μλ(y + x - y)| λ} = {μλ (x)| λ}∀ x, y∈ N.

(V) Again μλ ((x + a)y -xy) ≥ μ(a),∀ x, y, a ∈ N and ∀ λ .

∴ {μλ(x+a)y-xy)| λ} ≥ {μλ (a)| λ}))∀ x, y, a ∈ N.

Hence {μλ | λ } TLI r (N).

As TLI r (N) L^N and L^N is a complete lattice with L-sub near-field space of N over regular delta near-ring set inclusion relation ≤.

∴ TLI r (N) is a complete lattice with L- sub near-field space of N over regular delta near-ring inclusion relation ≤.

Let μ, TLI r (N).Then ∀ x, y, a ∈ N.
(μT)(0) = μ (0) T (0) =1T1=1.

Clearly $(\mu T)(-x) \geq \mu(x)T(x) = (\mu T)(x)\ \forall\ x \in N$.
$(\mu T)(x+y) = \mu(x+y)T(x+y) \geq (\mu(x)T\mu(y))T((x)T(y)) = (\mu(x)T(x))T(\mu(x)T(x))$
$= (\mu T)(x)T(\mu T)(y)$.

Thus $(\mu T)(x+y) \geq (\mu T)(x)\, T\, (\mu T)(y)\ \forall\ x, y \in N$.

Now $(\mu T)(y+x-y) = \mu(y+x-y)\, T\, (y+x-y) \geq \mu(x)T(x) = (\mu T)(x)\ \forall\ x, y \in N$.

Thus $(\mu T)(y+x-y) \geq (\mu T)(x)\ \forall\ x, y \in N$.

Again $(\mu T)((x+a)y-xy) = \mu((x+a)y-xy)\, T\, ((x+a)y-xy) \geq \mu(a)T(a) = (\mu T)(a)$.

Therefore $(\mu T)((x+a)y-xy) \geq (\mu T)(a)\ \forall\ x, y, a \in N$.

Hence μT TLI r(N) and TLI r(N) is closed under L- sub near-field space of N over regular delta near-ring T-intersection.
$\langle\ \rangle$ A r}, for $x \neq 0$.

Finally 1N, 1{0} TLI r(N) and $1\{0\}(x) \leq \mu(x) \leq 1N(x)\ \forall\ x \in N$.
.
Hence 1N, 1{0} are the maximal and minimal elements of TLI r(N) respectively.

Iff Part: Let μλ TLI l (N), λ where is any non empty index set.

Then { μλ (x)|λ } is normal TL- sub near-field space of N over regular delta near-ring of (N,+).

Since $\mu\lambda(xy) \geq \mu\lambda(y)\ \forall\ x, y \in N$ and $\forall\ \lambda$.
$\therefore\ \{\mu\lambda(xy) \mid \lambda\} \geq \{\mu\lambda(y) \mid \lambda\}\ \forall\ x, y \in N$.

Hence {μλ | λ } TLIl (N).

As TLI l (N) $\subseteq L^N$ and L^N is a complete lattice with L-sub near-field space of N over regular delta near-ring inclusion relation $\leq$.

Therefore TLI l (N) is a complete lattice with L- sub near-field space of N over regular delta near-ring inclusion relation $\leq$.

Let μ, TLI l (N).Then for all $x, y \in N$,
$(\mu T)(xy) = \mu(xy)T(xy) \geq \mu(y)T(y) = (\mu T)(y)$ for all $x, y \in N$.

Hence μT TLI l(N) and TLIl (N) is closed under L- sub near-field space of N over regular delta near-ring T-intersection.

Clearly, 1_N, $1_{\{0\}}$ TLI_{l} (N) and $1_{\{0\}}(x) \leq \mu(x) \leq 1_R(x)\ \forall\ x \in N$.

Hence 1_N, $1_{\{0\}}$ are the maximal and minimal elements of TLI l(N) respectively. This completes the proof of the theorem.

Now we prove a property of the directed family {μi |i I} of TL-ideals of N:

3.24 Theorem: Let {μi | i I} be a directed family of TL-right (or TL-left) ideals near-field space of N over regular delta near-ring R where I is an index set.

Then { μi | i I } TLI r(N) (or { μi | i I } TLIl (N).

If N is a zero symmetric near-field space over a regular delta near-ring with unity then from the given L-sub near-field space of N over a regular delta near-ring μ over R we can obtain a TL-right ideal near-field space over a regular delta near-ring R generated by μ:

Theorem (3.25): Let N be a zero symmetric near-field space over a regular delta near-ring with unity. Let $\mu \in L^N$. Define $\gamma(x) = 1$ if $x = 0$, such that $\gamma(x) = \vee\{T_{y \in A}\ \mu(y) \mid A \subseteq N, 1 \leq |A| < \infty, x \in \langle A_r \rangle\}$ Then $\langle \mu \rangle_{rT} = \gamma$.

Proof: By definition of γ we observe that
(i) $\gamma(0) = 1$.

By taking A= {x} we get $\mu(x) \le \gamma(x)$ for all x.
$\therefore\ \mu \le \gamma$.
(ii) $\gamma(-x) = \{T_{y \in A}\ \mu(y) \mid A\ R,\ 1\le |A| < \infty,\ -x \langle A \rangle r\}$.
$\ge \{T_{y \in A}\ \mu(y) \mid A\ R,\ 1\le |A| < \infty,\ x \langle A \rangle r\}$.
$= \gamma(x)$

Thus, $\gamma(-x) \ge \gamma(x)\ \forall\ x\ N$.
(iii) Now we prove that $\gamma(x+y) \ge \gamma(x)T\gamma(y)$.for all x, y $\in$ N. For this consider the following cases:

Case (1): If x=0, y=0.

Then in this case x+y=0 and therefore we get $\gamma(x+y) = 1$,

Hence $\gamma(x+y) \ge \gamma(x)\ T\ \gamma(y)$.

Case (2): If x=0, y≠0.

Then in this case x+y=y and therefore we get $\gamma(x+y) \ge 1T\gamma(y) = \gamma(y)$

Case (3): If x≠0, y=0.

The proof is as case (2) and we get $\gamma(x+y) \ge \gamma(x)T\gamma(y)$.

Case (4): If $x \ne 0$, $y \ne 0$.Then $x + y \ne 0$.

Let $x \in \langle A\rangle_r$, $y \in \langle B\rangle_r$, where A, B are subsets of N, $1\le |A| < \infty$, $1\le |B| \le \infty$.

Then $x \in \langle A\rangle_r$, $y \in \langle B\rangle_r$, implies that x+y x $\in \langle A \cup B\rangle_r$.
$\gamma(x+y) \ge T_{z \in A \cup B}[\mu(z)]$.

$\ge (\{T\ \mu(u) \mid A \subseteq N, 1\le|A|<\infty, x \langle A \rangle r\})\ T\ (\{T\ \mu(v) \mid B \subseteq N, 1\le|B|<\infty, y \in 1 \langle B \rangle_r\})$.
$\forall\ u \in A$, $v \in B$. Hence in all the cases we get $\gamma(x+y) \ge \gamma(x)\ T\ \gamma(y)\ \forall\ x, y \in N$.
(iv) $\gamma(y+x-y) = \vee\{T\ \mu(z) \mid A \subseteq B \subseteq N, 1\le |A\ B| < \infty, y+x-y\ (A \subseteq B)r\}$, $z \in A \subseteq B$
$\ge \vee(\{T\ \mu(z) \mid A \subseteq N, 1\le |A| < \infty, x \in \langle A\rangle_r\}) = \gamma(x)$. $z \in A$

Hence $\gamma(y+x-y) \ge \gamma(x)$ for all x, y $\in$ N.

(v) Next let x, y $\in$ N and $i \in \langle B\rangle_r$. Then $(x+i)y - xy \in \langle B\rangle_r$.

$\gamma((x+i)y-xy) = \{T\ \mu(z) \mid B \subseteq N, 1\le|B|<\infty, (x+i)y - xy \in \langle B\rangle_r\}$. $z \in B$
$\ge \{T\ \mu(z) \mid B \subseteq N, 1\le|B| < \infty, i\ B\ r\} = \gamma(i)$. $z \in B$

Thus $\gamma((x+i)y-xy) \ge \gamma(i)$ for all x, y, i $\in$ N.

Hence $\gamma \in TLIr(N)$ and $\mu \le \gamma$.

Let $\xi \in TLIr(N)$ and $\mu \le \xi$.

First we prove that $\gamma \le \xi$.

If x=0, then $\xi(x) = \gamma(x) = 1$.

Let us consider $x \ne 0$.

Let A= $\{a_1, a_2, \text{-------}, a_n\}$ N and x $\in$N.

x= ∑ [(xi + ai) yi – xi yi] for some x_1, x_2, -------, x_n, ,y_1, y_2, -------, y_n R.

Since ξ TLIr (N) , we can write

ξ(x) = ξ(∑[(xi + ai) yi – xi yi]) ≥ T{ξ [(xi + ai) yi – xi yi]}≥ T ξ(ai).

∴ ξ(x) = {T ξ(z) | A ⊆R,1≤|A|< ∞, x ∈ x∈ A r}. z∈A

As μ ≤ ξ, we get
ξ(x) = {T ξ(z) | A ⊆ N,1≤ |A|< ∞, x ∈ A r}. z∈A

ξ(x) ≥ {T μ (z) | A ⊆N, 1≤ |A| < ∞, x∈ A r} = γ (x). z∈A

Thus γ (x) ≤ ξ(x) for all x ∈N.

Hence γ ≤ ξ and therefore γ = ‹μ› rT.

This completes the proof of the theorem.

Similarly we can obtain the following theorem:

Theorem (3.26): Let R be a zero symmetric near-ring with unity. Let μ LR. Define γ (x) = 1 if x = 0,
= {T μ(y) | y A, A R,1≤|A|<∞ , x ⟨A⟩ l }, for all x R. Then ‹ μ›lT = γ.

Following theorem is an easy consequence of the above theorem.
(2) Let μ, γ TL(R) and x, y R.

Theorem (3.27): Let N be a zero symmetric near-field space over a regular delta near-ring with multiplicative identity element. Let μ, γ ∈ L^N.

1. If μ, γ TL(R), then μ+ T TL (R).
2. If μ TLI l(R) and γ TL (R), then μ+ T TL (R).

Proof: obvious

Theorem (3.28): Let R be an abelian near-ring and μ TLI (R). Then

1. (R/μ, + T, *) is a near-ring isomorphic to the quotient near-ring (R/R μ, + T, *) where the binary operations + T , * are defined by (x+ μ) + T (y+ μ) = (x+y)+ μ and (x+ μ) * (y+ μ) = xy+ μ for all x, y R.
2. If an L-subset μ of R/μ is defined by μ (x+ μ) = μ(x), x R then μ TLI(R).

Proof: Obvious.

SECTION 4: HOMOMORPHISM OF TL-IDEALS OF NEAR-FIELD SPACES OVER REGULAR DELTA NEAR-RINGS: MAIN RESULT

In this section, I deduced theorem and is extended to near-field spaces with the in depth study over regular delta near-rings in near-field space N. Further to prove that the homomorphic image of TL-ideal of near-field space N over regular delta near – ring R is a TL- ideal of S:

4.1 Theorem (4.1): Let f: R S be a homomorphism of a near-field space N over a regular delta near-ring R onto a near-field space M over a regular delta near-ring S, μ ∈ TLI r(N) (resp.μ ∈ TLI l(N)). Then f (μ)∈TLI r(S)) (resp. μ ∈ TLI l(N)).

Proof: Let f: N → M be an onto homomorphism of near-field spaces over regular delta near-rings R and S and x, y ∈ S.

Part (I): Let μ TLI r(N).
(i) Clearly f (μ)(0') = 1.
(ii) f(μ)(-x) = ∨{μ(w)|w ∈ N,f(w) = -x} = ∨{μ(-w)|-w ∈ N, f(-w) = x}.
≥∨{μ(w)|w ∈N,f(w) = x} = f(μ)(x) ∀ x ∈ S.
(ii) f(μ)(x-y) = ∨{μ(w)|w ∈ N, f(w) = x-y} ≥ ∨{μ(u-v)|u, v ∈ N, f(u) = x, f(v)=y}
≥ ∨{μ(u)T μ (v)|u, v ∈ N, f(u) = x, f(v)=y}
≥ (∨{μ(u))|u ∈ N, f(u) =x })T(∨{μ (v)| v ∈ N, f(v)=y} = f (μ)(x)T f(μ)(y) ∀ x, y∈S.

(iii) f (μ) (y+x-y) = $\vee$ {μ(w)|w $\in$ N, f(w) = y+x-y}
= $\vee$ {μ(v+u-v) |u, v $\in$ N, f(u) =x, f(v)=y}.
$\geq$ $\vee$ {μ(u) |u $\in$ N, f(u) =x}. $\therefore$ f(μ)(y+x-y) $\geq$ $\vee$ {μ(u)|u $\in$ N, f(u) = x}.
Hence f(μ)(y+x-y) $\geq$ f(μ)(x) $\forall$ x, y $\in$ S.
(v) f(μ)((x+i)y-xy) = $\vee$ {μ(w)|w $\in$ N, f(w) = (x+i)y-xy}.
$\geq$ $\vee$ {μ((u+t)v-uv)|w $\in$ N, f(u) = x, f(v) = y, f(t) = i}.
$\geq$ $\vee$ {μ(t)| t $\in$ N, f(t)=i} = f(μ)(i) for all x, y, i $\in$ S. Hence f(μ) $\in$ TLI r(S).

Part (II): Let μ $\in$ TLI l (N).Then clearly
(vi) Now f (μ)(xy) = $\vee$ {μ(w)|w $\in$ N, f(w) =xy}
$\geq$ $\vee$ {μ(uv)|u, v $\in$ N, f(u) = x, f(v) =y }
$\geq$ $\vee$ {μ(v)| v $\in$ N, f(v) =y }$\geq$ f(μ)(y) $\forall$ x, y $\in$ S.
Hence from (i), (ii),(iii), (iv) and (vi); f(μ) $\in$ TLI l(S).

About the inverse images of TL- ideals of a near-field space M over regular delta near-ring S:

4.2 Theorem: Let f: N(R) $\to$ M(S) be a homomorphism of a near-field space N over regular delta near-ring R into a near-field space M over regular delta near-ring S and TLIr(N) (resp. TLI l (N)). Then, f -1 ( ) TLI r (M) (resp. f -1 ( ) TLI l(M)).

Proof: Let f: N(R) $\to$ M(S) be a homomorphism of a near-field space N over regular delta near-ring R into a near-field space M over regular delta near-ring S.

If Part: Let TLI r(S).
(i) f -1 (γ)(0) = (f)(0) = (0') = 1.
(ii) f -1 (γ)(-x) = (f)(-x) = (-f(x)) $\geq$ (f (x)) = f -1 (γ)(x) for all x $\in$ N(R).
(iii) f -1 (γ)(x+y) = (f(x+y)) = (f(x)+f(y)) $\geq$ (f(x))T (f(y)). = (f -1(γ) (x))T (f -1 (γ)(y)) $\forall$ x, y$\in$N(R).
(iv) f -1(γ)(y-x+y) = (f(y+x-y)) $\geq$ (f (x)) = f -1 (γ)(x) for all x, y $\in$ N(R)..
(v) f -1 (γ) ((x+i)y-xy) = (f((x+i)y-xy)) = ((f(x)-f(i))f(y)- f(x) f(y) $\geq$ f(i). = f-1 (i) for all x, y, i $\in$ N(R).
Hence f -1 (γ) TLIr (N).

Iff Part: Let TLI l (S).
(vi) f -1 (γ) (xy) = (f (xy)) = (f(x) f(y)) $\geq$ f(y)) = f -1 (γ)(y) for all x, y$\in$N(R). Hence from (i), (ii), (iii), (iv) and (vi) f -1(γ) TLI l(N).

For μ LR and TL(N), f (μ) is a TL-left ideal of near-field space N over a regular delta near-ring R as f (γ):

4.3 Theorem: Let f: N(R) $\to$ M(S) be a homomorphism of a near-field space N over regular delta near-ring R into a near-field space M over regular delta near-ring S. Let TL(N) and μ be a TL- left ideal of a near-field space N over regular delta near-ring R. Then f(μ) is a TL-left ideal of a near-field space M over regular delta near-ring S f (γ).

Proof: Part (I): Let μ be a TL- right ideal of a near-field space N over regular delta near-ring R. Then f (μ) and f (γ) are TL-sub near-field spaces groups of (N, +).Let x, y $\in$ S. Then
f (μ)(y+x-y) = $\vee$ {μ(w)|w $\in$N, f(w) =y+x-y}.
$\geq$ $\vee$ {μ(v+u-v)|u, v $\in$N, f(u) = x, f(v) = y}.
= $\vee$ {μ (u)|u N, f(u) =x}= f (μ)(x) for all x, y $\in$N.

f(μ)((x+i)y-xy) = $\vee$ {μ(w)|w $\in$N, f(w) =(x+i)y-xy}
$\geq$ $\vee$ {μ((u+t)v-uv)|u, v, t $\in$N, f((u+t)v-uv)) = (x+i)y-xy}
$\geq$ $\vee$ {μ(t)T (v) |t, y $\in$ N, f(t)=i, f(y)=v}
= ($\vee$ {μ(t)) |t $\in$N, f(t)=i})T($\vee$ {(v) |y $\in$N, f(y)=v}).
= f (μ)(i) T f (γ)(y).
Thus f (μ)((x+i)y-xy) $\geq$ f (μ)(i) T f ( )(y), for all x, y, i $\in$N.

Hence f (μ) is a TL-right ideal of a near-field space N over a regular delta near-ring R and f (γ) is a TL-right ideal of a near-field space M over regular delta near-ring S.

Part (II): Let μ be a TL- left ideal of a near-field space N over a regular delta near-ring R.
f(μ)(xy) = $\vee$ {μ(w)|w $\in$ N, f(w) =xy}.
$\geq$ $\vee$ {μ(uv)|u,v $\in$ N,f(u) =x, f(v) = y}.

Then (i) f -1 ≥∨{(u)T μ(v)|u, v ∈N, f(u) =x, f(v) = y}. =(∨{(u)| u ∈ N, f(u) =x})T(∨{μ(v) | v ∈ N, f(v) =y}) = f (γ)(x) T f (μ) (y) for all x, y ∈N. Hence f (μ) is a TL-left ideal of f ( ).

If μ LN, TL (N) then f (μ) is a TL-left ideal of near-field space N over regular delta near-ring R and f (γ) is proved in the following theorem:

4.4 Theorem: Let f: N(R) → M(S) be a homomorphism of a near-field space N over regular delta near-ring R into a near-field space M over regular delta near-ring S. Let TL(N) and and TL (M). If μ is a TL-right (resp.left) ideal of a near-field space M over regular delta near-ring S. Then f-1(μ) is a TL-right (resp.left) ideal of a near-field space M over regular delta near-ring S. f-1 (γ).

Proof: Let f: N(R) → M(S) be a homomorphism of a near-field space N over regular delta near-ring R into a near-field space M over regular delta near-ring S.

Clearly, μ≤ ⇒ f -1(μ) ≤ f-1(γ).

∴ f -1 (μ) and f-1 (γ) are TL-subgroups of R and f -1 (μ) ≤ f-1 (γ).

If Part: Let μ be a TL- right ideal of a near-field space N over regular delta near-ring R and x, y ∈ N.
(μ)(0) = μ (f)(0) = (0') = 1.
(ii)f -1 (μ)(-x) = μ (f)(-x) = μ (-f (x)) ≥ μ (f (x)) = f -1 (μ)(x) for all x ∈ N.

(iii)f -1 (μ)(x-y) = μ (f)(x-y) = μ (f (x-y)) = μ (f (x)-f(y))
≥ μ (f (x)) T μ (f (y)) = f -1 (μ)(x) T f -1 (μ)(y). Therefore f -1 (μ)(x-y) ≥ f -1 (μ)(x) T f -1 (μ)(y)
for all x, y ∈ N.
(iv)f -1 (μ)(y+x-y) = μ (f)(y+x-y) = μ (f(y)+f (x)-f(y))≥μ (f(x)) = f -1 (μ)(x). Therefore f -1 (μ)(y+x-y) ≥ f -1 (μ)(x) for all x,y ∈ N.
(v)f -1 (μ)((x+i)y-xy) = μ (f)((x+i)y-xy) = μ((f(x) + (f(i))(f(y)- (f(x)(f(x)).≥ μ (f (i)) = f -1 (μ)(i).
∴ f -1(μ)((x+i)y-xy) ≥ μ(f(i)) = f -1(μ)(i) for all x, y, i ∈ N.

Hence f -1(μ) is a right ideal of a near-field space N over regular delta near-ring R and f-1 (γ) is a right ideal of a near-field space M over regular delta near-ring S..

Iff Part: Let μ be a TL- left ideal of a near-field space N over regular delta near-ring R. Then

(vi)f-1(μ)(xy)=μ (f(xy)) = μ(f(x)f(y))≥ (f (x) T μ (f(y))= (f -1 ( )(x)) T (f -1 (μ)(y)).
∴ f -1 (μ)(xy) ≥ (f -1 ( )(x)) T (f -1 (μ)(y)) for all x, y ∈N.

Hence from (i), (ii) (iii) (iv) and (vi) f -1 (μ) is a TL-left ideal of a near-field space N over regular delta near-ring R and f-1 (γ) is a TL-left ideal of a near-field space N over regular delta near-ring S. This completes the proof of the theorem.

Theorem (4.5): If an L-sub near-field space μ of N/μ is defined by μ (x+ μ) = μ(x) for all x ∈N then μ ∈ TLI(N).

Proof: Let us define a function f: N/μ → N/Nμ by f(x+μ) = x+N μ for all x∈N. We shall prove that f is an onto isomorphism.

(1) f((x+ μ) + T (y+ μ)) = f((x+y)+μ) = (x+y)+N μ = (x+ N μ) + (y+ N μ) =f(x+ μ) +f (y+ μ)).
(2) f((x+ μ)*(y+ μ)) = f(xy+μ) = xy+ N μ = (x+ N μ) * (y+ N μ) =f(x+ μ)*f(y+ μ).
(3) Let f(x+ μ) = f(y+ μ) where x,y ∈ N.
Then f(x+ μ) = f(y+ μ) ⇒ x + N μ = y + N μ ⇒ x+ μ = y+ μ.
Hence f is an isomorphism.
(4) Clearly f is an onto isomorphism.
(5) An L-sub near-field space N over a regular delta near-ring μ of N/μ is defined by μ (x+ μ) = μ(x) for all x∈N. Then
(i) μ (μ) = μ (0+ μ) = μ(0) =1.
(ii) μ (-x+ μ) = μ(-x) ≥ μ(x) = μ (x+ μ) for all x ∈N.
(iii) μ ((x+ μ) + T (y+ μ)) = μ ((x+ y) + μ) = μ(x+y) ≥ μ(x)T μ(y) =μ (x+ μ)T μ (y+ μ) ∀ x, y ∈N.
(iv) μ ((y+ μ) + T (x+ μ) + T(-y+ μ)) = μ ((y+ x) + μ) + T(-y+ μ)).
=μ ((y+ x) + μ) + T(-y+ μ)).
=μ ((y+x-y)+μ)=μ(y+x-y)
≥ μ (x) = μ (x+ μ) for all x, y ∈N.
(v) μ ((x+ μ)*(y+ μ)) = μ (xy+ μ) = μ(xy) ≥ μ(y) = μ (y+ μ) ∀ x, y ∈N.

(vi) μ [((x+ μ) + T (a+ μ)))*(y+ μ)- (x+ μ)*(y+ μ)] = μ [(x+a)*(y+ μ)- (x+ μ)*(y+ μ)].
= μ [(x+a)y+ μ)- (xy+μ)].
= μ [((x+a)y – xy)+μ)].
= μ ((x+a)y– xy).
≥ μ (a).
= μ (a+ μ) ∀ x, y, a ∈N.

Therefore μ is a TL- ideal of a near-field space over regular delta near-ring R. Hence μ ∈ TLI (N).

SECTION 5: CONCLUSION

Near-field space theory over regular delta near-rings under algebra of mathematics has many applications in the study of permutation groups, block schemes and projective geometry. Near-field spaces over regular delta near-rings provide a non-linear analogue to the development of Linear Algebra, combinatorial problems and useful for agricultural experiments. In this paper I presented the notion of TL- ideals of near-field spaces over regular delta near-rings and derived the properties of these ideals of near-field spaces over regular delta near-rings.

REFERENCES

1. S.Abou-Zaid, On Fuzzy Subnear-rings and Ideals, Fuzzy Sets and Systems 44 (1991), 139 - 146.
2. M.Akgul, Properties of Fuzzy Groups, Journal of Mathematical Analysis and Applications 133(1988), 93-100.
3. J.M.Anthoy and H.Sherwood, Fuzzy Groups Redefined, Journal of Mathematical Analysis and Applications 69(1979), 124-130.
4. M.Asaad and S.Abou-Zaid, A Contribution to the Theory of Fuzzy Subgroups, Fuzzy Sets and Systems 77(1996), 355-369.
5. S.C.Cheng, John N.Mordeson and Yu Yandong, Lecture Notes in Fuzzy Mathematics and Computer Science.
6. P.S.Das, Fuzzy Groups and Level Subgroups, Journal of Mathematical Analysis and Applications 84(1981), 264-269.
7. V.N.Dixit, S.K.Bhambri and Pratibha Kumar, Level Subgroups and Union of Fuzzy Subgroups, Fuzzy Sets and Systems 37(1990), 359-371.
8. T. K.Dutta and B. K. Biswas, Fuzzy Ideals of Near-Ring, Bull. Cal. Math Soc. 89 (1997), 447 - 456.
9. J.A.Goguen, L-Fuzzy Sets, Jour. of Math Analysis & Appl.18 (1967), 145-174.
10. Pilz Gunter, Near-Rings, North-Holland, Amsterdam, 1983.
11. W. Liu, Fuzzy Invariant Subgroups and Fuzzy Ideals, Fuzzy Sets and Systems 8(1982), 133-139.
12. J.D.P.Meldrum, Near-Rings and their Links with Groups, Pitman, London, 85.
13. A.Rosenfeld, Fuzzy Groups, J. of Math. Analysis & Appl, 35(1971), 512-517.
14. L.A.Zadeh, Fuzzy Sets, Inform. and Control 8(1965), 338-353.
15.] N V Nagendram,T V Pradeep Kumar and Y V Reddy On "Semi Noetherian Regular Matrix δ–Near Rings and their extensions", International Journal of Advances in Algebra (IJAA) Jordan, ISSN 0973-6964, Vol.4, No.1, (2011), pp.51-55.
16. N V Nagendram,T V Pradeep Kumar and Y V Reddy "A Note on Bounded Matrices over a Noetherian Regular Delta Near Rings", (BMNR-delta-NR) published in International Journal of Contemporary Mathematics, Vol.2, No.1, June 2011, Copyright @ MindReader Publications, ISSNNo:0973-6298, pp.13-19.
17. N V Nagendram,T V Pradeep Kumar and Y V Reddy "A Note on Boolean Regular Near-Rings and Boolean Regular δ–Near Rings", (BR-delta-NR) published in International Journal of Contemporary Mathematics, IJCM Int. J. of Contemporary Mathematics ,Vol. 2, No. 1, June 2011, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp. 29 - 34.
18. N V Nagendram,T V Pradeep Kumar and Y V Reddy "on p-Regular δ–Near-Rings and their extensions", (PR-delta-NR) accepted and to be published in int. J. Contemporary Mathematics (IJCM),0973-6298,vol.1, no.2, pp.81-85, June 2011.
19. N V Nagendram, T V Pradeep Kumar and Y V Reddy "On Strongly Semi –Prime over Noetherian Regular δ–Near Rings and their extensions", (SSPNR-delta-NR) published in International Journal of Contemporary Mathematics, Vol.2, No.1, June 2011, pp.83-90.
20. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Structure Theory and Planar of Noetherian Regular δ-Near–Rings (STPLNR-delta-NR)", International Journal of Contemporary Mathematics, IJCM, published by IJSMA, pp.79-83, Dec, 2011.
21. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Matrix's Maps over Planar of Noetherian Regular δ-Near–Rings (MMPLNR-delta-NR)", International Journal of Contemporary Mathematics, IJCM, published by IJSMA, pp.203-211, Dec, 2011.
22. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On IFP Ideals on Noetherian Regular-δ- Near Rings(IFPINR-delta-NR)", Int. J. of Contemporary Mathematics, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, Vol. 2, No. 1, pp.53-58, June 2011.

23. N V Nagendram, B Ramesh paper "A Note on Asymptotic value of the Maximal size of a Graph with rainbow connection number 2*(AVM-SGR-CN2*)" published in an International Journal of Advances in Algebra(IJAA) Jordan @ Research India Publications, Rohini, New Delhi, ISSN 0973-6964 Volume 5, Number 2 (2012), pp. 103-112.
24. N V Nagendram and B Ramesh on "Polynomials over Euclidean Domain in Noetherian Regular Delta Near Ring Some Problems related to Near Fields of Mappings (PED-NR-Delta-NR & SPR-NF)" Accepted and published in an International Journal of Mathematical Archive (IJMA), An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN 2229-5046, vol.3, no.8, pp no. 2998-3002,2012.
25. N V Nagendram research paper on "Near Left Almost Near-Fields (N-LA-NF)" communicated to for 2nd intenational conference by International Journal of Mathematical Sciences and Applications, IJMSA @ mindreader publications, New Delhi on 23-04-2012 also for publication.
26. N V Nagendram, T Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy "A Generalized Near Fields and (m, n) Bi-Ideals over Noetherian regular Delta-near rings (GNF-(m, n) BI-NR-delta-NR)", published in an International Journal of Theoretical Mathematics and Applications (TMA), Greece, Athens, dated 08-04-2012.
27. N V Nagendram, Smt.T.Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy "Applications of Linear Programming on optimization of cool freezers(ALP-on-OCF)" Published in International Journal of Pure and Applied Mathematics, IJPAM-2012-17-670 ISSN-1314-0744 Vol-75 No-3(2011).
28. N V Nagendram "A Note on Algebra to spatial objects and Data Models (ASO-DM)" Published in international Journal American Journal of Mathematics and Mathematical Sciences, AJMMS, USA, Copyright @ Mind Reader Publications, Rohini, New Delhi, ISSN. 2250-3102, Vol.1, No.2 (Dec. 2012), pp. 233 – 247.
29. N V Nagendram, Ch Padma, Dr T V Pradeep Kumar and Dr Y V Reddy "A Note on Pi-Regularity and Pi-S-Unitality over Noetherian Regular Delta Near Rings (PI-R-PI-S-U-NR-Delta-NR)" Published in International Electronic Journal of Pure and Applied Mathematics, IeJPAM-2012-17-669 ISSN-1314-0744 Vol-75 No-4(2011).
30. N V Nagendram, Ch Padma, Dr T V Pradeep Kumar and Dr Y V Reddy "Ideal Comparability over Noetherian Regular Delta Near Rings (IC-NR-Delta-NR)" Published in International Journal of Advances in Algebra (IJAA, Jordan),ISSN 0973-6964 Vol:5,NO:1(2012), pp.43-53@ Research India publications, Rohini, New Delhi.
31. N. V. Nagendram, S. Venu Madava Sarma and T. V. Pradeep Kumar, "A Note On Sufficient Condition Of Hamiltonian Path To Complete Grphs (SC-HPCG)", IJMA-2(11), 2011, pp.1-6.
32. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Noetherian Regular Delta Near Rings and their Extensions (NR-delta-NR)", IJCMS, Bulgaria, IJCMS-5-8-2011,Vol.6, 2011, No.6, 255-262.
33. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Semi Noehterian Regular Matrix Delta Near Rings and their Extensions (SNRM-delta-R)", Jordan,@Research India Publications, Advancesin Algebra ISSN 0973-6964 Volume 4, Number 1 (2011), pp.51-55© Research India Publicationspp.51-55.
34. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Boolean Noetherian Regular Delta Near Ring(BNR-delta-NR)s", International Journal of Contemporary Mathematics, IJCM Int. J. of Contemporary Mathematics, Vol. 2, No. 1-2, Jan-Dec 2011, Mind Reader Publications, ISSN No: 0973-6298, pp. 23-27.
35. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Bounded Matrix over a Noetherian Regular Delta Near Rings (BMNR-delta-NR)", Int. J. of Contemporary Mathematics,Vol. 2, No. 1-2, Jan-Dec 2011, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp.11-16
36. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Strongly Semi Prime over Noetherian Regular Delta Near Rings and their Extensions (SSPNR-delta-NR)", Int. J. of Contemporary Mathematics, Vol. 2, No. 1, Jan-Dec 2011, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp.69-74.
37. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On IFP Ideals on Noetherian Regular Delta Near Rings (IFPINR-delta-NR)", Int. J. of Contemporary Mathematics, Vol. 2, No. 1-2, Jan-Dec 2011, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp.43-46.
38. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Structure Thoery and Planar of Noetherian Regular delta-Near–Rings (STPLNR-delta-NR)", International Journal of Contemporary Mathematics, IJCM, accepted for international conference conducted by IJSMA, New Delhi December 18, 2011, pp:79-83, Copyright @ Mind Reader Publications and to be published in the month of Jan 2011.
39. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "On Matrix's Maps over Planar of Noetherian Regular delta-Near–Rings (MMPLNR-delta-NR)", International Journal of Contemporary Mathematics, IJCM, accepted for international conference conducted by IJSMA, New Delhi December 18,2011, pp:203-211, Copyright @ Mind Reader Publications and to be published in the month of Jan 2011.
40. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "Some Fundamental Results on P- Regular delta-Near–Rings and their extensions (PNR-delta-NR)",International Journal of Contemporary Mathematics, IJCM, Jan-December' 2011, Copyright@MindReader Publications,ISSN:0973-6298, vol.2,No.1-2, PP.81-85.
41. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "A Generalized ideal based-zero divisor graphs of Noetherian regular Delta-near rings (GIBDNR- d-NR)" , International Journal of Theoretical Mathematics and Applications (TMA) accepted and published by TMA, Greece, Athens, ISSN:1792- 9687 (print), vol.1, no.1, 2011, 59-71, 1792-9709 (online), International Scientific Press, 2011.

42. N V Nagendram, Dr T V Pradeep Kumar and Dr Y V Reddy "Inversive Localization of Noetherian regular Delta-near rings (ILNR- Delta-NR)", International Journal of Pure And Applied Mathematics published by IJPAM-2012-17-668, ISSN.1314-0744 vol-75 No-3, SOFIA, Bulgaria.
43. N V Nagendram, S V M Sarma, Dr T V Pradeep Kumar "A note on Relations between Barnette and Sparse Graphs" publishd in an International Journal of Mathematical Archive (IJMA), An International Peer Review Journal for Mathematical, Science & Computing Professionals, 2(12),2011, pg no.2538-2542, ISSN 2229 – 5046.
44. N V Nagendram "On Semi Modules over Artinian Regular Delta Near Rings (S Modules-AR-Delta-NR) Accepted and published in an International Journal of Mathematical Archive (IJMA)", An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN 2229-5046, IJMA-3-474, 2012.
45. N VNagendram, N Chandra Sekhara Rao "Optical Near field Mapping of Plasmonic Nano Prisms over Noetherian Regular Delta Near Fiedls (ONFMPN-NR-Delta-NR)" accepted for 2nd international Conference by International Journal of Mathematical Sciences and Applications, IJMSA @ mind reader publications, New Delhi going to conduct on 15 – 16 th December 2012 also for publication.
46. N V Nagendram, K V S K Murthy (Yoga), "A Note on Present Trends on Yoga Apart From Medicine Usage and Its Applications (PTYAFMUIA)" pubished by the International Association of Journal of Yoga Therapy, IAYT 18 th August, 2012.
47. N V Nagendram, B Ramesh, Ch Padma, T Radha Rani and S V M Sarma research article "A Note on Finite Pseudo Artinian Regular Delta Near Fields (FP AR-Delta-NF)" communicated to International Journal of Advances in Algebra, IJAA, Jordan on 22 nd August 2012.
48. Dr. N V Nagendram, A Text Book "Algebraic topology over near-fields & Semi simple near-fields and Its Applications" published by Research India Publications RIP, Rohini, Sector 16, New Delhi 110 089, Sept 2015.

Source of support: Nil, Conflict of interest: None Declared