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ON INTEGRAL PERFECT FACTOGRAPH AND INTEGRAL BI-FACTOGRAPH

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ABSTRACT

Using the theorem of unique factorization for integers, every positive integer z can be written in the canonical form $z = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_r^{\alpha_r}$, where $p_1, p_2, \ldots p_r$ are distinct primes, $\alpha_1, \alpha_2, \ldots \alpha_r$ are positive integers. We can construct a graph G which is associated with this z. Integral divisors of z being a vertex set V, two distinct vertices of V are adjacent in G if their product is in V and the corresponding graph is called Integral Factograph. For given z, we have introduced two new classes of graphs namely, Integral Perfect Factograph and Integral Bi-Factograph with respect to the values $r = 1$ and $r = 2$, attempt to find their degree sequence and clique number.

Keywords: *Factograph, Integral Factograph, Integral Perfect factograph, Integral Bi-factograph, clique number.*

1. INTRODUCTION

Graph indicates a finite undirected, non-trivial graph without loops and multiple edges. The order and size of a graph is denoted by p and q respectively. For terms not defined here we refer to Frank Harary[5].The concept of facto graph was introduced in [2][3]. In this paper we extent the concept to Integral Factograph. For a positive integer z, an Integral Factograph is represented as G=(V, E) where V= {v_i, -v_i / v_i and -v_i are factors of z} and two distinct vertices v_i and v_j are adjacent if and only if their product is in V. A clique of a graph G is a complete subgraph of G. A clique of G is a maximal clique if it is not properly contained in another clique of G. Number of vertices in the maximal clique of G is called the clique number of G and is denoted by ω(G). For v ∈ V, d(v) is the number of edges incident with v.

2. INTEGRAL PERFECT FACTOGRAPH

Definition: 2.1 An Integral factograph G with z =$p_1{}^{\alpha_1}$, where p_1 is a prime and α_1 is a positive integer is called Integral perfect factograph.

Theorem: 2.2 An Integral perfect factograph G is of order 2(α_1+1) and the degree sequence is
(i) s_1: 2α_1+1, 2α_1+1, 2α_1-1,2α_1-1,…, α_1 +1, α_1+1, α_1 , α_1 ,…,4,4,2,2, when α_1 is even.
(ii) s_2: 2α_1+1, 2α_1+1, 2α_1-1, 2α_1-1,…, $2\left\lceil\frac{\alpha_1}{2}\right\rceil + 1$, $2\left\lceil\frac{\alpha_1}{2}\right\rceil + 1$,$2\left\lceil\frac{\alpha_1}{2}\right\rceil$, $2\left\lceil\frac{\alpha_1}{2}\right\rceil$,…,4,4,2,2, when α_1 is odd.

Proof:

Case (i): When α_1 is even,

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Let G= (V, E) be an Integral factograph with z =${p_1}^{\alpha_1}$. Number of integral divisors of ${p_1}^{\alpha_1}$ is $2(\alpha_1$+1) and hence the order of G is $2(\alpha_1$+1). Let V= {p_1^0,- p_1^0, p_1^1, -p_1^1, p_1^2,-p_1^2,⋯, ${p_1}^{\alpha_1}$, -${p_1}^{\alpha_1}$} be the vertex set of G. For convenience split the vertex set V into two classes such that A= { p_1^0, p_1^1, p_1^2,…, ${p_1}^{\alpha_1}$} and B= {- p_1^0,- p_1^1,-p_1^2,…, -${p_1}^{\alpha_1}$}. In G, We observe that for i≠j, the vertex p_1^i (or -p_1^i) is adjacent to p_1^j (or -p_1^j) if and only if i+j $\leq \alpha_1$. Thus the vertex p_1^0 adjacent with rest of the vertices, which implies *d(*p_1^0*)*= $2\alpha_1$+1 and *d(*$-p_1^0$*)*=$2\alpha_1$+1. The vertex p_1^1 adjacent with α_1-1 vertices from A and α_1 vertices from B implies that *d(*p_1^1*)*= $2\alpha_1$-1. Also, *d(-*p_1^1*)*= $2\alpha_1$-1. Consider the vertex $p_1^{\frac{\alpha_1}{2}}$ in A, it is adjacent to $\frac{\alpha_1}{2}$ vertices of A and $\frac{\alpha_1}{2}$+1 vertices of B and which implies that *d(*$p_1^{\frac{\alpha_1}{2}}$ *)*= α_1+1 and *d(-*$p_1^{\frac{\alpha_1}{2}}$ *)*= α_1+1. Likewise ${p_1}^{\alpha_1}$ is adjacent only with p_1^0 and -p_1^0, which gives *d(* ${p_1}^{\alpha_1}$*)*= 2 and *d(-*${p_1}^{\alpha_1}$*)*= 2. Hence the degree sequence of G is s: $2\alpha_1$+1, $2\alpha_1$+1, $2\alpha_1$-1,$2\alpha_1$-1,…, α_1+1, α_1+1, α_1, α_1 ,…,4,4,2,2.

Similar manner we can prove for the odd case.

Theorem: 2.3 For an Integral perfect factograph *G*, the clique number is given by,

$$\omega(G) = \begin{cases} \left(\frac{\alpha_1}{2}+1\right)+\left(\frac{\alpha_1}{2}+1\right), & \text{if } \alpha_1 \text{ is even} \\ \left(\left\lceil\frac{\alpha_1}{2}\right\rceil+1\right)+\left(\left\lfloor\frac{\alpha_1}{2}\right\rfloor+1\right), & \text{if } \alpha_1 \text{ is odd} \end{cases}$$

Proof: When α_1 is even,

Let G= (V, E) be an Integral perfect factograph with z =${p_1}^{\alpha_1}$. Let V= {p_1^0,- p_1^0, p_1^1, -p_1^1, p_1^2, -p_1^2,⋯, ${p_1}^{\alpha_1}$, -${p_1}^{\alpha_1}$} be the vertex set of G and we consider the set S={p_1^x,- p_1^x/0≤ x ≤ $\frac{\alpha_1}{2}$}, which is a proper subset of V. We seek to prove that the subgraph of G induced by S is the maximal clique of G.

Claim: ⟨S⟩ is a clique of G

In a perfect factograph G, we observe that for i≠j, a vertex p_1^i (or -p_1^i) is adjacent to p_1^j (or -p_1^j) if and only if i+j $\leq \alpha_1$. We try to prove that every pair of distinct vertices in S is adjacent. Consider two arbitrary vertices p_1^a(or -p_1^a) and p_1^b (or -p_1^b) in S. Since $a + b \leq \alpha_1$, we have $p_1^a$and p_1^b are adjacent. Thus the subgraph induced by S is a complete subgraph of G. It remains to prove that ⟨S⟩ is the maximal clique of G.Take any arbitrary vertex v in V which is not in S. By the Integral facto graph condition, S+ {v} is not a clique of G implies that S is the maximal clique of G and $\omega(G)$ = $|S|$ = $\left(\frac{\alpha_1}{2}+1\right)$+$\left(\frac{\alpha_1}{2}+1\right)$.

Similar manner, we can prove for the odd case.

Example: 2.4 consider an Integral perfect factograph G with z =${p_1}^3$. Here order of G is 8.

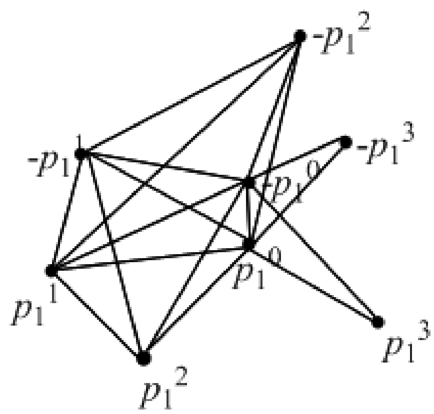

Figure: 1

We observe that, the degree sequence of G is s: 7,7,5,5,4,4,2,2 and ω(G) is 5.

3. INTEGRAL BI-FACTOGRAPH

Definition: 3.1 An Integral factograph G with z=${p_1}^{\alpha_1}{p_2}^{\alpha_2}$, where p_1, p_2 are distinct primes and α_1, α_2 are positive integers is called Integral Bi-factograph.

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Theorem: 3.2 Let α_1 and α_2 be two positive integers, p_1 and p_2 be two distinct primes. An Integral Bi-factograph G has order $2(\alpha_1+1)(\alpha_2+1)$ and the degree sequence is given by,

(i) s_1: $2(\alpha_1+1)(\alpha_2+1)$-1, $2(\alpha_1+1)(\alpha_2+1)$-1, $2(\alpha_1+1)\alpha_2$-1, $2(\alpha_1+1)\alpha_2$-1,⋯, $2(\alpha_1+1)(\frac{\alpha_2}{2})$, $2(\alpha_1+1)(\frac{\alpha_2}{2})$,⋯, $2(\alpha_1+1)$, $2(\alpha_1+1)$,⋯, $2(\frac{\alpha_1}{2})(\alpha_2+1)$, $2(\frac{\alpha_1}{2})(\alpha_2+1)$,⋯,2,2, where α_1 and α_2 are even.

(ii) s_2: $2(\alpha_1+1)(\alpha_2+1)$-1, $2(\alpha_1+1)(\alpha_2+1)$-1,⋯, $2(\alpha_1+1)\left\lfloor\frac{\alpha_2}{2}\right\rfloor$, $2(\alpha_1+1)\left\lfloor\frac{\alpha_2}{2}\right\rfloor$,⋯, $2\left\lfloor\frac{\alpha_1}{2}\right\rfloor(\alpha_2+1)$, $2\left\lfloor\frac{\alpha_1}{2}\right\rfloor(\alpha_2+1)$,⋯, 2, 2, where α_1 and α_2 are odd.

(iii) s_3: $2(\alpha_1+1)(\alpha_2+1)$-1,$2(\alpha_1+1)(\alpha_2+1)$-1,⋯,, $2(\alpha_1+1)(\frac{\alpha_2}{2})$,$2(\alpha_1+1)(\frac{\alpha_2}{2})$,⋯,$2\left\lfloor\frac{\alpha_1}{2}\right\rfloor(\alpha_2+1)$,$2\left\lfloor\frac{\alpha_1}{2}\right\rfloor(\alpha_2+1)$,⋯, 2, 2, where α_1 is odd and α_2 is even.

(iv) s_4: $2(\alpha_1+1)(\alpha_2+1)$-1, $2(\alpha_1+1)(\alpha_2+1)$-1,⋯, $2(\alpha_1+1)\left\lfloor\frac{\alpha_2}{2}\right\rfloor$, $2(\alpha_1+1)\left\lfloor\frac{\alpha_2}{2}\right\rfloor$,⋯, $(\frac{\alpha_1}{2})(\alpha_2+1)$, $(\frac{\alpha_1}{2})(\alpha_2+1)$,⋯,2,2, where α_1 is even and α_2 is odd.

Proof: ***Case* (i):** When α_1 and α_2 are even,

Let G= (V, E) be an Integral Bi-factograph. If z=$p_1^{\alpha_1}\ p_2^{\alpha_2}\ \cdots\ p_r^{\alpha_r}$, then the number integral divisors of z is $2\prod_{i=1}^{r}(\alpha_i+1)$. Therefore, number of Integral divisors of z $= p_1^{\alpha_1}p_2^{\alpha_2}$ is $2(\alpha_1+1)(\alpha_2+1)$ so that the order of G is $2(\alpha_1+1)(\alpha_2+1)$. Let V={$p_1^0p_2^0$,-$p_1^0p_2^0$,$p_1^0p_2^1$, -$p_1^0p_2^1$, ⋯, $p_1^0p_2^{\frac{\alpha_2}{2}+1}$,-$p_1^0p_2^{\frac{\alpha_2}{2}+1}$, ⋯,$p_1^0p_2^{\alpha_2}$, -$p_1^0p_2^{\alpha_2}$ ⋯,$p_1^{\frac{\alpha_1}{2}+1}p_2^{\alpha_2}$, -$p_1^{\frac{\alpha_1}{2}+1}p_2^{\alpha_2}$, ⋯, $p_1^{\alpha_1}p_2^{\alpha_2}$, - $p_1^{\alpha_1}p_2^{\alpha_2}$}. In G, we observe that for i≠k, j≠l, the vertex $p_1^ip_2^j$ (or - $p_1^ip_2^j$) is adjacent with $p_1^kp_2^l$ if and only if i+k $\leq \alpha_1$ and j+l $\leq \alpha_2$. Thus the vertex $p_1^0p_2^0$ adjacent to rest of the vertices which implies d($p_1^0p_2^0$) =$2(\alpha_1+1)(\alpha_2+1)$-1 and d(-$p_1^0p_2^0$) =$2(\alpha_1+1)(\alpha_2+1)$-1. Proceeding like that we have the degree sequence as follows d($p_1^0p_2^1$)=$2(\alpha_1+1)\alpha_2$-1, d(-$p_1^0p_2^1$)=$2(\alpha_1+1)\alpha_2$-1 ,⋯, d($p_1^0p_2^{\frac{\alpha_2}{2}+1}$)=$2(\alpha_1+1)(\frac{\alpha_2}{2})$, d(-$p_1^0p_2^{\frac{\alpha_2}{2}+1}$) $=2(\alpha_1+1)\left(\frac{\alpha_2}{2}\right)$,⋯, d($p_1^0p_1^{\alpha_2}$)=$2(\alpha_1+1)$, d(-$p_1^0p_1^{\alpha_2}$)=$2(\alpha_1+1)$, ⋯, d($p_1^{\frac{\alpha_1}{2}+1}p_2^0$)=$2(\frac{\alpha_1}{2})(\alpha_2+1)$, d(-$p_1^{\frac{\alpha_1}{2}+1}p_2^0$)=$2(\frac{\alpha_1}{2})(\alpha_2+1)$, ⋯, d($p_1^{\alpha_1}p_2^{\alpha_2}$)=2, d(-$p_1^{\alpha_1}p_2^{\alpha_2}$)=2.

Similarly we can prove for the cases (ii), (iii), (iv).

Theorem: 3.3 The clique number of an Integral Bi-fact graph G is
(i) $\omega(G)$= $2(\frac{\alpha_1}{2}+1)\ (\frac{\alpha_2}{2}+1)$, when α_1 and α_2 are even.
(ii) $\omega(G)$= $2\left[\left(\left\lfloor\frac{\alpha_1}{2}\right\rfloor+1\right)\left(\left\lfloor\frac{\alpha_2}{2}\right\rfloor+1\right)\right]+2$, when α_1 and α_2 are odd.
(iii) $\omega(G)$= $2\left[(\left\lfloor\frac{\alpha_1}{2}\right\rfloor+1)\ (\frac{\alpha_2}{2}+1)\right]+1$, when α_1 is odd and α_2 is even.
(iv) $\omega(G)$= $2\left[(\frac{\alpha_1}{2}+1)\ (\left\lfloor\frac{\alpha_2}{2}\right\rfloor+1)\right]+1$, when α_1 is even and α_2 is odd.

Proof: Case (i): When α_1 and α_2 are even,

Let G= (V, E) be an Integral Bi-factograph and V= {$p_1^0p_2^0$, -$p_1^0p_2^0$, ⋯ , $p_1^{\alpha_1}p_2^{\alpha_1}$ -$p_1^{\alpha_1}p_2^{\alpha_1}$} be the vertex set of G. We consider the set S= {$p_1^xp_2^y$,-$p_1^xp_2^y$ / $0\leq x\leq\frac{\alpha_1}{2}, 0\leq y\leq\frac{\alpha_2}{2}$} which is a proper subset of V.We seek to prove that the subgraph of G induced by S is the maximal clique of G. In G, we observe that for i≠k,j≠l,the vertex $p_1^ip_2^j$ (or - $p_1^ip_2^j$) is adjacent with $p_1^kp_2^l$ (or -$p_1^kp_2^l$) in G if and only if i+k$\leq \alpha_1$ and j+l $\leq \alpha_2$.We have to prove that every pair of distinct vertices in S are adjacent.Take two arbitray vertices $p_1^ap_2^b$ and $p_1^cp_2^d$ in S, since the maximum range of a,c and b,d are less than are equal to $\frac{\alpha_1}{2}$ and $\frac{\alpha_1}{2}$ respectively. Therefore, by the integral factograph condition $p_1^ap_2^b$ and $p_1^cp_2^d$ are adjacent.Thus $\langle S\rangle$ is a clique of G.It remains to show that $\langle S\rangle$ is the maximal clique of G. Take an arbitrary vertex v ∈ V\S, v cannot be adjacent to $p_1^{\frac{\alpha_1}{2}}p_2^{\frac{\alpha_2}{2}}$ in S, which implies $\langle S\rangle+\{v\}$ cannot be a clique of G. Therefore $\langle S\rangle$ is the maximal clique of G and $|S|$=$2(\frac{\alpha_1}{2}+1)(\frac{\alpha_2}{2}+1)$, which implies that $\omega(G)$ = $2(\frac{\alpha_1}{2}+1)(\frac{\alpha_2}{2}+1)$.

Similarly we can prove the cases (ii), (iii), (iv).

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Example: 3.4 Consider an Integral Bi-factograph G with z= $p_1{}^1p_2{}^1$ which is depicted in figure.2.

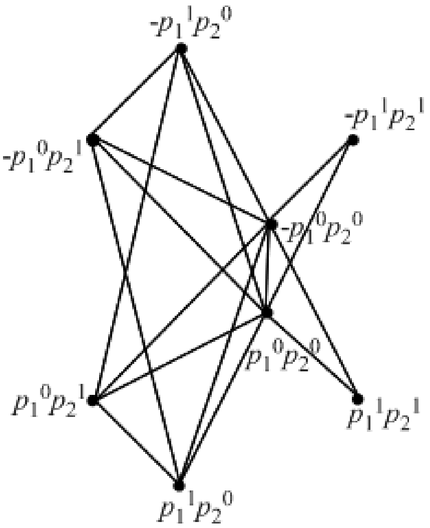

Figure: 2

We observe that, the order of G is 8, the degree sequence is s:7,7,4,4,4,4,2,2 and ω(G) is 6.

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