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BASIC PROPERTIES OF TOTAL BLOCK-EDGE TRANSFORMATION GRAPHS G^{abc}

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ABSTRACT

In this paper, we investigate some basic properties such as connectedness, graph equations and diameters of total block-edge transformation graphs.

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Keywords: *line graph, block graph, jump graph, qlick graph, total block-edge transformation graphs* G^{abc}.

1. INTRODUCTION

Throughout the paper we only consider simple graphs without isolated vertices. We refer to [8] for unexplained terminology and notation. A *block* of a graph is connected nontrivial graph having no cutvertices. Let $G = (V, E)$ be a graph with block set $U(G)$={B_i; B_i is a block of G}. If a block $B \in U(G)$ with the edge set $\{e_1, e_2, \ldots, e_r; r \geq 1\}$, then we say that an edge e_i and a block B are incident with each other, where $1 \leq i \leq r$. The *line graph* $L(G)$ of a graph G is the graph with vertex set as the edge set of G and two vertices of $L(G)$ are adjacent whenever the corresponding edges in G have a vertex in common. The *jump graph* $J(G)$ of a graph G is the graph whose the vertex set is the edge set of G and two vertices of $J(G)$ are adjacent if and only if the corresponding edges in G are not adjacent in G. The *block graph* $B(G)$ of a graph G is the graph whose vertices are the blocks of G and in which two vertices are adjacent whenever the corresponding blocks have a cutvertex in common.

The edges and blocks of G are called *members of* G. The *qlick graph* $Q(G)$ of a graph G is the graph whose set of vertices is the union of the set of edges and blocks of G and in which two vertices are adjacent if and only if the corresponding member of G are adjacent or incident. This concept is introduced by V. R. Kulli [10] and was studied in [4, 5, 12].

In [16], Wu and Meng generalized the concept of total graph and introduced the total transformation graphs and defined as follows:

Definition: Let $G = (V, E)$ be a graph, and x, y, z be three variables taking values $+$ or $-$. The *transformation graph* G^{xyz} is the graph having $V(G) \cup E(G)$ as the vertex set, and for α, $\beta \in V(G) \cup E(G)$, α and β are adjacent in G^{xyz} if and only if one of the following holds:

(i) $\alpha, \beta \in V(G)$. α and β are adjacent in G if $x = +$; α and β are not adjacent in G if $x = -$.
(ii) $\alpha, \beta \in E(G)$. α and β are adjacent in G if $y = +$; α and β are not adjacent in G if $y = -$.
(iii) $\alpha \in V(G)$, $\beta \in E(G)$. α and β are incident in G if $z = +$; α and β are not incident in G if $z = -$.

In [2], B. Basavanagoud et. al generalized the concept of total block graph and introduced the block-transformation graphs and defined as follows:

Definition: Let $G = (V, E)$ be a graph with block set $U(G)$, and let α, β, γ be three variables taking values 0 or 1. The *block-transformation graph* $G^{\alpha\beta\gamma}$ is the graph having $V(G) \cup U(G)$ as the vertex set. For any two vertices x and $y \in V(G) \cup U(G)$ we define α, β, γ as follows:

(i) Suppose x, y are in $V(G)$. α=1 if x and y are adjacent in G. α=0 if x and y are not adjacent in G.
(ii) Suppose x, y are in $U(G)$. β=1 if x and y are adjacent in G. β=0 if x and y are not adjacent in G.
(iii) $x \in V(G)$ and $y \in U(G)$. γ=1 if x and y are incident with each other in G. γ=0 if x and y are not incident with each other in G.

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Inspired by the definition of total transformation graphs [16] and block-transformation graphs [2], Basavanagoud [1] generalized the concept of qlick graph and obtained the four pairs of transformation graphs namely total block-edge transformation graphs.

Definition: Let $G = (V, E)$ be a graph with a block set $U(G)$ and $a,\ b,\ c$ be three variables taking values $+$ or $-$. The *total block-edge transformation graph* G^{abc} is a graph whose vertex set is $E(G) \cup U(G)$, and two vertices x and y of G^{abc} are joined by an edge if and only if one of the following holds:

(i) $x, y \in E(G)$. x and y are adjacent in G if $a = +$; x and y are not adjacent in G if $a = -$.

(ii) $x, y \in U(G)$. x and y are adjacent in G if $b = +$; x and y are not adjacent in G if $b = -$.

(iii) $x \in E(G),\ y \in U(G)$. x and y are incident with each other in G if $c = +$; x and y are not incident with each other in G if $c = -$.

Thus, we obtain eight kinds of total block-edge transformation graphs, in which G^{+++} is the qlick graph $Q(G)$ of G and G^{---} is its complement. Also G^{--+}, G^{-+-} and G^{-++} are the complements of G^{++-}, G^{+-+} and G^{+--} respectively. Some other graph valued functions were studied in [2, 3, 6, 7, 9, 11, 13, 14, 16]. The vertex $e_i^{'}$ ($B_i^{'}$) of G^{abc} corresponding to edge e_i (block B_i) of G and is referred as edge (block)-vertex.

The following will be useful in the proof of our results.

Remark 1.1: *$L(G)$ is an induced subgraph of G^{+bc}.*

Remark 1.2: *$J(G)$ is an induced subgraph of G^{-bc}.*

Remark 1.3: *$B(G)$ is an induced subgraph of G^{a+c}.*

Remark 1.4: *$\overline{B(G)}$ is an induced subgraph of G^{a-c}.*

Remark 1.5: *[7] If a disconnected graph G has no isolated vertices, then $J(G)$ is connected.*

Theorem 1.1: *[8] If G is connected, then $L(G)$ is connected.*

Theorem 1.2: *[8] If G is connected, then $B(G)$ is connected.*

Theorem 1.3: *[17] Let G be a graph of size $q \geq 1$. Then $J(G)$ is connected if and only if G contains no edge that is adjacent to every other edge of G unless $G = K_4$ or C_4.*

In this paper, we investigate some basic properties of these eight kinds of total block-edge transformation graphs.

2. CONNECTEDNESS OF G^{abc}

The first theorem is obvious from the notion of G^{abc}.

Theorem 2.1: *For a given graph G, G^{+++} is connected if and only if G is connected.*

Theorem 2.2: *For a given graph G, G^{++-} is connected if and only if $G \neq B_i \cup B_j$ is not a block, where B_i and B_j are blocks.*

Proof: Suppose $G \neq B_i \cup B_j$ is not a block. Then we consider the following cases:

Case-1. Suppose G is connected. Then it has at least two blocks. Hence by Theorem 1.2 and Remark 1.3, $B(G)$ is a connected induced subgraph of G^{++-}, and also each edge-vertex $e_i^{'}$ in G^{++-} is adjacent to at least one block-vertex $B_x^{'}$, where B_x is not incident with e_i in G. Therefore for every pair of vertices in G^{++-} are connected. Thus G^{++-} is connected.

Case-2. Suppose G is disconnected. Then it has at least three blocks. If e_i and e_j are adjacent edges in G, then $e_i^{'}$ and $e_j^{'}$ are adjacent in G^{++-}. If e_i and e_j are not adjacent edges in G, then $e_i^{'}$ and $e_j^{'}$ are connected through the block-vertex $B_x^{'}$, where B_x is not incident with e_i and e_j in G. If B_x and B_y are adjacent blocks in G, then $B_x^{'}$ and $B_y^{'}$ are adjacent in G^{++-}. If B_x and B_y are not adjacent blocks in G, then $B_x^{'}$ and $B_y^{'}$ are connected through the edge-vertex $e_i^{'}$, where e_i is not incident with B_x and B_y in G. If e is not incident with B in G, then $e^{'}$ and $B^{'}$ are adjacent in G^{++-}. If e is incident with B in G, then there exists not incident edge e_1 and block B_1 are not incident with B and e respectively such that $e^{'}$ and $B^{'}$ are connected in G^{++-}. Otherwise, there is a block B_1 is not incident with e, and is adjacent to B, such that $e^{'}$ and $B^{'}$ are connected in G^{++-}. Since in such a case, there is a path between any two vertices of G^{++-}. Hence G^{++-} is connected.

Conversely, suppose G^{++-} is connected. If G is a block, then $G^{++-} = L(G) \cup K_1$ is disconnected, a contradiction. If $G = B_i \cup B_j$, then G^{++-}=$(L(B_i) + K_1) \cup (L(B_j) + K_1)$ is a disconnected graph, a contradiction.

Theorem 2.3: G^{+-+} *is connected for any graph* G.

Proof: If G is connected, then by Remark 1.1 and Theorem 1.1, $L(G)$ is a connected induced subgraph of G^{+-+}, and each block-vertex $B_x^{'}$ in G^{+-+} is adjacent to at least one edge-vertex $e_i^{'}$, where e_i is incident with B_x in G. Thus G^{+-+} is connected.

If G is disconnected, then $\overline{B(G)}$ is a connected induced subgraph of G^{+-+}, and each edge-vertex $e_i^{'}$ in G^{+-+} is adjacent to exactly one block-vertex $B_x^{'}$, where B_x is incident with e_i in G. Thus G^{+-+} is connected.

Theorem 2.4: *For a given graph* G, G^{+--} *is connected if and only if* G *is not a block.*

Proof: If G is a connected graph with at least two blocks, then by Remark 1.1 and Theorem 1.1, $L(G)$ is a connected induced subgraph of G^{+--}, and in G^{+--}, each block-vertex $B_x^{'}$ is adjacent to at least one edge-vertex $e_i^{'}$, where e_i is not incident with B_x in G. Thus G^{+--} is connected.

If G is disconnected, then $\overline{B(G)}$ is a connected induced subgraph of G^{+--}, and in G^{+--}, each edge-vertex $e_i^{'}$ is adjacent to at least one block-vertex $B_x^{'}$, where B_x not is incident with e_i in G. Thus G^{+--} is connected.

Conversely, if G is a block, then $G^{+--} = L(G) \cup K_1$ is disconnected, a contradiction.

Theorem 2.5: G^{-++} *is connected for any graph* G.

Proof: If G is connected, then by Remark 1.3 and Theorem 1.2, $B(G)$ is a connected induced subgraph of G^{-++}, and each edge-vertex $e_i^{'}$ in G^{-++} is adjacent to exactly one block-vertex $B_x^{'}$, where B_x is incident with e_i in G. Thus G^{-++} is connected.

If G is disconnected, then by Remarks 1.2 and 1.5, $J(G)$ is a connected induced subgraph of G^{-++}, and each block-vertex $B_x^{'}$ in G^{-++} is adjacent to at least one edge-vertex $e_i^{'}$, where e_i is incident with B_x in G. Thus G^{-++} is connected.

Theorem 2.6: *For a given graph* G, G^{-+-} *is connected if and only if* G *is not a block.*

Proof: If G is a connected graph with at least two blocks, then by Remark 1.3 and Theorem 1.2, $B(G)$ is a connected induced subgraph of G^{-+-}, and in G^{-+-}, each edge-vertex $e_i^{'}$ is adjacent to at least one block-vertex $B_x^{'}$, where B_x is not incident with e_i in G. Thus G^{-+-} is connected.

If G is disconnected, then by Remarks 1.2 and 1.5, $J(G)$ is a connected induced subgraph of G^{-+-}, and in G^{-+-}, each block-vertex $B_x^{'}$ is adjacent to at least one edge-vertex $e_i^{'}$, where e_i is not incident with B_x in G. Thus G^{-+-} is connected.

Conversely, if G is a block, then $G^{-+-} = J(G) \cup K_1$ is disconnected, a contradiction.

Theorem 2.7: *For a given graph* G, G^{--+} *is connected if and only if* G *contains no block* K_2 *that is adjacent to every other edge of* G.

Proof: Suppose a graph G contains no block K_2 that is adjacent to every other edge of G. If G is a block, then $G^{--+} = J(G) + K_1$ is connected. If G has more than one block, then we consider the following two cases:

Case-1. Suppose G contains no edge that is adjacent to every other edge of G. Then by Remark 1.2 and Theorem 1.3, $J(G)$ is a connected induced subgraph of G^{--+}, and each block-vertex $B_x^{'}$ is adjacent to at least one edge-vertex $e_i^{'}$ in G^{--+}, where e_i is incident with B_x. Thus G^{--+} is connected.

Case-2. Suppose G contains an edge e that is adjacent to every other edge of G. Then e is incident with a block B of size more than 2 and $e^{'}$ is isolated vertex in $J(G)$ such that $e^{'}, B^{'}, e^{'}$ is a path in G^{--+}, where e_1 is incident with B. Therefore every pair of edge-vertices are connected in G^{--+} and each block-vertices $B_x^{'}$ is adjacent to at least one edge-vertex $e_i^{'}$ in G^{--+}, where e_i is incident with B_x in G. Thus G^{--+} is connected.

Conversely, suppose G^{--+} is connected. Assume G contains a block K_2, say e, that is adjacent to every other edge of G. Then it is easy to see that G^{--+}=$(G - e)^{--+} \cup K_2$ is disconnected, a contradiction.

Theorem 2.8: *For a given graph G, G^{---} is connected if and only if $G \neq P_3$ is not a block.*

Proof: Suppose $G \neq P_3$ is not a block. We consider the following two cases:

Case-1. Suppose G contains no edge that is adjacent to every other edge of G. Then by Remark 1.2 and Theorem 1.3, $J(G)$ is a connected induced subgraph of G^{---}, and each block-vertex $B_x^{'}$ is adjacent to at least one edge-vertex $e_i^{'}$ in G^{---}, where e_i is not incident with B_x in G. Thus G^{---} is connected.

Case-2. Suppose G contains an edge e that is adjacent to all other edge of G. Then by definition of G^{---}, each edge-vertex $e_i^{'}$ is adjacent to edge-vertex $e_k^{'}$ and to at least one block-vertex $B_j^{'}$, where B_j is not incident with e_i, and e_k is not adjacent to e_i in G. And also each block-vertex $B_x^{'}$ is adjacent to block-vertex B_y and to at least one edge-vertex $e_i^{'}$, where e_i is not incident with B_x, and B_y not adjacent to B_x in G. Hence there is a path between any two vertices of G^{---}. Therefore G^{---} is connected.

Conversely, suppose G^{---} is connected. If G is a block, then $G^{---} = J(G) \cup K_1$ is disconnected, a contradiction. If $G = P_3$, then $G^{---} = 2K_2$ is disconnected, a contradiction.

3. GRAPH EQUATIONS AND ITERATIONS OF G^{abc}

For a given graph operator Φ, which graph is fixed under Φ ?, that is $\Phi(G) = G$. It is well known in [15] that for a given graph G, the interchange graph $G^{'} = G$ if and only if G is a 2-regular graph.

For a given total block-edge transformation graph G^{abc}, we define the iteration of G^{abc} as follows:
(1). $G^{(abc)^1} = G^{abc}$ (2). $G^{(abc)^n} = [G^{(abc)^{n-1}}]^{abc}$ for $n \geq 2$.

Theorem 3.1: *Let G be a connected graph. The graphs G and G^{ab+} are isomorphic if and only if $G = K_2$.*

Proof: Suppose $G^{ab+} = G$. Assume G is a connected graph with $p \geq 3$ vertices. We consider the following two cases:

Case-1. Suppose G is not a tree with p vertices. Then G has at least p edges and at least one block. Thus G^{ab+} has at least $p+1$ vertices. Hence $G^{ab+} \neq G$, a contradiction.

Case-2. Suppose G is a tree with p vertices. Then it has $p-1$ edges and $p-1$ blocks. Thus G^{ab+} has $2p-2$ vertices. Hence $|V(G)| < |V(G^{ab+})|$. Therefore $G^{ab+} \neq G$, a contradiction.

Conversely, suppose $G = K_2$. Then it is easy to see that $G^{ab+} = K_2 = G$.

Corollary 3.2: *Let G be a connected graph. The graphs G and $G^{(ab+)^n}$ are isomorphic if and only if $G = K_2$.*

Theorem 3.3: *The graphs G and G^{++-} are isomorphic if and only if $G = 2K_2$.*

Proof: Suppose $G^{++-} = G$. Assume $G \neq 2K_2$. We consider the following two cases:

Case-1. Suppose G is a block. Then clearly $G^{++-} = L(G) \cup K_1$ is disconnected. Thus $G^{++-} \neq G$, a contradiction.

Case-2. Suppose G has at least two blocks with q edges. Then G^{++-} has at least $2q-1$ edges. Hence the number of edges in G is less than that in G^{++-}. Thus $G^{++-} \neq G$, a contradiction.

Conversely, suppose $G = 2K_2$. Then it is easy to see that $G^{++-} = 2K_2 = G$.

Corollary 3.4: *The graphs G and $G^{(++-)^n}$ are isomorphic if and only if $G = 2K_2$.*

Theorem 3.5: *For any graph G, $G^{ab-} \neq G$, where $G^{ab-} \neq G^{++-}$.*

Proof: If $G = K_2$, then $G^{ab-} = 2K_1 \neq G$. We consider the following two cases:

Case-1. Suppose $G \neq K_2$ is a connected graph. By the definitions of G^{ab+} and G^{ab-}, we have $|V(G^{ab+})| = |V(G^{ab-})|$. By proof of the Theorem 3.1, we have $|V(G)| \neq |V(G^{ab+})|$. Hence $|V(G)| \neq |V(G^{ab-})|$. Therefore $G^{ab-} \neq G$.

Case-2. Suppose G is a disconnected graph with q edges. Then G^{ab-} has at least $q+1$ edges. Hence $|E(G)| \neq |E(G^{ab-})|$. Therefore $G^{ab-} \neq G$. From all the above two cases, we have $G^{ab-} \neq G$.

Corollary 3.6: *For any graph G, $G^{(ab-)^n} \neq G$, where $G^{(ab-)^n} \neq G^{(++-)^n}$.*

4 DIAMETERS OF G^{abc}

The distance between two vertices v_i and v_j, denoted by $d(v_i, v_j)$, is the length of the shortest path between the vertices v_i and v_j in G. The shortest $v_i - v_j$ path is often called *geodesic*. The *diameter* of a connected graph G, denoted by $diam(G)$, is the length of any longest geodesic.

In this section, we consider the diameters of G^{abc}.

Theorem 4.1: *If G is a connected graph, then $diam(G^{+++}) \leq diam(G) + 1$.*

Proof: Let G be a connected graph. We consider the following three cases:

Case-1. Assume G is a tree. Then it is easy to see that $diam(G^{+++}) = diam(G)$.

Case-2. Assume G is a cycle C_n for $n \geq 3$. Then $G^{+++} = W_{n+1}$ and $diam(G^{+++}) < diam(G) + 1$.

Case-3. Assume G contains a cycle C_n for $n \geq 3$. Corresponding to cycle C_n, W_{n+1} appears as subgraph in G^{+++}. Therefore $diam(G^{+++}) \leq diam(G) + 1$.

From all the above three cases, we have $diam(G^{+++}) \leq diam(G) + 1$.

Theorem 4.2: *If G is neither a block nor a union of two blocks, then $diam(G^{++-}) \leq 3$.*

Proof: Let e_1', e_2' be the two edge-vertices of G^{++-}. If e_1 and e_2 are adjacent edges in G, then e_1' and e_2' are adjacent in G^{++-}. If e_1 and e_2 are not adjacent edges in G, then we have following cases:

Case-1. If e_1 and e_2 are incident with same block, then there exists a block B is incident with neither e_1 nor e_2 such that e_1', B', e_2' is a path of length 2 in G^{++-}.

Case-2. If e_1 and e_2 are incident with different blocks B_1 and B_2 in G respectively, then we have the following subcases:

Subcase-2.1. If B_1 and B_2 are adjacent in G, then e_1', B_2', B_1', e_2' is a path of length 3 in G^{++-}.

Subcase-2.2. If B_1 and B_2 are not adjacent in G, then there exists a block B_3 is incident with neither e_1 nor e_2 such that e_1', B_3', e_2' is a path of length 2 in G^{++-}.

Let B_1', B_2' be the two block-vertices of G^{++-}. If B_1 and B_2 are not adjacent blocks in G, then B_1' and B_2' are adjacent in G^{++-}. If B_1 and B_2 are adjacent blocks in G, then there exists an edge e is incident with neither B_1 nor B_2 such that B_1', e', B_2' is a path in G^{++-} of length 2.

Let e' and B' be the edge-vertex and block-vertex of G^{++-} respectively. If e is not incident with B in G, then e' and B' are adjacent in G^{++-}. If e is incident with B in G, then there exists not incident edge e_1 and block B_1 are not incident with B and e respectively such that e', B_1', e_1', B' is a path in G^{++-} of length 3. Otherwise, there is a block B_1 is not incident with e, and is adjacent to B, such that e', B_1', B' is a path in G^{++-} of length 2.

Theorem 4.3: *For a given graph G, $diam(G^{+-+}) \leq 5$.*

Proof: Let e_1', e_2' be the two edge-vertices of G^{+-+}. If e_1 and e_2 are adjacent edges in G, then e_1' and e_2' are adjacent in G^{+-+}. If e_1 and e_2 are not adjacent edges in G, then we have the following cases:

Case-1. If e_1 and e_2 are incident with same block B, then e_1', B', e_2' is a path of length 2 in G^{+-+}.

Case-2. If e_1 and e_2 are incident with different blocks B_1 and B_2 in G respectively, then we have the following subcases:

Subcase-2.1. If B_1 and B_2 are adjacent, then there exists two adjacent edges e_3 in B_1 and e_4 in B_2, then $e_1', B_1', e_3', e_4', B_2', e_2'$ is a path of length at most 5 in G^{+-+}.

Subcase-2.2. If B_1 and B_2 are not adjacent, then e_1', B_1', B_2', e_2' is a path of length 3 in G^{+-+}.

Let $B_1^{'}$, $B_2^{'}$ be the two block-vertices of G^{+-+}. If B_1 and B_2 are not adjacent blocks in G, then $B_1^{'}$ and $B_2^{'}$ are adjacent in G^{+-+}. If B_1 and B_2 are adjacent blocks in G, then there exists two adjacent edges e_1 in B_1 and e_2 in B_2 such that $B_1^{'}, e_1^{'}, e_2^{'}, B_2^{'}$ is a path in G^{+-+} of length 3.

Let $e_1^{'}$ and $B_2^{'}$ be the edge-vertex and block-vertex of G^{+-+} respectively. If e_1 is incident with B_2 in G, then $e_1^{'}$ and $B_2^{'}$ are adjacent in G^{+-+}. If e_1 in B_1, is not incident with B_2 in G, then we have the following cases:

Case-1. If B_1 and B_2 are adjacent in G, then there exists two adjacent edges e in B_1 and e_2 in B_2 such that $e_1^{'}, B_1^{'}, e^{'}, e_2^{'}, B_2^{'}$ is a path in G^{+-+} of length at most 4.

Case-2. If B_1 and B_2 are not adjacent in G, then $e_1^{'}, B_1^{'}, B_2^{'}$ is a path in G^{+-+} of length 2.

Theorem 4.4: *If G is neither a block nor a connected graph with two blocks, then $diam(G^{+--}) \leq 3$.*

Proof: Let $e_1^{'}$, $e_2^{'}$ be the two edge-vertices of G^{+--}. If e_1 and e_2 are adjacent edges in G, then $e_1^{'}$ and $e_2^{'}$ are adjacent in G^{+--}. If e_1 and e_2 are not adjacent edges in G, then we have following cases:

Case-1. If e_1 and e_2 are incident with same block, then there exists a block B is incident with neither e_1 nor e_2 such that $e_1^{'}, B^{'}, e_2^{'}$ is a path of length 2 in G^{+--}.

Case-2. If e_1 and e_2 are incident with different blocks B_1 and B_2 in G respectively, then we have the following subcases:

Subcase-2.1. If B_1 and B_2 are not adjacent in G, then $e_1^{'}, B_2^{'}, B_1^{'}, e_2^{'}$ is a path of length 3 in G^{+--}.

Subcase-2.2. If B_1 and B_2 are adjacent in G, then there exists a block B is incident with neither e_1 nor e_2 such that $e_1^{'}, B^{'}, e_2^{'}$ is a path of length 2 in G^{+--}.

Let $B_1^{'}$, $B_2^{'}$ be the two block-vertices of G^{+--}. If B_1 and B_2 are not adjacent blocks in G, then $B_1^{'}$ and $B_2^{'}$ are adjacent in G^{+--}. If B_1 and B_2 are adjacent blocks in G, then there exists an edge e is incident with neither B_1 nor B_2 such that $B_1^{'}, e^{'}, B_2^{'}$ is a path in G^{+--} of length 2.

Let $e^{'}$ and $B^{'}$ be the edge-vertex and block-vertex of G^{+--} respectively. If e is not incident with B in G, then $e^{'}$ and $B^{'}$ are adjacent in G^{+--}. If e is incident with B in G, then we have the following cases:

Case-1. If there is a block B_1 is not adjacent to B, and is not incident with e, then $e^{'}, e_1^{'}, B^{'}$ is a path in G^{+--} of length 2.

Case-2. If there is an edge e_1 is adjacent to e, and is not incident with B, then $e^{'}, B_1^{'}, B^{'}$ is a path in G^{+--} of length 2.

Lemma 4.5: *If a connected graph G has two blocks, then $diam(G^{+--}) \leq 5$.*

Proof: Suppose G is a connected graph with two blocks B_1 and B_2 of size q_1 and q_2 respectively. Then K_{1,q_1} and K_{1,q_2} are two edge-disjoint subgraphs of G^{+--}. And there exists at least one edge $e^{'}$ in G^{+--} is incident with exactly one pendant vertex of K_{1,q_1} and K_{1,q_2}. It is easy that see that the diameter of star is at most 2.

Hence $diam(G^{+--}) = diam(K_{1,q_1}) + diam(K_{1,q_2}) + 1 \leq 2 + 2 + 1 = 5$.

Theorem 4.6: *For a given graph G, $diam(G^{-++}) \leq 3$.*

Proof: Let $e_1^{'}$, $e_2^{'}$ be the two edge-vertices of G^{-++}. If e_1 and e_2 are not adjacent edges in G, then $e_1^{'}$ and $e_2^{'}$ are adjacent in G^{-++}. If e_1 and e_2 are adjacent edges in G, then we have the following cases:

Case-1. If there is an edge e is not adjacent to both e_1 and e_2 in G, then $e_1^{'}, e^{'}, e_2^{'}$ is a path in G^{-++} of length 2.

Case-2. If e_1 and e_2 are incident with same block B, then $e_1^{'}, B^{'}, e_2^{'}$ is a path of length 2 in G^{-++}.

Case-3. If e_1 and e_2 are incident with different blocks B_1 and B_2 respectively, then $e_1^{'}, B_1^{'}, B_2^{'}, e_2^{'}$ is a path of length 3 in G^{-++}.

Let $B_1^{'}$, $B_2^{'}$ be the two block-vertices of G^{-++}. If B_1 and B_2 are adjacent blocks in G, then $B_1^{'}$ and $B_2^{'}$ are adjacent in G^{-++}. If B_1 and B_2 are not adjacent blocks in G, then we have two cases:

Case-1. If there is a block B is adjacent to both B_1 and B_2 in G, then $B_1^{'}, B^{'}, B_2^{'}$ is a path in G^{-++} of length 2.

Case-2. If there are two not adjacent edges e_1 in B_1 and e_2 in B_2, then $B_1^{'}, e_1^{'}, e_2^{'}, B_2^{'}$ is a path in G^{-++} of length 3.

Let $e^{'}$ and $B^{'}$ be the edge-vertex and block-vertex of G^{-++} respectively. If e is incident with B in G, then $e^{'}$ and $B^{'}$ are adjacent in G^{-++}. If e is not incident with B in G, then we consider the following two cases:

Case-1. If there is a block B_1 is incident with e, and is adjacent to B, then $e^{'}, B_1^{'}, B^{'}$ is a path in G^{-++} of length 2.

Case-2. If there is an edge e_1 is incident with B, and is not adjacent to e, then $e^{'}, e_1{'}, b^{'}$ is a path in G^{-++} of length 2.

Theorem 4.7: *If a graph G is not a block, then $diam(G^{-+-}) \leq 3$.*

Proof: Let $e_1^{'}$, $e_2^{'}$ be the two edge-vertices of G^{-+-}. If e_1 and e_2 are not adjacent edges in G, then $e_1^{'}$ and $e_2^{'}$ are adjacent in G^{-+-}. If e_1 and e_2 are adjacent edges in G, then we have one of the following case:

Case-1. If e_1 and e_2 are incident with same block, then there exists a block B is incident with neither e_1 nor e_2 such that $e_1^{'}, B^{'}, e_2^{'}$ is a path of length 2 in G^{-+-}.

Case-2. If e_1 and e_2 are incident with different blocks B_1 and B_2 respectively in G, then $e_1^{'}, B_2^{'}, B_1^{'}, e_2^{'}$ is a path in G^{-+-} of length 3.

Let $B_1^{'}$, $B_2^{'}$ be two block-vertices of G^{-+-}. If B_1 and B_2 are adjacent in G, then $B_1^{'}$ and $B_2^{'}$ are adjacent in G^{-+-}. If B_1 and B_2 are not adjacent in G, then there exists two not adjacent edges e_1 and e_2 are incident with B_1 and B_2 respectively such that $B_1^{'}, e_2^{'}, e_1^{'}, B_2^{'}$ is a path of length 3 in G^{-+-}. Otherwise, there is an edge e is incident with neither B_1 nor B_2, then $B_1^{'}, e^{'}, B_2^{'}$ is a path of length 2 in G^{-+-}.

Let $e^{'}$ and $B^{'}$ be the edge-vertex and block-vertex of G^{-+-} respectively. If e is not incident with B in G, then $e^{'}$ and $B^{'}$ are adjacent in G^{-+-}. If e is incident with B in G, then we have the following cases:

Case-1. If there is an edge e_1 is incident with B, and is not adjacent to edge e in G, then $e^{'}, e_1^{'}, B^{'}$ is a path in G^{-+-} of length 2.

Case-2. If there is a block B_1 which is incident with B, and is not adjacent to an edge e, then $e^{'}, B_2^{'}, B^{'}$ is a path of length 2 in G^{-+-}.

Theorem 4.8: *If a graph G contains no block K_2 that is adjacent to other edge, then $diam(G^{--+}) \leq 4$.*

Proof: Let $e_1^{'}$, $e_2^{'}$ be the two edge-vertices of G^{--+}. If e_1 and e_2 are not adjacent edges in G, then $e_1^{'}$ and $e_2^{'}$ are adjacent in G^{--+}. If e_1 and e_2 are adjacent edges in G, then we have one of the following case:

Case-1. If e_1 and e_2 are incident with same block B, then $e_1^{'}, B^{'}, e_2^{'}$ is a path of length 2 in G^{--+}.

Case-2. If e_1 and e_2 are incident with different blocks B_1 and B_2 respectively, then we have the following subcases:

Subcase-2.1. If there is an edge e which is adjacent to neither e_1 nor e_2 in G, then $e_1^{'}, e^{'}, e_2^{'}$ is a path in G^{--+} of length 2.

Subcase-2.2. If there is an edge e which is incident with B_2, and is not adjacent to e_1, then $e_1^{'}, e^{'}, B_2^{'}, e_2^{'}$ is a path in G^{--+} of length 3.

Subcase-2.3. If there are two not adjacent edges e_3 and e_4, where e_3 and e_4 are not adjacent to e_1 and e_2 respectively, then $e_1^{'}, e_3^{'}, e_4^{'}, e_2^{'}$ is a path in G^{--+} of length 3.

Let $B_1^{'}$, $B_2^{'}$ be the two block-vertices of G^{--+}. If B_1 and B_2 are not adjacent blocks in G, then $B_1^{'}$ and $B_2^{'}$ are adjacent in G^{-+-}. If B_1 and B_2 are adjacent blocks in G and are incident with e_1 and e_2 respectively,then we have the following cases:
Case-1. If e_1 and e_2 are not adjacent in G, then $B_1^{'}, e_1^{'}, e_2^{'}, B_2^{'}$ is a path of length 3 in G^{--+}. Otherwise, there is an edge e is not adjacent to e_1 and e_2 such that $B_1^{'}, e_1^{'}, e^{'}, e_2^{'}, B_2^{'}$ is a path of length 4 in G^{--+} .

Case-2. If there is a block B is adjacent to neither B_1 nor B_2 in G, then $B_1^{'}, B^{'}, B_2^{'}$ is a path of length 2 in G^{--+}. Otherwise, there are two not adjacent blocks B_3 and B_4, are not adjacent to B_2 and B_1 respectively such that $B_1^{'}, B_4^{'}, B_3^{'}, B_2^{'}$ is a path in G^{--+} of length 3.

Let e_1' and B_2' be the edge-vertex and block-vertex of G^{--+} respectively. If e_1 is incident with B_2 in G, then e_1' and B_2' are adjacent in G^{--+}. If e_1 is not incident with B_2 in G, then we have the following cases:

Case-1. If there is an edge e_2 is incident with B_2, where e_2 is not adjacent to e_1 in G, then B_2', e_2', e_1' is a path in G^{--+} of length 2.

Case-2. If there are two not adjacent edges e_{2x} and e_{1x}, where e_{2x} and e_{1x} are incident with B_1 and B_2 respectively, and e_1 is adjacent to e_{2x} and e_{1x}, then $B_2', e_{2x}', e_{1x}', B_1, e_1'$ is a path in G^{--+} of length 4.

Case-3. If there is an edge e_3 is not adjacent to both e_1 and e_2, where e_1 in B_1 and e_2 in B_2, then B_2', e_2', e_3', e_1' is a path in G^{--+} of length 3.

Theorem 4.9: *If a graph $G \neq P_3$ is not a block, then $diam(G^{---}) \leq 4$.*

Proof: Let e_1', e_2' be the two edge-vertices of G^{---}. If e_1 and e_2 are not adjacent edges in G, then e_1' and e_2' are adjacent in G^{---}. If e_1 and e_2 are adjacent edges in G, then we have one of the following case:

Case-1. If e_1 and e_2 are incident with same block, then there exists a block B is incident with neither e_1 nor e_2 such that e_1', B', e_2' is a path of length 2 in G^{---}.

Case-2. If e_1 and e_2 are incident with different blocks B_1 and B_2 respectively in G, then we have the following subcases:

Subcase-2.1. If there is a block B which is incident with neither e_1 nor e_2 in G, then e_1', B', e_2' is a path in G^{---} of length 2.

Subcase-2.2. If there is an edge e is incident with block B_2, and is not adjacent to e_1, then e_2', B_1', e', e_1' is a path in G^{---} of length 3.

Subcase-2.3. If there is an edge e_3 which is adjacent to neither e_1 nor e_2, then e_1', e_3', e_2' is a path in G^{---} of length 2.

Let B_1', B_2' be two block-vertices of G^{---}. If B_1 and B_2 are not adjacent blocks in G, then B_1' and B_2' are adjacent in G^{---}. If B_1 and B_2 are adjacent blocks in G, then we have the following cases:

Case-1. If there is an edge e is incident with neither B_1 nor B_2, then B_1', e', B_2' is a path of length 2 in G^{---}.

Case-2. If there are two not adjacent edges e_1 and e_2 are incident with B_1 and B_2 respectively, then B_1', e_2', e_1', B_2' is a path of length 3 in G^{---}.

Let e' and B' be the edge-vertex and block-vertex of G^{---} respectively. If e is not incident with B in G, then e' and B' are adjacent in G^{---}. If e is incident with B in G, then we have the following cases:

Case-1. If there is an edge e_1 is not incident with B, and is not adjacent to edge e in G, then e', e_1', B' is a path in G^{---} of length 2.

Case-2. If there are not incident edge e_2 and block B_3, where e_2 is not incident with B, and B_3 is not incident to e, then e', B_3', e_2', B' is a path of length 3 in G^{---}.

Case-3. If there is an edge e_1 which is incident with B_1, and is not adjacent to an edge e_2, where e_2 is incident with B, then B', e_1', e_2', B_1', e' is a path of length 4 in G^{---}.

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