International Journal of Mathematical Archive-6(8), 2015, 7-11
Available online through www.ijma.info ISSN 2229 – 5046

WIENER INDEX AND WIENER POLYNOMIAL FOR SOME SPECIAL GRAPHS

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(Received On: 09-07-15; Revised & Accepted On: 13-08-15)

ABSTRACT

The Wiener Index is a graphical invariant that has found extensive application in all branches of science. It is a distance based topological index defined as the sum of the distance between all pairs of vertices in a graph. A generating function, which we call Wiener polynomial is also defined. In this paper, Wiener index and Wiener polynomial of Wheel graph, Friendship graph and Barbell graph are obtained.

Keywords: *Barbell graph, Friendship graph, Wheel graph, Wiener Index, Wiener Polynomial.*

Classification Number: *05C05, 05C12, 05C15, 05C75, 05C90.*

1. INTRODUCTION

All graphs considered in this paper are finite, undirected and simple graphs. We refer the reader to (Cha, Les; 1986) for terminology and notations. A **graph** G = (V,E) is a set of finite, non empty set of objects called **vertices** together with a set of unordered pair of distinct vertices of G called **edges**. The vertex set of G is denoted by V(G) and the edge set is denoted by E(G). If $e = \{u, v\}$ is an edge of G, then u and v are **adjacent** vertices, while u and e are **incident**, as are v and e. The **distance** $d_G(u,v)$, from a vertex u to a vertex v in a connected graph G is the length of the shortest $u-v$ path in G.

The **Wiener Index** is the first topological index to be used in Chemistry. It was introduced in 1947 by Harold Wiener. Wiener himself conceived 'W' for acyclic molecules and defined it in a different manner. The definition of the Wiener Index in terms of distances between vertices of a graph was first given by Hosoya. Wiener index has many applications in Chemistry and Communication Theory. Wiener showed that the Wiener Index is closely correlated with the boiling points of Alkane molecules. In his later work, he showed that it is also correlated with other quantities including the parameters of its critical point, the density, surface tension and viscosities of its liquid phase and surface area of the molecule.

Wiener Index is defined as the sum of the distances between all pairs of vertices of a graph (Wei, Li; 2014) and is denoted by $W(G)$. That is, $W(G) = \sum_{u,v \in V(G)} d(u,v).$

2. WHEEL GRAPH

A Wheel graph is a graph with n vertices, formed by connecting a single vertex to all vertices of (n-1) cycle (Buc, Fre; 1988).

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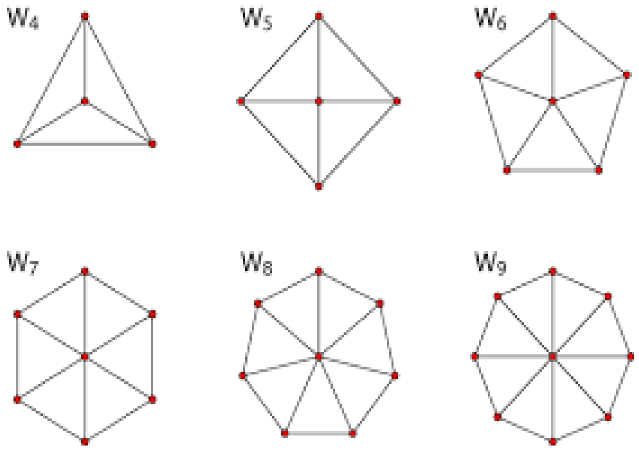


It is denoted as W_n. Wheel graphs are planar graphs and as such have a unique planar embedding. They are self dual and the planar dual of any wheel graph is an isometric graph. Any maximal planar graph, other than $K_4=W_4$, contain as a sub graph either W_5 or W_6. There is always a Hamiltonian cycle in the wheel graph and there are (n^2-3n+3) cycles in W_n [5].

Theorem 2.1: Let W_p be the wheel graph of order p where $p \geq 3$ and the wiener index of the wheel graph W_p is $W(W_p) = (p-2)(p-1)$.

Proof: The wiener index of W_p, for $p = 4,5,6$ etc can be computed as follows.

By virtue of definition of wiener index, we get
$W(W_4) = 6$
$W(W_5) = 12$
$W(W_6) = 20$
$W(W_7) = 30$ etc

Proceeding like this, we observe that $W(W_p) = (p-2)(p-1)$.

Remark 2.2: The wiener poynomial of W_P for $p = 4,5,6$ etc is given as follows.
$W(W_4,x) = 6x$
$W(W_5, x) = 2x^2 + 8x$
$W(W_6, x) = 5x^2+10x$
$W(W_7, x) = 9x^2+12x$

Proof: For W_P, $p = 4$, there are 6 pairs of vertices contribute distance one to the graph.
Therefore, $W(W_4, x) = 6x$

For W_P, $p = 5$, there are 8 pairs of vertices contribute distance one to the graph and 2 pairs of vertices contribute distance two to the graph.

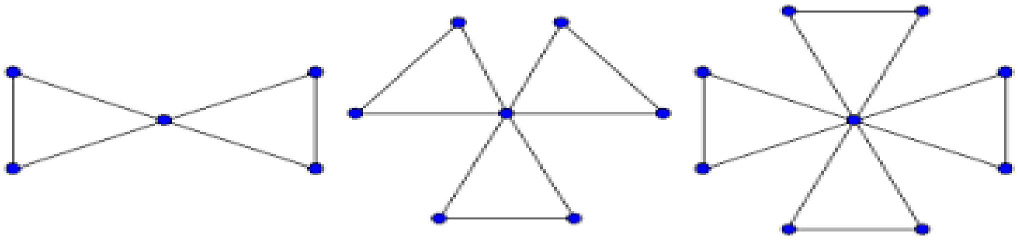
Therefore, $W(W_5, x) = 2x^2 + 8x.$

Similarly, $W(W_6, x)$ and $W(W_7, x)$ are found and are tabulated below.

	W_4	W_5	W_6	W_7
$W(G)$	6	12	20	30
$W(G,x)$	$6x$	$2x^2+8x$	$5x^2+10x$	$9x^2+12x$

3. FRIENDSHIP GRAPH

The friendship graph F_n can be constructed by joining n copies of triangle graph with a common vertex. It is denotes by F_n and it is a planar, undirected graph with (2n+1) vertices and 3n edges (Pau, Alf;1966).



In this graph every two vertices have exactly one neighbour in common. If a group of people has the property that every pair of people has exactly a friend in common, then there must be one person who is a friend to all others.[4]

Theorem 3.1: Let F_p be the friendship graph of order p where $p \geq 3$ and the wiener index of the friendship graph F_p is W(F_p) = $3p+2\left[\left(2p+1\right)C_2-3p\right]$.

Proof: We observe that the friendship graph F_2 has two copies of triangle graph K_3 with a common neighbour.

The friendship graph F_3 has three copies of triangle graph K_3 with a common neighbour and so on.

To find the distance between pairs of vertices in F_p, we classify the graph as follows.
(i) pairs of vertices in p copies of K_3.

The wiener index contributed by p copies of K_3 is given by 3p.

(ii) other pairs of vertices in the graph F_P.
The wiener index contributed by the remaining pairs of vertices in F_p is given by $2\left[\left(2p+1\right)C_2-3p\right]$.

Hence, the Wiener index of Friendship Graph F_P is $3p+2\left[\left(2p+1\right)C_2-3p\right]$.

Remark 3.2: The Wiener polynomial of F_P for p = 2,3,4 etc is given as follows.
$W(F_2,x) = 4x^2 + 6x$
$W(F_3,x) = 12x^2 + 9x$
$W(F_4,x) = 24x^2 + 12x$

Proof: For F_P , p=2, there are 6 pairs of vertices contributing distance one to the graph and 4 pairs of vertices contributing distance two to the graph.
Therefore, $W(F_2,x) = 4x^2 + 6x$

For F_P , p = 3, there are 9 pairs of vertices contributing distance one to the graph and 12 pairs of vertices contributing distance two to the graph.

Therefore, $W(F_3,x) = 12x^2 + 9x$

For F_P , p=4, there are 12 pairs of vertices contributing distance one to the graph and 24 pairs of vertices contributing distance two to the graph.

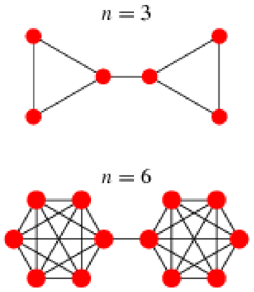
Therefore, $W(F_4,x) = 24x^2 + 12x$.

The Wiener Index and Wiener Polynomial of F_P for p = 2,3,4 are tabulated below.

	F_2	F_3	F_4
$W(G)$	14	33	60
$W(G,x)$	$4x^2+6x$	$12x^2+9x$	$24x^2+12x$

4. BARBELL GRAPH

A n-barbell graph is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge and it is denoted by $B(K_p, K_p)$[6].



Theorem 4.1: Let $B(K_p, K_p)$ be the Barbell graph of order p where $p \geq 3$ and the Wiener index of the barbell graph $B(K_p, K_p)$ is $W(B(K_p, K_p)) = (p^3 - p^2 + 4p - 3)$.

Proof: The Wiener index for W_p, p = 3,4,5 etc… can be computed as follows:

By virtue of definition of Wiener index, we get the following.
$W(B(K_3, K_3)) = 27$
$W(B(K_4, K_4)) = 52$
$W(B(K_5, K_5)) = 85$
$W(B(K_6, K_6)) = 126$

Proceeding like this, we observe that $W(B(K_p, K_p)) = (p^3 - p^2 + 4p - 3)$.

Remark 4.2: The Wiener Polynomial of $B(K_p, K_p)$ for p = 3,4,5,6 etc is given as follows:
$W(B(K_3, K_3),x) = 4x^3 + 4x^2 + 7x$
$W(B(K_4, K_4), x) = 9x^3 + 6x^2 + 13x$
$W(B(K_5, K_5), x) = 16x^3 + 8x^2 + 21x$
$W(B(K_6, K_6),x) = 25x^3 + 10x^2 + 31x$

Proof: For $B(K_3, K_3)$ for p = 3, there are 7 pairs of vertices contributing distance one to the graph, 4 pairs of vertices contributing distance two to the graph and 4 pairs of vertices contributing distance three to the graph.

Therefore, $W(B(K_3, K_3);x) = 4x^3 + 4x^2 + 7x.$

For $B(K_4, K_4)$ for p = 4, there are 13 pairs of vertices contributing distance one to the graph, 6 pairs of vertices contributing distance two to the graph and 9 pairs of vertices contributing distance three to the graph.

Therefore, $W(B(K_4, K_4), x) = 9x^3 + 6x^2 + 13x.$

For $B(K_5, K_5)$ for p = 5, there are 21 pairs of vertices contributing distance one to the graph, 8 pairs of vertices contributing distance two to the graph and 16 pairs of vertices contributing distance three to the graph.

Therefore $W(B(K_5, K_5); x) = 16x^3 + 8x^2 + 21x.$

For $B(K_6, K_6)$, p = 6 there are 31 pairs of vertices contributing distance one to the graph,10 pairs of vertices contributing distance two to the graph and 25 pairs of vertices contributing distance three to the graph.

Therefore, $W(B(K_6, K_6);x) = 25x^3 + 10x^2 + 31x.$

Similarly, a Wiener polynomial of W_P, $p > 7$ can be computed.Also it can be generalised as

$$W(B(K_p, K_p);x) = \left(p^2 - p + 1\right)x + \left(2p - 2\right)x^2 + p\left(p-1\right)^2 x^3.$$

The Wiener index and Wiener Polynomial of $B(K_3, K_3)$, $B(K_4, K_4)$, $B(K_5, K_5)$ and $B(K_6, K_6)$ are tabulated here.

	$B(K_3, K_3)$	$B(K_4, K_4)$	$B(K_5, K_5)$	$B(K_6, K_6)$
$W(G)$	27	52	85	126
$W(G,x)$	$4x^3+4x^2+7x$	$9x^3+6x^2+13x$	$16x^3+8x^2+21x$	$25x^3+10x^2+31x$

CONCLUSION

In this paper, Wiener index for Wheel graph, Friendship graph and Barbell graph were investigated. Also the Wiener Polynomial for above graphs were obtained. This work would be extended to find Wiener index of Steiner distance and its respective Polynomial for the above graphs.

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Source of support: Nil, Conflict of interest: None Declared