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ON THE NON-NEWTONIAN EFFECTS OF RABINOWITSCH FLUID ON THE SQUEEZE FILM CHARACTERISTICS BETWEEN PARALLEL STEPPED PLATES

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ABSTRACT

In this paper the theoretical analysis investigates the effect of non-Newtonian pseudoplastic and dilant lubricants on the squeezing characteristics between parallel stepped plates is made. The modified Reynolds type equation is derived to study the effects of non-Newtonian fluids in comparison with Newtonian fluid. The approximate closed form solution is obtained by using Perturbation method. The load carrying capacity and squeezing time characteristics have been calculated. From the results obtained, it is clear that, the pressure, load carrying capacity and squeeze film time increases for dilants fluids as compared to corresponding Newtonian fluids. Also it is observe that the response time decreases as step height increases.

Keywords: *Rabinowitsch fluid model, squeeze film, non-Newtonian fluids, parallel stepped plates.*

INTRODUCTION

In recent years, the use of squeeze film lubrication has received much attention. This type of lubrication is observed in many applications such as clutches, gears, bearings and machine tools etc. The relevant literature on squeeze film lubrication can be found in Moore [1] and Archibald [2]. The squeeze film lubrication between two infinitely long parallel plates is studied by Cameron [3]. The flow of an incompressible fluid between two parallel plates due to normal motion of the plates is investigated by Bujurke [4]. The squeeze film with Newtonian lubricants has been studied by Jackson [5], Gupta and Gupta [6] investigated the squeezing flow between parallel plates.

It has been found that the use of additives can stabilize fluid properties and minimize the sensitivity of change in shearing strain rate [7].For effective improvement in the bearing characteristics as compared to the Newtonian lubricants, the use of Newtonian fluids blended with various additives increases. The use of oils blended with high molecular weight as lubricants has received attention of several researchers.

The present theoretical analysis investigates the effect of non-Newtonian pseudoplastic and dilant lubricants on the squeezing characteristics between parallel stepped plate. There are several fluid models to study the non-Newtonian properties of the lubricants such as power law, couple stress and micropolar fluid model. The squeeze film between finite plates lubricated with couple stress fluids was studied by Ramanaiah [8]. The squeeze film lubrication between circular stepped plates of couple stress fluids was studied by Naduvinamani and Siddangouda [9]. It is found that the influence of couple stresses is to enhances the squeeze film pressure, load carrying capacity and decreases the response time as compared to the classical Newtonian lubricants. The squeeze film lubrication between parallel stepped plates with couple stress fluids was studied by Kashinath[10].

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Rabinowitsch fluid model is one of the models to establish the non-linear relationship between the shearing stress and shearing strain rate which can be described for one dimensional fluid flow as follows.

$$\tau_{xy} + \kappa\tau_{xy}^3 = \mu\frac{\partial u}{\partial y} \tag{1}$$

Where μ denote the zero shear rate viscocity and κ denotes the non linear factor which describes the non-Newtonian effects of the lubricant which will be referred to as coefficient of pseudo plasticity. This model can be applicable to Newtonian, dilant and pseudoplastic lubricants for

$\kappa = 0,\ \kappa < 0$ and $\kappa > 0,$ respectively. The advantage of this model lies in the fact that the theoretical analysis for present model was verified with experimental justification by Wada and Hayashi [11]. Recently several researchers have investigated the non-Newtonian effect of Rabinowitsch lubricants on various type of bearings. Lin *et.al* [12] studied the non-Newtonian effect of Rabinowitsch fluid model on the slider bearings and parallel annular disks [13]. The effect of non-Newtonian Rabinowitsch fluids in wide parallel rectangular squeeze film plates is studied by Lin *et al.* [14]. The Variational principle for non-Newtonian lubrication of Rabinowitsch fluid model was analyzed by Him [15]. Singh *et.al* [16, 17] investigated the effect of Rabinowitsch fluid model on the hydrostatic thrust bearings and the squeeze film characteristics between a long cylinder and a flat plate and a sphere and flat plate is also studied by Singh *et. al* [18, 19].

In this paper an attempt is made to study the squeeze characteristics between parallel stepped plates lubricated with Rabinowitsch fluid.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a squeeze film between two parallel stepped plates approaching each other with a normal velocity V $\left(=\frac{dh}{dt}\right)$ as shown in the fig.1. The lubricant in the film region is considered as non-Newtonian Rabinowitsch fluid.

The assumptions of thin film lubrications are assumed to be applicable. The basic equations governing the flow are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xy}}{\partial y} \tag{3}$$

$$\frac{\partial p}{\partial y} = 0 \tag{4}$$

The relevant boundary conditions for velocity components are

i. At the upper surface $y = h$, u = o and $v = -V = \left(-\frac{dh}{dt}\right)$ 5(a)

ii. At the lower surface y = 0, u = 0 and v = 0 5(b)

Integrating equation (3) with respect to y subject to the boundary conditions 5(a) and 5(b) and using constitutive equation (1) the expression for velocity component is obtained as

$$u = \frac{1}{\mu}\left[\frac{1}{2}fF_1 + \kappa^* f^3 F_2\right] \tag{6}$$

where $F_1 = y(\mathrm{y-h})$, $F_2 = \frac{1}{4}y^4 - \frac{1}{2}y^3h + \frac{3}{8}y^2h^2 - \frac{1}{8}yh^3$ and f = $\frac{\partial p}{\partial x}$ κ^* = non-linear factor of lubricants.

Using equation (6) in the continuity equation (2) and integrating with respect to y under the relevant boundary conditions 5(a) and 5(b) for y, the Reynold's type equation for non-Newtonian Rabinowitsch fluid is obtained in the form

$$\frac{\partial}{\partial x}\left[h^3\left(\frac{\partial p}{\partial x}\right) + \frac{3}{20}\kappa^* h^5\left(\frac{\partial p}{\partial x}\right)^3\right] = -12\mu\frac{\partial h}{\partial t} \tag{7}$$

Equation (7) is a non- linear equation and it is not easy to find it's solution in closed form using analytical methods. Hence the classical perturbation method is used to find it's solution.

The squeeze film pressure can be perturbed as

$$p = p_0 + \kappa^* p_1 \tag{8}$$

Substituting into the Reynold's type equation (7) and neglecting the higher order terms of κ^*, the two separated equations governing the squeeze film pressure p_0 and p_1 can be derived respectively

$$\frac{\partial}{\partial x}\left[h^3 \frac{\partial p_0}{\partial x}\right] = -12\mu \frac{\partial h}{\partial t} \tag{9}$$

$$\frac{\partial}{\partial x}\left[\frac{3}{20} h^5 \left(\frac{\partial p_0}{\partial x}\right)^3 + h^3 \left(\frac{\partial p_1}{\partial x}\right)\right] = 0 \tag{10}$$

The modified Reynold's equation for determining the squeeze film pressure is obtained from of equation (9) and (10) in the form.

$$\frac{dp_{0i}}{dx} = \frac{-12\mu V x}{h_i^3} \tag{11}$$

$$\frac{dp_{1i}}{dx} = \frac{1296\mu V x^3}{5h_i^7} \tag{12}$$

Where $h_i = h_1$ for $0 \le x \le KL$ 13(a)

$= h_2$ for $KL \le x \le L$ 13(b)

The relevant boundary conditions for the pressure are

$p_1 = p_2$ at $x = KL$

$p_2 = 0$ at $x = L$

For the Region I:

$$p_1 = 6\mu V \left[\frac{K^2L^2 - x^2}{h_1^3} + \frac{L^2(1-\mathrm{K}^2)}{h_2^3}\right] + \kappa^* \mu V \frac{324}{5}\left[\frac{x^4 - K^4L^4}{h_1^7} + \frac{L^4(1-\mathrm{K}^4)}{h_2^7}\right] \tag{14}$$

and for the Region II:

$$p_2 = 6\mu V \left[\frac{R^2 - x^2}{h_2^3}\right] + \kappa^* \mu V \frac{324}{5}\left[\frac{L^4 - x^4)}{h_2^7}\right] \tag{15}$$

The load carrying capacity W is obtained in the form

$$W = 2b\int_0^{KL} p_1 dx + 2b\int_{KL}^{L} p_2 dx \tag{16}$$

which in the non-dimensional form is

$$W^* = \frac{Wh_2^3}{8b\mu VL^3} = \left\{\frac{K^3}{H^{*3}} + \frac{(1-\mathrm{K}^3)}{1}\right\} + \frac{324}{25}\alpha\left\{\frac{-K^5}{H^{*3}} + \frac{(1-\mathrm{K}^5)}{1}\right\}\left(= \frac{Wh_2^3}{8b\mu VL^3}\right) \tag{17}$$

where $H^* = \dfrac{h_1}{h_2}$ and $\alpha = \kappa^*\left(\dfrac{L}{h_2}\right)^2$

writing $V = \dfrac{-dh_2}{dt}$ in the equation (17) the squeezing time for reducing the film thickness from the initial value h_0 of h_2 to a final value h_f is given by

$$t = \frac{-8\mu b L^3}{W} = \int_{h_0}^{h_f}\left[\left\{\frac{K^3}{h_1^3} + \frac{1-K^3}{h_2^3}\right\} + \frac{324}{25}\kappa^* L^3\left\{\frac{-K^5}{h_1^7} + \frac{1-K^5}{h_2^7}\right\}\right]dh_2 \tag{18}$$

which in the non- dimensional form is

$$t^* = \frac{Wh_0^2 t}{8\mu b^{\ 3}} = \frac{1}{L}\int_{h_f^*}^{1}\left[\left\{\frac{K^3}{(h_2^*+h_s^*)^3}+\frac{1-K^3}{h_2^{*3}}\right\}+\frac{324}{2}\alpha\left\{\frac{-K^5}{(h_2^*+h_s^*)^7}+\frac{1-K^5}{h_2^{*7}}\right\}\right]dh_2^* \tag{19}$$

Where $h_f^* = \dfrac{h_f}{h_0}$; $h_2^* = \dfrac{h_2}{h_0}$; $h_s^* = \dfrac{h_s}{h_0}$

In the limiting case of $\alpha \to 0$, the equation (17) and (19) reduces to their corresponding Newtonian case presented by Naduvinamani and Siddanagouda [9] (When the couple stress parameter tends to zero)

3. RESULTS AND DISCUSSIONS

Based on the Rabinowitsch fluid model the effect of non-Newtonian rheology on the squeeze film characteristics between parallel stepped plates are investigated. We can study the characteristics between parallel stepped plates lubricated with the Newtonian fluids (α = 0), dilant fluids (α < 0) and pseudoplastic fluids (α > 0).

3.1. Load carrying capacity

From the figure2, it is observed that the dimensionless load carrying capacity W^* increases as the height of the fluid film thickness decreases. It is also observed that the maximum load is delivered for dilant fluids for smaller values of H^*. The increase in $W^*$is more accentuated for psudoplastic fluids as compared to the Newtonian fluids. But for the dilant fluids the reverse trend is observed. The dotted curve represents the Newtonian case. The relative percentage increase in W^*. $R_{W^*}\left[=\left(\frac{W^*_{Rabinowitsch} - W^*_{Newtonian}}{W^*_{Newtonian}}\right)\times 100\right]$. For different values of K is given in the Table -1.

3.2. Time – height relationship

The most important characteristics of the squeeze film bearing is the squeeze film time, that is the time required for reducing the initial film thickness h_0 of h_2 to a final value h_f. The figure 4 depicts the variation of non-dimensional squeeze-film time t^*as a function of h_f^* for different values of α the response time t^*increases for decreasing film thickness. For α = - 0.01 and α = - 0.005, as the film thickness h_f^* decreases the response time also decreases. An increase in t^* is more accentuated for dilant fluids than the corresponding Newtonian fluids. The relative increase in non-dimensional squeeze film time t^*, $R_{t^*}\left[=\left(\frac{t^*_{Rabinowitsch} \pm t^*_{Newtonian}}{t^*_{Newtonian}}\right)\times 100\right]$ for different values of K and α is given in the Table1. It is found that an increase of nearly 30% is observed for the dilant fluids.

The variation of t^* with h_f^* for different values of K for both Newtonian lubricants (α = 0), dilant lubricants (α < 0) and pseudoplastic lubricants (α > 0) is depicted in the figure 5. It is observed that t^* increases as the value K decreases.

The variation of t^* with h_f^* for different values of step height h_s^* with K = 0.7 is shown in the figure 6. It is observed that t^* decreases for increasing value of step height $h_{s.}^*$ The decrease in t^* is observed more for dilant fluids than Newtonian fluids where as reverse trend is observed for psudoplastic fluids.

K	α	R_{w*}	R_{t*}
0.5	-0.01	-14.093294	-31.4265239
	-0.005	-7.0466448	-15.7132618
	0.01	7.0466470	31.4265259
	0.005	14.093289	15.7132633
0.7	-0.01	-15.381026	-28.1837266
	-0.005	-7.6905162	-14.0918644
	0.01	7.6905132	28.18372438
	0.005	15.381032	14.0918622
0.9	-0.01	-14.490749	-10.28214628
	-0.005	-7.2453738	-5.141071788
	0.01	7.2453744	10.28215439
	0.005	14.490748	5.141079898

Table-1: The variation of load (R_{w*}) and relative time (R_{t*}) for different values of α and K

4. CONCLUSIONS

Based on the Rabinowitsch fluid model this paper predicts the non-Newtonian effects on squeeze film lubrication between parallel stepped plates. Based on the present theoretical analysis and the presented one can draw the following conclusions:

1. The effect of dilant fluids is to increase the load carrying capacity as compared to the corresponding Newtonian fluids, whereas the reverse trend is observed for the pseudoplastic lubricants.
2. The relative load response time Rt_* and relative load carrying capacity R_{w*} are dependent on the step size (K) and the non-linear factor of the lubricant (α).
3. As the squeeze film thickness decreases the non-dimensional response time t^* increases for dilant lubricants.

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NOMENCLATURE

b	: bearing width
H^*	: non-dimensional mean film thickness $\left(=\frac{h_1}{h_2}\right)$
h_0	: Initial value of minimum film thickness
h_1	: maximum film thickness
h_2	: minimum film thickness
h_s	: Step height
h_s^*	: Step height ratio $\left(=\frac{h_s}{h_o}\right)$
KL	: the position of the step; $0 < K < 1$
P_1	: fluid film pressure in the region $0 \le x \le KL$
P_2	: fluid film pressure in the region $KL \le x \le L$
x, y	: horizontal and vertical coordinates
t	: time of approach
t^*	: non-dimensional time of approach $\left(=\frac{Wh_0^2 t}{8\mu bL^3}\right)$
u, v	: velocity components of lubricant in the x and y directions respectively
V	: velocity of approach $\left(=\frac{dh}{dt}\right)$
W	: load- carrying capacity
W^*	: non-dimensional load- carrying capacity $\left(=\frac{Wh_2^3}{8b\mu VL^3}\right)$
α	: dimensionless non-linear factor of lubricants
μ	: initial viscocity of a Newtonian fluid
κ	: non-linear factor of lubricants
τ_{xy}	: shear stress component

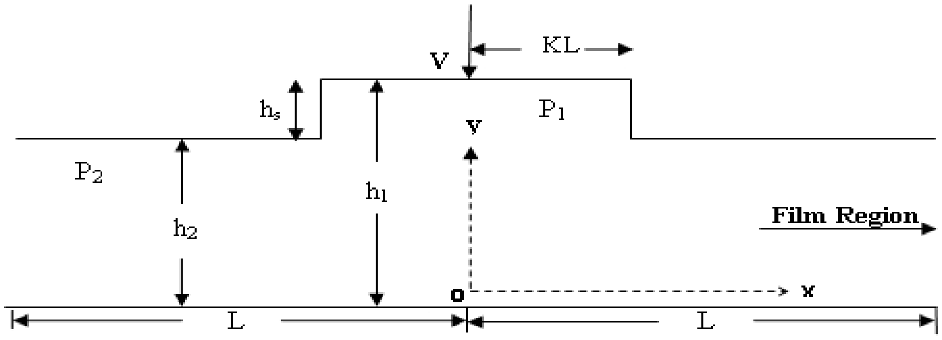

Figure 1. Squeeze film between parallel stepped plates.

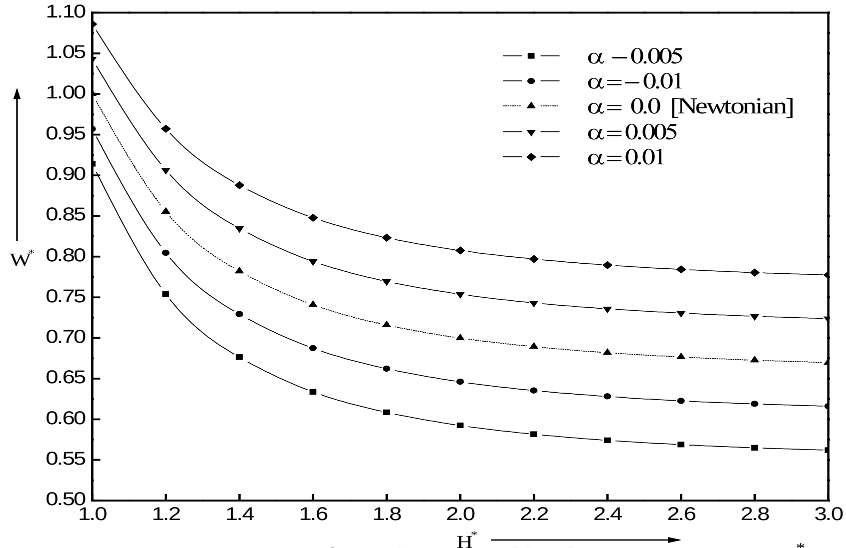

Figure 2. Variation of non-dimensional load carrying capacity W^* with H^* for different values of α with $K = 0.7$.

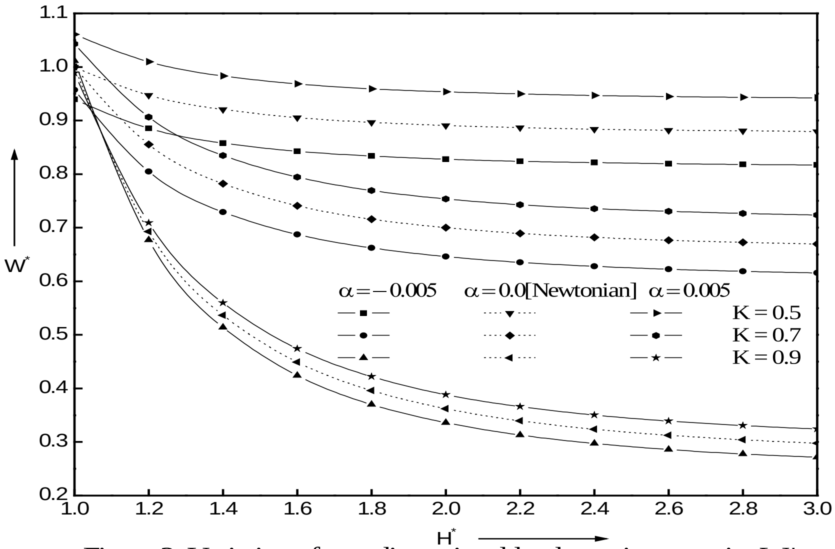

Figure 3. Variation of non-dimensional load carrying capacity W* with H* for different values of K

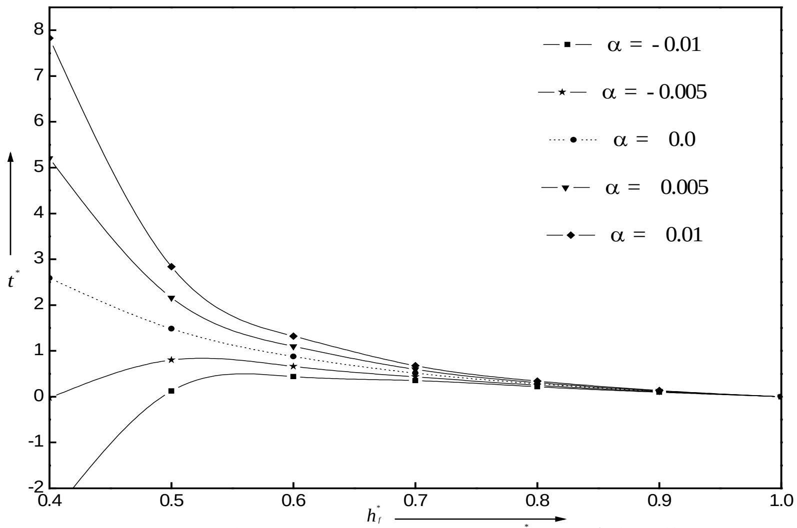

Figure 4.Variation of non-dimensional response time t^* with h_f^* for different values of α with $h_s^* = 0.15$

h_f^*

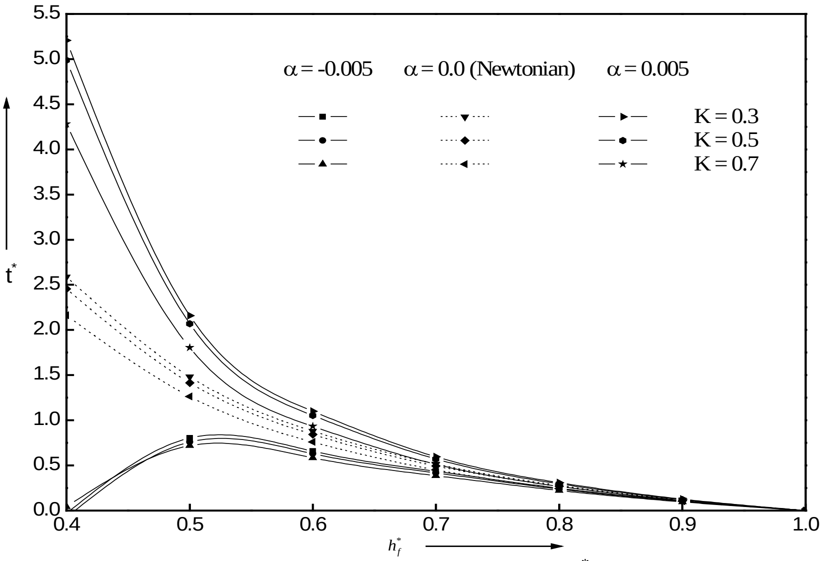

Figure 5. Variation of non-dimensional response time t^* with h_f^* for different values of K with h_s^* = 0.15.

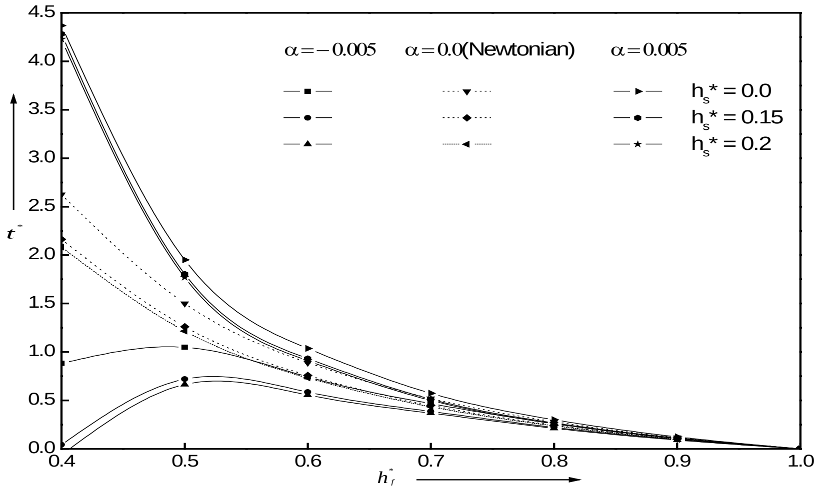

Figure 6. Variation of non-dimensional response time t^* with h_f^* for different values of h_s^* with K = 0.7

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