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BLOOD FLOW THROUGH STENOSED INCLINED TUBES WITH PERIODIC BODY ACCELERATION IN THE PRESENCE OF MAGNETIC FIELD AND WITH OUT POROUS MEDIA

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ABSTRACT

This is about the mathematical model for blood flow through stenosed inclined tubes with periodic body acceleration and magnetic field without porous medium. It is observed that the velocity decreases with increasing ξ for different time t and volumetric flow rate, shear stress for different time, phase angle are studied for particular parameters.

Key Words: *Blood flow, Stenosis, Periodic body acceleration, magnetic field, porous media, inclined tubes.*

INTRODUCTION

Chaturani , P. and Palanasamy . V, [1] studied "Pulsatile flow of blood with periodic body acceleration". Sud, V.K.and Sekhon,G.S.,[2] studied "Arterial flow under periodic body acceleration". Rathod and Gopichand[3]studied "Pulsatile flow of blood through a stenosed tube under periodic body acceleration with magnetic field".Rathod et al[4] sudied "Pulsatile flow of blood under the periodic body acceleration with magnetic field .ElShahawey,E.F., Elsayed, *et al* [5]studied "MHD flow of an elastic-viscous fluid under periodic body acceleration. El-Shahawey. *et al.* studied, "Pulsatile flow of blood through a porous medium under periodic body acceleration". Coklet.G.R[7] stuydied., "The Rheology of Human blood..Vardanyan.V.A, [8]studied"Effect of magnetic field on blood flow". Bhuvan.B.C.and Hazarika .G.C.[9] studied, "Effect of magnetic field on Pulsatile flow of blood in a porous channel".Chaturani.P.and Biswas[10] studied, "A Comparative study of two layered blood flow models with different boundary conditions". Berger.S.A.,Jou.L.D studied, "Flows in stenotic vessels" .Young.d.F [12] studied ,Fluid mechanics of arterial stenosis.K.Das and G.C. Saha [13],studied "Arterial MHD Pulsatile flow of blood under the periodic body acceleration.C.Sanyal, K. Das and S. Debnath[14],studied the "Effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration". Gaurav Mishra, Ravindra Kumar and K.K.Singh[15]studied "A study of Oscillatory blood flow through porous medium in a stenosed artery

In this paper, using finite Hankel and Laplace transforms, analytical expressions for velocity profile, volumetric flow rate and wall shear stress have been obtained and their natures are portrayed graphically for different parameters such as Hartmann number, phase angle, time etc.in an inclined tube under stenoses.

MATHEMATICAL FORMULATION

Let us consider the axially symmetric and fully developed pulsatile flow of blood through a stenosed porous circular artery with body acceleration under the influence of uniform transverse magnetic field. Blood is assumed to be Newtonian and incompressible fluid. Also for mathematical model, we take the artery to be a long cylindrical tube with the axis along z-axis. The pressure gradient and body acceleration are respectively given by

$$-\frac{\partial P}{\partial z} = A_0 + A_1 \cos(\omega_P t) \tag{1}$$

$$G = a_0 \cos(\omega_b t + \phi) \tag{2}$$

where A_0 and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_0 is the amplitude of body acceleration, $\omega_P = 2\pi f_p$, $\omega_b = 2\pi f_b$ with f_p is the pulse frequency and f_b is body acceleration frequency, ϕ is the phase angle of body acceleration G with respect to pressure gradient and t is time.

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The governing equation of motion for flow in cylindrical polar coordinates is given by

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial P}{\partial z}+\rho G+\mu\nabla^2 u-\sigma B_0 u+\rho g\sin\theta \tag{3}$$

where u is the axial velocity of blood; P, blood pressure; $\frac{\partial P}{\partial z}$,pressure gradient; ρ , density of blood; μ , the viscosity of blood; k , the permeability of the isotropic porous medium; B_0 , the external magnetic field along the radial direction and σ is the conductivity of blood.

The geometry of stenosis is shown in figure-1.

$$R(z)=\begin{cases} a-\delta(1+\cos\frac{\pi z}{2z_0}),-2z_0\le z\le 2z_0 \\ a, otherwise\end{cases}$$

where $R(z)$ is the radius of the stenosed artery, a is the radius of artery, $4z_0$ is the length of stenosis and 2δ is the maximum protuberance of the stenotic form of the artery wall.

$$\xi=\frac{r}{R(z)}$$

where $R(z)$ depends on δ .

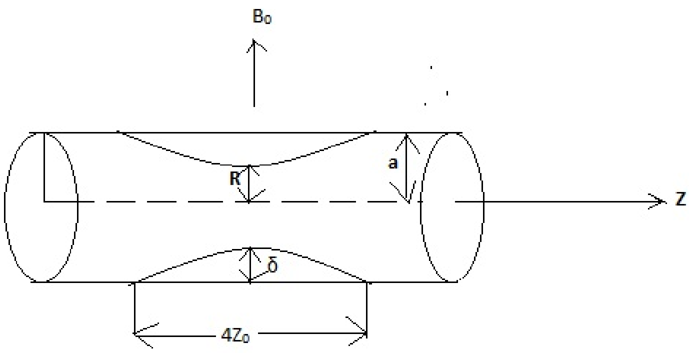

Fig.1.Geometry of artery with stenosis

The equation (3) becomes

$$\rho\frac{\partial u}{\partial t}=A_0+A_1\cos(\omega_P t)+\rho a_0\cos(\omega_b t+\phi)+\frac{\mu}{R^2}\left[\frac{\partial^2 u}{\partial\xi^2}+\frac{1}{\xi}\frac{\delta u}{\delta\xi}\right]-\mu C^2 u+\rho g\sin\theta \tag{4}$$

where

$C=\sqrt{\frac{M^2}{R^2}}$, $M=\sqrt{\frac{\sigma}{\mu}}RB_0$ (Hartmann number)

We assumed that t<0 only the pumping action of the heart is present and at t=0, the flow in the artery corresponds to the instantaneous pressure gradient i.e.,

$$-\frac{\partial P}{\partial z}=A_0+A_1$$

As a result, the flow velocity at t=0 is given by

$$u(\xi,0)=\frac{A_0+A_1}{\mu C^2}\left[1-\frac{I_0(CR\xi)}{I_0(CR)}\right] \tag{5}$$

where I_0 is modified Bessel function of first kind of order zero.

The initial and boundary conditions to the problem are

$$u(\xi,0)=\frac{A_0+A_1}{\mu C^2}\left[1-\frac{I_0(CR\xi)}{I_0(CR)}\right], u=0 \quad at \quad \xi=1, u \quad is \quad finite \quad at\ \xi=0 \tag{6}$$

Solutions: Applying Laplace transform to equation (4) and first boundary condition of (6), we get

$$\rho s\bar{u}-\frac{\rho(A_0+A_1)}{\mu C^2}\left[1-\frac{I_0(CR\xi)}{I_0(CR)}\right]=\frac{A_0}{s}+\frac{A_1}{(s^2+{\omega_P}^2)}+\frac{\rho a_0(s\cos\phi-\omega_b\sin\phi)}{(s^2+{\omega_b}^2)}$$
$$+\frac{\mu}{R^2}\left[\frac{\partial^2\bar{u}}{\partial\xi^2}+\frac{1}{\xi}\frac{\delta\bar{u}}{\delta\xi}\right]-\mu C^2\bar{u}+\frac{\rho g\sin\theta}{s} \tag{7}$$

where $\bar{u}(\xi,s)=\int_0^\infty e^{-st}u(\xi,t)dt(s\succ 0)$

Then applying the finite Hankel transform to equation (7), we obtain

$$\bar{u}^*(\lambda_n,s)=\frac{J_1(\lambda_n)R^2}{\lambda_n[\rho sR^2+\mu(C^2R^2+{\lambda_n}^2)]}\left[\frac{A_0}{s}+\frac{A_1}{(s^2+{\omega_P}^2)}+\frac{\rho a_0(s\cos\phi-\omega_b\sin\phi)}{(s^2+{\omega_b}^2)}+\frac{\rho(A_0+A_1)R^2}{\mu(C^2R^2+{\lambda_n}^2)}+\frac{\rho g\sin\theta}{s}\right] \tag{8}$$

Where $\bar{u}^*(\lambda_n,s)=\int_0^1 ru(r,s)J_0(r\lambda_n)dr$ and λ_n are zeros of J_0, Bessel function of first kind and $\nu=\frac{\mu}{\rho}$

The Laplace and Hankel inversions of equation (8) give the final solution for blood velocity as

$$u(\xi,t)=2\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{J_1(\lambda_n)}\left[\begin{array}{l}\left\{\begin{array}{l}\frac{(A_0+g\sin\theta)R^2}{\mu({\lambda_n}^2+C^2R^2)}+\frac{A_1R^2[\nu({\lambda_n}^2+C^2R^2)\cos\omega_Pt+\omega_PR^2\sin\omega_Pt]}{\rho[R^4{\omega_P}^2+\nu^2({\lambda_n}^2+C^2R^2)^2]}\\+\frac{a_0R^2[\nu({\lambda_n}^2+C^2R^2)\cos(\omega_bt+\phi)+\omega_bR^2\sin(\omega_bt+\phi)]}{R^4{\omega_P}^2+\nu^2({\lambda_n}^2+C^2R^2)^2}\end{array}\right\}\\-e^{-\left(\frac{\nu}{R^2}\right)({\lambda_n}^2+C^2R^2)t}\left\{\begin{array}{l}\frac{-A_1{\omega_P}^2R^6}{\mu({\lambda_n}^2+C^2R^2)[R^4{\omega_P}^2+\nu^2({\lambda_n}^2+C^2R^2)^2]}\\+\frac{a_0R^2[\nu({\lambda_n}^2+C^2R^2)\cos\phi+\omega_bR^2\sin\phi]}{R^4{\omega_b}^2+\nu^2({\lambda_n}^2+C^2R^2)^2}\\+\frac{g\sin\theta}{\left(\frac{\nu}{R^2}\right)({\lambda_n}^2+C^2R^2)}\end{array}\right\}\end{array}\right] \tag{9}$$

$$u(\xi,t)=\frac{2A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{J_1(\lambda_n)}\left[\begin{array}{l}\left\{\begin{array}{l}\left\{\frac{A_0+g\sin\theta}{A_0\,({\lambda_n}^2+C^2R^2)}+\frac{\varepsilon({\lambda_n}^2+C^2R^2)\cos\omega_Pt+\alpha^2\sin\omega_Pt}{({\lambda_n}^2+C^2R^2)^2+\alpha^4}\right\}\\+\frac{\rho a_0}{A_0}\left\{\frac{({\lambda_n}^2+C^2R^2)\cos(\omega_bt+\phi)+\beta^2\sin(\omega_bt+\phi)}{({\lambda_n}^2+C^2R^2)^2+\beta^4}\right\}\end{array}\right\}\\-e^{-\left(\frac{\nu}{R^2}\right)({\lambda_n}^2+C^2R^2)t}\left\{\begin{array}{l}\frac{\varepsilon\alpha^4}{({\lambda_n}^2+C^2R^2)[\alpha^4+({\lambda_n}^2+C^2R^2)^2]}\\+\frac{\frac{\rho a_0}{A_0}\{({\lambda_n}^2+C^2R^2)\cos\phi+\beta^2\sin\phi\}}{({\lambda_n}^2+C^2R^2)^2+\beta^4}\\+\frac{g\sin\theta}{\left(\frac{\nu}{R^2}\right)({\lambda_n}^2+C^2R^2)}\frac{\mu}{A_0R^2}\end{array}\right\}\end{array}\right] \tag{10}$$

where $\alpha^2 = \frac{\omega_P R^2}{\nu} = \mathrm{Re}_P$, $\beta^2 = \frac{\omega_b R^2}{\nu} = \mathrm{Re}_b$, $\varepsilon = \frac{A_1}{A_0}$

The analytical expression of u consists of four parts. The first and second parts correspond to steady and oscillatory parts of pressure gradient, the third term indicates body acceleration and the last term is the transient term. As $t \to \infty$, the transient term approaches to zero. Then from equation (10), we get

$$u(\xi,t) = \frac{2A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{J_1(\lambda_n)}\left[\begin{array}{l}\left\{\dfrac{A_0+g\sin\theta}{A_0\,(\lambda_n{}^2+C^2R^2)}+\dfrac{\varepsilon(\lambda_n{}^2+C^2R^2)\cos\omega_P t+\alpha^2\sin\omega_P t}{(\lambda_n{}^2+C^2R^2)^2+\alpha^4}\right\}\\ +\dfrac{\rho a_0}{A_0}\left\{\dfrac{(\lambda_n{}^2+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\right\}\end{array}\right] \quad (11)$$

The volumetric flow rate Q is given by

$$Q(\xi,t) = 2\pi\int_0^R ru\,dr$$

$$Q(\xi,t) = \frac{4\pi A_0R^4}{\mu}\sum_{n=0}^{\infty}\frac{1}{\lambda_n{}^2}\left[\begin{array}{l}\left\{\dfrac{A_0+g\sin\theta}{A_0\,(\lambda_n{}^2+C^2R^2)}+\dfrac{\varepsilon(\lambda_n{}^2+C^2R^2)\cos\omega_P t+\alpha^2\sin\omega_P t}{(\lambda_n{}^2+C^2R^2)^2+\alpha^4}\right\}\\ +\dfrac{\rho a_0}{A_0}\left\{\dfrac{(\lambda_n{}^2+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\right\}\end{array}\right] \quad (12)$$

The fluid acceleration F is given by

$$F(\xi,t) = \frac{\partial u}{\partial t}$$

$$F(\xi,t) = \frac{2a_0}{\rho}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{J_1(\lambda_n)}\left[\begin{array}{l}\left\{\dfrac{\alpha^2\{-\varepsilon(\lambda_n{}^2+C^2R^2)\sin\omega_P t+\alpha^2\cos\omega_P t\}}{(\lambda_n{}^2+C^2R^2)^2+\alpha^4}\right\}\\ +\dfrac{\rho a_0\beta^2}{A_0}\left\{\dfrac{-(\lambda_n{}^2+C^2R^2)\sin(\omega_b t+\phi)+\beta^2\cos(\omega_b t+\phi)}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\right\}\end{array}\right] \quad (13)$$

The expression for the wall shear stress τ_w can be obtained from

$$\tau_w = \mu\left(\frac{\partial u}{\partial r}\right)_{r=R}$$

$$\tau_w(\xi,t) = -2A_0R\sum_{n=1}^{\infty}\left[\begin{array}{l}\left\{\dfrac{A_0+g\sin\theta}{A_0\,(\lambda_n{}^2+C^2R^2)}+\dfrac{\varepsilon(\lambda_n{}^2+C^2R^2)\cos\omega_P t+\alpha^2\sin\omega_P t}{(\lambda_n{}^2+C^2R^2)^2+\alpha^4}\right\}\\ +\dfrac{\rho a_0}{A_0}\left\{\dfrac{(\lambda_n{}^2+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\right\}\end{array}\right] \quad (14)$$

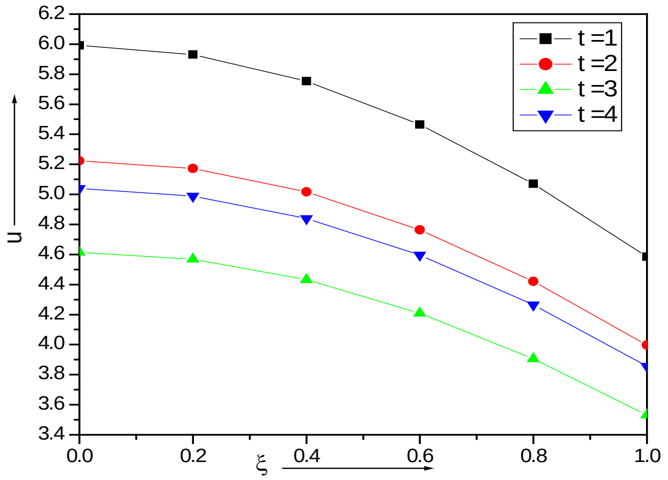

Fig.2.Variation of velocity profiles for aorta artery against ξ with $\phi=45^0,\theta=30^0$,M=2.0

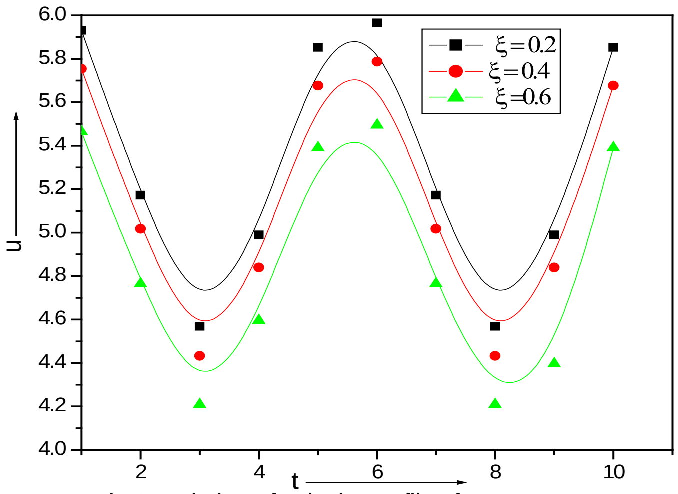

Fig.3.Variation of velocity profiles for aorta artery against t with $\phi=45^0,\theta=30^0$,M=2.0

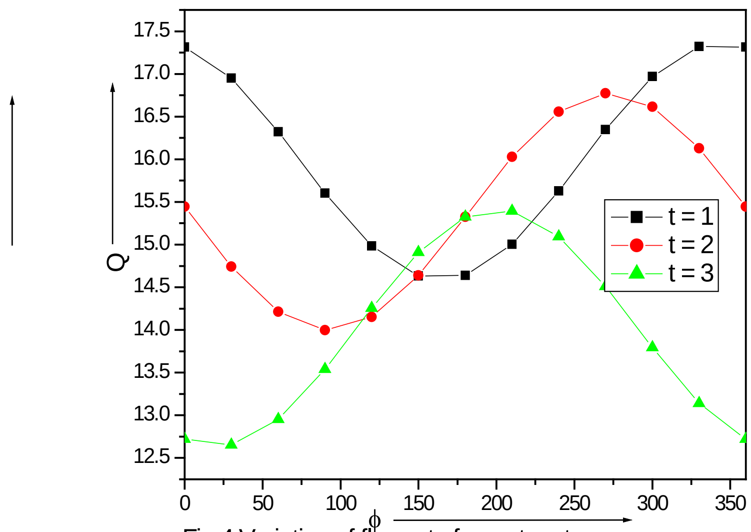

Fig.4.Variation of flow rate for aorta artery against ϕ for $\theta=30^0$,g=9.8,M=2.0

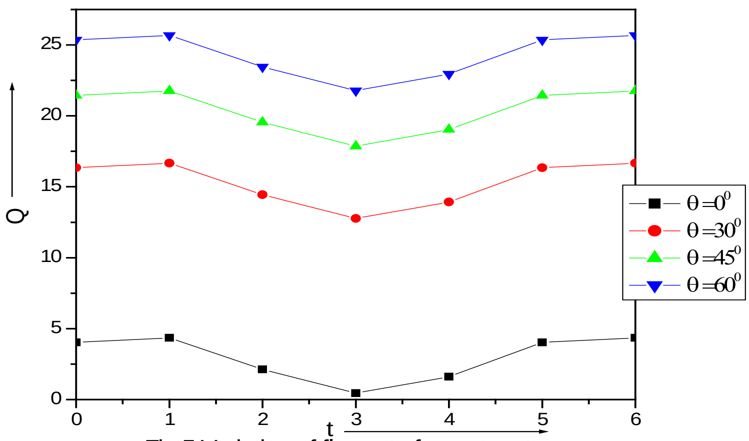

Fig.5.Variation of flow rate for aorta artery against t for $\theta=30^0$,g=9.8,M=2.0,$\phi=45^0$

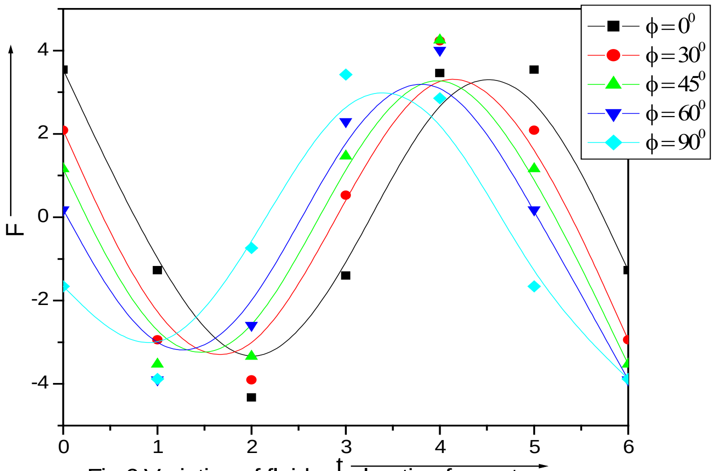

Fig.6.Variation of fluid accelaration for aorta artery against ξ=0.2,M=2.0,$\theta=30^0$, g = 9.8

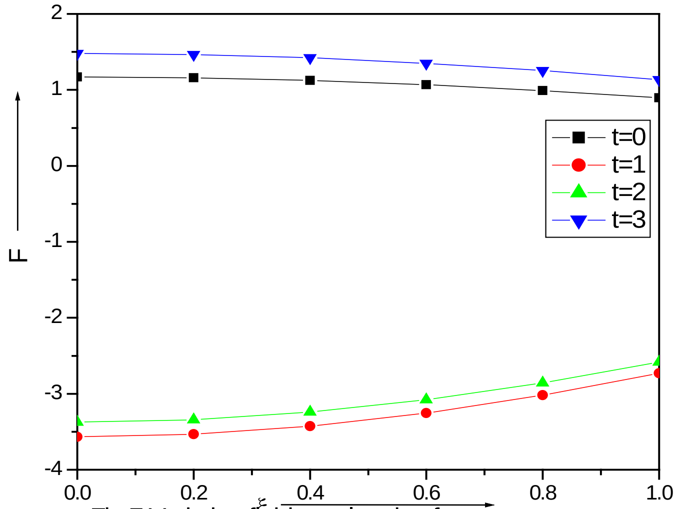

Fig.7.Variation fluid accelaration for aorta artery against ξ for M=2.0,$\theta=60^0$, g = 9.8,$\phi=45^0$

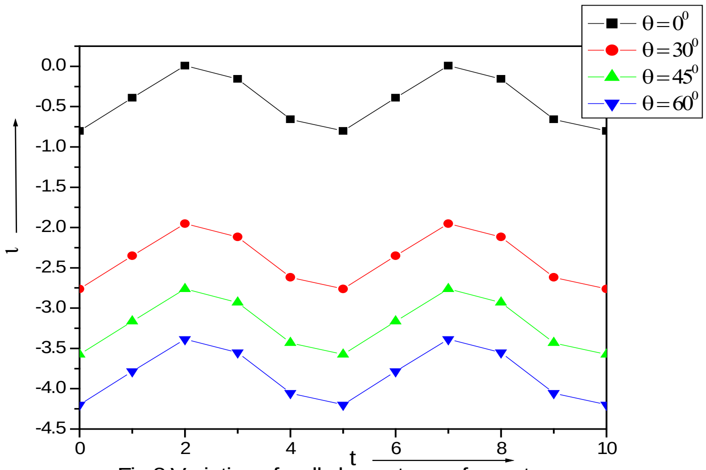

Fig.8.Variation of wall shear stress ι for aorta artery against t when $\phi=45^0$,M=2.0,g=9.8,$\alpha=\beta=1$

In fig (2), If we plot u vurses ξ for fixed values of $\phi = 45^0$, $\theta = 30^0$, M=2.0, u decreaseswith increasing ξ for different t.

In fig (3), for fixed values of $\phi = 45^0$, $\theta = 30^0$, M=2.0, if we plot u vurses t,we get the oscilllatory nature of cuves for different ξ.

In fig (4), for $\theta = 30^0$, g=9.8, M=2.0, if we plot Q vurses ϕ for different t,we get the cureves as shown in figure(4).

In fig (5), for fixed $\theta = 30^0$, g=9.8, M=2.0, $\phi = 45^0$, if we plot Q vurses t for different θ, as θ increases Q is also increases.

In fig (6), for fixed ξ=0.2, M=2.0, $\theta = 30^0$, g=9.8, if we plot F vursest for increasing ϕ, we get oscillatory nature of curves.

In fig (7), for fixed M=2.0, $\theta = 60^0$,g=9.8, $\phi = 45^0$, if we plot F vurses ξ for different t, we get the curves as shown in fig(7).

In fig (8), for fixed $\phi = 45^0$, M=2.0,g=9.8, $\alpha = \beta = 1$, if we plot τ vurses t for θ, as θ increases, τ decreases for increasing t.

REFERENCES

1. Chaturani, P. and Palanasamy. V, "Pulsatile flow of blood with periodic body acceleration", Int. J. Engng. Sci. 29 (1), 113-121(1991).
2. Sud, V.K. and Sekhon,G.S., "Arterial flow under periodic body acceleration", Bull of Math.Biol.47(1), 35-52(1985).
3. Rathod and Gopichand, "Pulsatile flow of blood through a stenosed tube under periodic body acceleration with magnetic field", Ultra Scientist of Physical Science, Vol(1)M,pp109-118(2005).
4. Rathod.V.P, Shakara Tanveer, Itagi Sheeba Rani,G.G.Rajput, "Pulsatile flow of blood under the periodic body acceleration with magnetic field", Ultra Scientist of Physical Sciences,17(1)M, 7-16(2005).
5. El-Shahawey, E.F., Elsayed,M.E.Ealbarbary, Afifi, N.A.S and Mostafa Elshahed, "MHD flow of an elastic-viscous fluid under periodic body acceleration", Int. J.& Math.Sci.23(11), 795-799(2000).
6. El-Shahawey , E.F. Elsayed, M.E.et al., "Pulsatile flow of blood through a porous medium under periodic body acceleration", Int. J .Theoretical Physics39(1),183-188(2000).
7. Coklet.G.R., "The Rheology of Human blood", Biomechanics-63-103(1972).
8. Vardanyan.V.A, "Effect of magnetic field on blood flow", Bio.Physics, 18,515(1973).
9. Bhuvan.B.C. and Hazarika .G.C., "Effect of magnetic field on Pulsatile flow of blood in a porous channel", Bio-Science research Bulletin.17 (2), 105-111(2001).

10. Chaturani.P. and Biswas, “A Comparative study of two layered blood flow models with different boundary conditions”, Bio-Math., N101, 47-56(1988).
11. Berger.S.A., Jou.L.D,2000, “Flows in stenotic vessels”. Annual review of fluid mechanics 32,347-382.
12. Young.d.F 1979, Fluid mechanics of arterial stenosis, J. of Bio Mech. Engng (Trans AMES).101, 157-175.
13. K.Das and G.C. Saha, “Arterial MHD Pulsatile flow of blood under the periodic body acceleration”, Bull. Soc. Math. Banja Luka, Vol.16 (2009), 21-42.
14. D.C.Sanyal, K. Das and S. Debnath, “Effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration”, Journal of Physical Sciences, Vol.11, 2007, 43-56.
15. Gaurav Mishra, Ravindra Kumar and K.K.Singh, “A study of Oscillatory blood flow through porous medium in a stenosed artery, Ultra Scientist. Vol.24 (2) A, 369-373(2012).

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