International Journal of Mathematical Archive-6(7), 2015, 209-212
Available online through www.ijma.info ISSN 2229 – 5046

ZONE HAMILTONIAN CIRCUIT AND ZONE HAMILTONIAN PATH FUNCTIONS OF HAMILTONIAN GRAPHS

R. ARANGULAVAN*
Department of Mathematics,
Bharathidasan University Constituent College, Perambalur-621 107, India.

M. KAMARAJ
Associate Professor of Mathematics,
Government Arts and Science College, Sivakasi-626 124, India.

(Received On: 18-06-15; Revised & Accepted On: 11-07-15)

ABSTRACT

A function f: A →{0, 1} on A of a Hamiltonian graph G(V, A) is called Zone(zero-one) Hamiltonian Circuit Function (ZHCF) if for any e ∈ H_c, f (e) = 1 and f (e) = 0 for e ∉ H_c. *A function f: A →{0, 1} on A of a Hamiltonian graph G(V, A) is called Zone(zero-one) Hamiltonian Path Function (ZHPF)* if for any e ∈ H_p, f (e) = 1 and f (e) = 0 for e ∉ H_p. *In this paper, we introduce zone (zero-one) Hamiltonian circuit function, Hamiltonian path function and study them.*

***Keywords**: Graphs, Hamiltonian graphs, Hamiltonian path, arc dominating set, arc dominating number.*

***2010 Mathematics Subject Classification:** 05C20.*

INTRODUCTION

We refer 'Graph Theory with Application' by J.A. Bondy and U.S.R. Murty for basic notations and definitions [3] and [4] for basic terminology in domination related concepts in graph theory. Let G(V, E) be a graph. A set S⊆V(G) is a *dominating set* if the function f : V(D) →{0, 1} with S = {v : $f(v)$ = 1} satisfy condition that, for every v ∈ V(G), f(N[v]) ≥ 1. A Dominating set S⊆V(G) is called an *efficient dominating set* if for every vertex u ∈ V(G), |N[u] ∩ S| = 1. Benge *et al*. [2] introduced the following efficiency measure for a graph. The *efficient domination number* of a graph, denoted F(G), is the maximum number of vertices that can be dominated by a set S that dominates each vertex at most once. A graph G of order n = |V(G)| has an efficient dominating set if and only if F(G) = n.

Lutz Volkmann [5] defined *a signed dominating function* on a finite simple digraph D to be a two-valued function f : V(D) →{-1, 1}. If $\sum_{x \in N^-[v]} f(x) \geq 1$ for each v ∈ V(D), where $N^-[v]$ consists of v and all vertices of D from which arcs go into v, then f is a *signed dominating function* on D. The sum f (V(D)) is called the weight $w(f)$ of f . The *minimum of weights* $w(f)$, taken over all signed dominating functions f on D, is the *signed domination number* γs(D) of D. A set {f_1, f_2,…, f_d} of signed dominating function on D with the property that $\sum_{i=1}^{d} f_i(x) \leq 1$ for each x ∈ V(D), is called a signed dominating family (of function) on D. The maximum number of functions in a signed dominating family on D is the domatic number of D, denoted by d_S (D).

K. Muthu Pandian *et al.* [6, 7, 8] defined a *Twin Dominating Function* (TDF) as follows, Let D(V, A) be any digraph. A function f : V → [0, 1] is called a twin dominating function if the sum of its function values over any closed out-neighborhood is at least one as well as the sum of its function values over any closed in-neighborhood is at least one. A TDF f of D is called a *minimal* TDF if there is no TDF g of D such that $g(v) \leq f(v)$ for all v∈V and $g(v_0) \neq f(v_0)$ for some v_0 ∈ V. An *in-dominating function* (IDF) of a digraph D(V, A) is a function f : V → [0, 1] such that $\sum_{u \in N^-[v]} f(u) \geq 1$ for all v ∈V, where $N^-[v]$ = $N^-(v) \cup \{v\}$ and $N^-(v)$ denote the set of all vertices of D which are adjacent to v. In this paper, we focus our study on zone in-degree efficient dominating numbers for directed graphs.

Corresponding Author: R. Arangulavan*, Department of Mathematics,
Bharathidasan University Constituent College, Perambalur-621 107, India.

1. PRELIMINARIES

A graph G(V, A) with vertex set V and arc set A is considered. A Hamiltonian circuit in a connected graph G is defined as a closed walk that traverses every vertex of G exactly once, except of course the starting vertex, at which the walk also terminates. If we remove any one arc from a Hamiltonian circuit, we left a path. This path is called a Hamiltonian path. A set F ⊆ A is an arc dominating set if each arc in A is either in F or is adjacent to an arc in F. The arc domination number $\gamma'(G)$ is the smallest cardinality among all minimal arc dominating sets. For a graph G(V, A), a subset F of A is independent if no two arcs in F are adjacent.

2. ZONE HAMILTONIAN CIRCUIT FUNCTION AND HAMILTONIAN PATH FUNCTION

Definition 2.1: Let G(V, A) be a Hamiltonian graph and C be a Hamiltonian circuit, the set of all arcs in Hamiltonian circuit C of G is denoted by $H_c = \{e : e$ is arc in Hamiltonian circuit$\}$.

Definition 2.2: Let G(V, A) be a Hamiltonian graph and P be a Hamiltonian path, the set of all arcs in Hamiltonian path P of G is denoted by $H_p = \{ e : e$ is arc in Hamiltonian path$\}$.

Definition 2.3: Let G(V, A) be a Hamiltonian graph. A function f: A →{0, 1} is said to be *zone Hamiltonian circuit function* (ZHCF) of G if $f(e) = 1$ for $e \in H_c$ and $f(e) = 0$ for $e \notin H_c$.

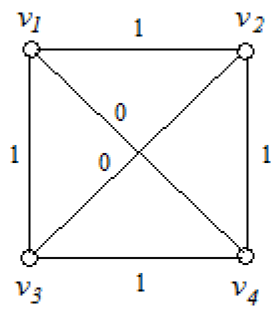
Definition 2.4: Let G(V, A) be a Hamiltonian graph. A function f: A →{0, 1} is said to be *zone Hamiltonian path function* (ZHPF) of G if $f(e) = 1$ for $e \in H_p$ and $f(e) = 0$ for $e \notin H_p$.

Definition 2.5: Let G(V, A) be a Hamiltonian graph and f : A →{0, 1} be the Hamiltonian function defined on G, the *weight* of f : A →{0, 1} is defined as $|f| = \sum_{e\in A} f(e) = f(A)$.

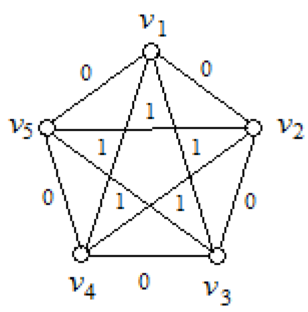
Definition 2.6: A set F ⊆ A is a *Hamiltonian arc dominating set* if each arc in H_c is either in F or is adjacent to an arc in F.

Definition 2.7: The *Hamiltonian arc domination number* $\gamma'(H_c)$ is defined as the smallest cardinality among all minimal Hamiltonian arc dominating sets in Hamiltonian circuit.

Example 2.8:



$|f| = 4 = |V|$



$|f| = 5 = |V|$

Proposition 2.9: Let G(V, A) be a Hamiltonian graph and f : A →{0, 1} be a zone Hamiltonian circuit function defined on G then $|f| = |V|$.

Proof: Since a Hamiltonian circuit in G is a closed walk that traverses every vertex of G exactly once so that $|f| = |V|$. By definition of zone Hamiltonian circuit function, $f(e) = 1$ for $e \in H_c$ otherwise the weight of each arc is zero hence $\sum_{e\in A} f(e) = |f| = |V|$.

Theorem 2.10: Let G(V, A) be a graph of Hamiltonian path of order n and f : A →{0, 1} be a zone Hamiltonian path function defined on G then $|f| = n-1$

Proof: We remove any one arc from a Hamiltonian circuit we left with a path (this path is called Hamiltonian path). Since by definition of zone Hamiltonian path function, assuming numerical value 1 for each arc in H_p and 0 for arcs not in H_p. Clearly, a Hamiltonian path in a graph G traverses every vertex of G. Hence the sum of the numerical values is equal to the length of the Hamiltonian path so that $|f| = n-1$

Theorem 2.11: In a complete graph with n (n is odd $n \geq 3$) vertices there are $\frac{n-1}{2}$ distinct zone Hamiltonian circuit function such that each function consists arc – disjoint Hamiltonian circuit.

Proof: There are $\frac{n(n-1)}{2}$ arcs in a complete graph G and a Hamiltonian circuit in G consists of n arcs. So that the number of arc - disjoint Hamiltonian circuit in G cannot exceed $\frac{n-1}{2}$. That there are $\frac{n-1}{2}$ arc – disjoint Hamiltonian circuits, when n is odd, can be shown as follows;

The subgraph of a complete graph of n vertices in fig.1.1 is a Hamiltonian circuit. Keeping the vertices fixed on a circle, rotate the polygonal pattern clockwise by $\frac{360}{n-1}, \frac{2\times360}{n-1}, \frac{3\times360}{n-1},\ldots.\frac{n-3}{2}\,\frac{360}{n-1}$ degrees.

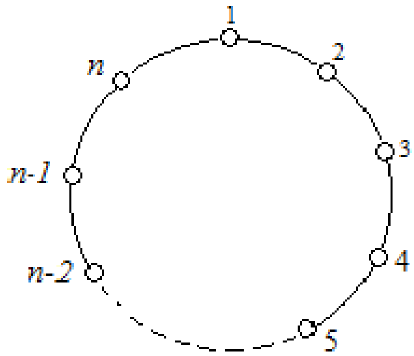


fig.1.1

Observe that each rotation produces a Hamiltonian circuit that has no arc in common with any of the previous once. Thus we have $\frac{n-3}{2}$ new Hamiltonian circuit, all arc – disjoint from the one in fig.1.1 and also arc – disjoint among themselves. Hence we can consists $\frac{n-1}{2}$ distinct zone Hamiltonian functions.

Theorem 2.12: Let G(V, A) be a Hamiltonian graph of order n (n is multiple number of 3) and $f : A \rightarrow \{0,1\}$ be a ZHCF defined on G then $\frac{|f|}{3} = \gamma'(H_c)$.

Proof: It is trivially true for $n = 3$ and $\gamma'(H_c) = 1$ since each Hamiltonian circuit in G has n arcs and n is multiple of 3. Let $H_c = \{ e_1, e_2, e_3,\ldots.e_n\}$, e_1 is adjacent to e_2, e_2 is adjacent to e_3, and so on e_n is adjacent to e_1 denoted as $e_1\rightarrow e_2 \rightarrow e_3 \rightarrow \ldots..\rightarrow e_n \rightarrow e_1$. Construct the set F from H_c as follows; $F = \{e_{1+3r} : 1+3r < n$ and $r = 0, 1, 2,\ldots.m\}$. Clearly, F is the Hamiltonian arc dominating set with smallest cardinality and $3 \times \gamma'(H_c) = |f|$ hence the theorem.

Theorem 2.13: Let G(V, A) be a Hamiltonian graph of order $n = 4+6r$ and $f : A \rightarrow \{0,1\}$ be a ZHCF defined on G then $\frac{|f|-2r}{2} = \gamma'(H_c)$, $r = 0, 1, 2,\ldots..m$.

Proof: Let $H_c = \{ e_1, e_2, e_3,\ldots.e_n\}$ and $e_1\rightarrow e_2 \rightarrow e_3 \rightarrow \ldots..\rightarrow e_n \rightarrow e_1$. Construct the set F in H_c as follows; $F = \{e_1, e_{3i} : 3i < n, i = 1, 2, 3,\ldots..k\}$. Clearly the cardinalities of F are 2, 4, 6, and so on for $i =1, 2, \ldots.k$. Hence $|F| = 2+2r$ for $r = 0, 1, 2,\ldots..m$. and $\gamma'(H_c) = 2+2r$. Given $n = 4+6r$ therefore $|f| = 4+6r$, i.e., $\frac{4+6r-2r}{2} = \frac{4+4r}{2} = 2+2r$ and hence the proof.

Theorem 2.14: Let G(V, A) be a complete graph of order n ($n \geq 4$ and n is even) and $f : A \rightarrow \{0,1\}$ be a ZHCF defined on G then $\frac{|f|}{2} = \gamma'(G)$.

Proof: It is trivially true for $n = 4$ and $\gamma'(G) = 2$. Decompose the graph G into two subgraphs G_1 and G_2 such that G_1 is a complete graph of n-2 vertices and G_2 is a graph of n vertices and $5+4r$ edges $r = 0, 1, 2, \ldots\ldots m$. It can be show that G_2 is connected and symmetric by an edge say $e_i \in G_2$ such that e_i is adjacent to all other edges in G_2 (and no edge in G_2 not adjacent to $e_i \in G_2$). For $n = 6$, G_1 of order n-2 has arc domination number 2 and the arc domination number G_2 of order n and $5+4r$ edges has 1 for $r = 1$. For $n = 8$, G_1 of order n-2 has arc domination number 3 and the arc domination number G_2 of order n and $5+4r$ edges has 1 for $r = 2$. Similarly it can be prove for all $n > 8$ (n is even).

REFERENCES

1. S. Arumugam, K.Karuppasamy and I. Sahul Hamid, *Fractional Global Domination In Graphs*, Discussiones Mathematicae, Graph Theory 30 (2010) 33-44.
2. D. W. Bange, A. E. Barkauskas, and P. J. Slater, *Efficient dominating sets in graphs*, in R. D. Ringeisen and F. S. Roverts (Eds.) Applications of Discrete Mathematics, pages 189-199, SIAM, Philadelphia, PA, 1988.
3. J.A. Bondy and U.S.R. Murty (Eds) *Graph Theory with Application* (The McMillan press Ltd, London, 1976).
4. Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater (Eds) *Fundamentals of Domination in Graphs* (Marcel Dekker, New York, 1998).
5. Lutz Volkmann, *Signed Domination and Signed Domatic Numbers of Digraphs*, Discussiones Mathematicae, Graph Theory 31 (2011) 415-427.
6. K. Muthu Pandian and K.Kamaraj, *Fractional Twin Domination Number in Digraphs*, International Journal of Mathematical Archive-3 (6) (2012) 2375-2379.

Source of support: Nil, Conflict of interest: None Declared