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b-H-OPEN SETS AND DECOMPOSITION OF CONTINUITY VIA HEREDITARY CLASSES

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ABSTRACT

In this paper, we introduce and study the notions of b-H-open sets and (b_H, λ)-continuity in hereditary generalized topological spaces. We also find the decomposition of (μ, λ)-continuity and (σ_H, λ) -continuity.

Keywords: *generalized topology, Hereditary classesand b-H-open.*

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1. INTRODUCTION AND PRELIMINARIES

A family μ of subsets of X is called a generalized topology (GT)[1], if $\emptyset \in \mu$ and μ is closed under arbitrary union. The generalized topology μ is said to be strong [7], if $X \in \mu$. A hereditary class H of X is a non-empty collection of subsets of X such that $A \subset B$, $B \in H$ implies $A \in H$ [2]. If μ is a GT on X and $A \subset X$, $x \in X$ then $x \in A_\mu^*$[2] iff $x \in M \in \mu \Rightarrow M \cap A \notin H$. A function f: (X, μ, H) $\rightarrow$ (Y, λ) is called (μ, λ) - continuous [1] (resp. (α_H, λ) -continuous [9] , (π_H, λ) -continuous[9], (σ_H, λ) -continuous[9], (δ_H, λ) --continuous [8]) if the inverse image of each λ -open set in Y is μ-open (resp. α - H -open π - H -open, σ - H -open, δ - H -open) .

Definition 1.1: A subset A of a hereditary generalized topological space (X, μ, H) is said to be

(a) α - H –open [2] if $A \subseteq i_\mu (c_\mu^*(i_\mu (A)))$,
(b) β - H –open [2] if $A \subseteq c_\mu(i_\mu (c_\mu^*(A)))$,
(c) σ - H –open [2] if $A \subseteq c_\mu^*(i_\mu (A))$,
(d) π - H –open [2] if $A \subseteq i_\mu (c_\mu^*(A))$,
(e) δ - H –open [2] if $i_\mu (c_\mu^*(A)) \subseteq c_\mu^*(i_\mu (A))$,
(f) strongβ - H -open [2] if $A \subseteq c_\mu^*(i_\mu (c_\mu^*(A)))$,
(g) S - H -set [10] if $i_\mu (A) = c_\mu^*(i_\mu (A))$,
(h) t - H -set [10] if $i_\mu (c_\mu^*(A)) = i_\mu (A)$,
(i) B_H -set [10] if $A = U \cap V$, where $U \in \mu$ and V is t - H -set.

Lemma 1.2: [6] Let (X, μ, H) be a hereditary generalized topological space and A, B be subsets of X. Then the following holds:

(a) If $A \subseteq B$, then $A^* \subseteq B^*$.
(b) If $G \in \mu$, then $G \cap A^* \subseteq (G \cap A)^*$.
(c) $A^* = c_\mu(A^*) \subseteq c_\mu(A)$.

2. b- H-OPEN SETS

Definition 2.1: A subset A of a hereditary generalized topological space (X, μ, H) is said to be b - H- open, if $A \subseteq i_\mu(c^*_\mu (A)) \cup c^*_\mu(i_\mu (A))$.

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Proposition 2.2: Let (X, μ, H) be a hereditary generalized topological space. Then the following holds:
(a) Every σ - H -open set is b - H -open.
(b) Every π - H -open set is b - H -open.
(c) Every b - H -open set is strong β - H -open.
(d) Every b - H -open set is β - H -open.

Proof:
(a) Let A be σ - H -open. Then $A \subseteq c^*_\mu(i_\mu(A)) \subseteq i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A))$. Hence A is b - H–open.
(b) Let A be π - H -open. Then $A \subseteq i_\mu(c_\mu^*(A)) \subseteq i_\mu(c_\mu^*(A)) \cup c_\mu^*(i_\mu(A))$. Hence A is b - H -open.
(c) Let A be b-H-open.
Then $A \subset i_\mu(c_\mu^*(A)) \cup c_\mu^*(i_\mu(A)) \subset c_\mu^*(i_\mu(c_\mu^*(A))) \cup c^*_\mu(i_\mu(A)) = c^*_\mu(i_\mu(c^*_\mu(A)) \cup i_\mu(A)) \subset c^*_\mu(i_\mu(c^*_\mu(A) \cup A)) = c^*_\mu(i_\mu(c^*_\mu(A)))$. Hence A is strong β - H –open.
(d) Let A be b - H -open.
Then $A \subset i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A)) \subset c_\mu(i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A))) = c_\mu(i_\mu(c^*_\mu(A))) \cup c_\mu(c^*_\mu(i_\mu(A))) = c_\mu(i_\mu(c^*_\mu(A))) \cup c_\mu((i_\mu(A))^* \cup i_\mu(A)) \subset c_\mu(i_\mu(c^*_\mu(A))) \cup c_\mu(c_\mu(i_\mu(A)) \cup i_\mu(A)) = c_\mu(i_\mu(c^*_\mu(A))) \cup c_\mu(i_\mu(A)) \subset c_\mu(i_\mu(c^*_\mu(A)))$. Hence A is β - H -open.

Remark 2.3: The following examples show that the converse of Proposition 2.2 need not be true.

Example 2.4: Let X = {a, b, c, d}, μ = {∅, {b},{b, c},{c, d},{b, c, d}} and H = {∅,{a},{b}}. Then
1. A = {c} is b - H -open set but not σ - H -open.
2. A = {a, b} is b - H -open set but not π - H -open.
3. A = {a, c} is β - H -open but not b - H -open.

Theorem 2.5: Let (X, μ, H) be a hereditary generalized topological space and A ⊆ X. If A is b - H -open and S - H -set, then A is π - H -open.

Proof: If A is S - H -set, then $i_\mu(A) = c_\mu^*(i_\mu(A))$. Since A is b - H -open, then
$A \subseteq i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A)) = i_\mu(c^*_\mu(A)) \cup i_\mu(A) = i_\mu(A \cup A^*) \cup i_\mu(A) \subseteq i_\mu(A \cup A^*) = i_\mu(c^*_\mu(A))$. Hence A is π - H -open.

Theorem 2.6: Let (X, μ, H) be a hereditary generalized topological space and A ⊆ X. If A is b - H -open and t - H -set, then A is σ - H -open.

Proof: If A is t - H -set, then $i_\mu(A) = i_\mu(c_\mu^*(A))$. Since A is b - H -open, then
$A \subseteq i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A)) = i_\mu(A) \cup c^*_\mu(i_\mu(A)) = i_\mu(A) \cup (i_\mu(A) \cup (i_\mu(A))^*) = i_\mu(A) \cup (i_\mu(A))^* = c^*_\mu(i_\mu(A))$. Hence A is σ - H -open.

Proposition 2.7: Let (X, μ, H) be a hereditary generalized topological space. Then the following are equivalent.
(a) Every β - H -open set is σ - H -open.
(b) Every b - H -open set is σ - H -open.
(c) Every π - H -open set is σ - H -open.

Proof: It follows from Proposition 2.2.

Proposition 2.8: Let (X, μ, H) be a hereditary generalized topological space. Then arbitrary union of b - H-open sets is b - H -open.

Proof: Let U_α be b - H -open for $\alpha \in \Delta$, we have $U_\alpha \subseteq i_\mu(c_\mu^*(U_\alpha)) \cup c_\mu^*(i_\mu(U_\alpha))$. Then Lemma 1.2. we have
$\cup_{\alpha\in\Delta} U_\alpha \subseteq \cup_{\alpha\in\Delta} (c_\mu^*(i_\mu(U_\alpha)) \cup i_\mu(c_\mu^*(U_\alpha))) = \cup_{\alpha\in\Delta} ((i_\mu(U_\alpha) \cup (i_\mu(U_\alpha))^*) \cup i_\mu(U_\alpha \cup U_\alpha^*)) \subseteq ((i_\mu(\cup_{\alpha\in\Delta} U_\alpha) \cup (\cup_{\alpha\in\Delta}(i_\mu(U_\alpha)))^*) \cup (i_\mu(\cup_{\alpha\in\Delta} U_\alpha) \cup (\cup_{\alpha\in\Delta} U_\alpha)^*)) \subseteq ((i_\mu(\cup_{\alpha\in\Delta} U_\alpha) \cup (i_\mu(\cup_{\alpha\in\Delta}(U_\alpha)))^*)) \cup (i_\mu(\cup_{\alpha\in\Delta} U_\alpha) \cup (\cup_{\alpha\in\Delta} U_\alpha)^*)) \subseteq i_\mu(c^*_\mu(\cup_{\alpha\in\Delta})) \cup c^*_\mu(i_\mu(\cup_{\alpha\in\Delta}))$. Hence $\cup_{\alpha\in\Delta} U_\alpha$ is b - H –open.

Proposition 2.9: Let (X, μ, H) be a hereditary generalized topological space and A, B be subsets of X . If A is b - H -open and B is μ -open, then A∩ B is b - H -open.

Proof: If A is b - H -open, then $A \subseteq i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A))$ and $A \cap B \subseteq (i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A))) \cap B = (i_\mu(c^*_\mu(A)) \cap B) \cup (c^*_\mu(i_\mu(A)) \cap B) = (i_\mu(A \cup A^*) \cap B) \cup (((i_\mu(A))^* \cup i_\mu(A)) \cap B) = (i_\mu(A \cup A^*) \cap i_\mu(B)) \cup (((i_\mu(A))^* \cap B) \cup (i_\mu(A) \cap B)) \subseteq (i_\mu((A \cup A^*) \cap B)) \cup ((i_\mu(A \cap B))^* \cup i_\mu(A \cap B)) = (i_\mu((A \cap B) \cup (A^* \cap B))) \cup ((i_\mu(A \cap B))^* \cup i_\mu(A \cap B)) \subseteq i_\mu((A \cap B) \cup (A \cap B)^*) \cup c^*_\mu(i_\mu(A \cap B)) = i_\mu(c^*_\mu(A \cap B)) \cup c^*_\mu(i_\mu(A \cap B))$. Hence A ∩ B is b - H –open.

Remark 2.10: The following examples show that the intersection of two b - H -open sets need not be b - H -open.

Example 2.11: Let X = {a, b, c, d}, μ = {∅,{b},{b, c},{c, d},{b, c, d}} and H = {∅,{a},{b}}.

Consider A = {a, b} and B = {a, c, d} are b - H -open sets, but A∩ B = {b} is not b - H -open set.

Proposition 2.12: Let (X, μ, H) be a hereditary generalized topological space and A ⊂ X. Then the following are equivalent:

(a) A is σ - H -open.
(b) A is both b - H -open and δ - H -open.

Proof:
(a) ⇒ (b): If A is σ - H –open, then $A \subset c_\mu^*(i_\mu(A))$. Now $i_\mu(c_\mu^*(A)) \subset c^*_\mu(A) \subset c^*_\mu(c^*_\mu(i_\mu(A))) \subset c^*_\mu(i_\mu(A))$. Hence A is δ - H -open. Obviously A is b - H - open.

(b) ⇒ (a): If A is b - H -open and δ - H -open, then $A \subset i_\mu(c_\mu^*(A)) \cup c_\mu^*(i_\mu(A))$ and $i_\mu(c_\mu^*(A)) \subset c_\mu^*(i_\mu(A))$, therefore $A \subset c_\mu^*(i_\mu(A))$. Hence A is σ - H -open.

Remark 2.13: The following example shows that the notions b - H -open and δ - H -open are independent.

Example 2.14: Let X = {a, b, c, d}, μ = {∅, {b}, {b, c}, {c, d}, {b, c, d}} and H = {∅, {a}, {b}}. Then

(a) A = {c} is b - H -open but not δ - H -open.
(b) A = {a} is δ - H -open but not b - H -open.

Proposition 2.15: Let (X, μ, H) be a hereditary generalized topological space and x ∈X. Then {x} is μ -open if and only if {x} is σ - H –open.

Proof: Let {x} be a μ -open. Then $\{x\} = i_\mu(\{x\}) \subseteq c_\mu^*(i_\mu(\{x\}))$. Hence {x} is σ - H -open. Conversely, assume that {x} is σ - H –open. Then $\{x\} \subseteq c_\mu^*(i_\mu(\{x\}))$. Now $i_\mu(\{x\})$ is either {x} or ∅. We have $c_\mu^*(\emptyset) = \emptyset$, but $\{x\} \subseteq c_\mu^*(i_\mu(\{x\}))$, so $i_\mu(\{x\}) \neq \emptyset$. Hence $i_\mu(\{x\}) = \{x\}$. Thus {x} is μ -open.

Lemma 2.16: Let (X, μ, H) be a hereditary generalized topological space, A ⊆ X and U ∈ μ. If A∩U = ∅, then $c^*_\mu(A) \cap U = \emptyset$.

Proposition 2.17: Let (X, μ, H) be a hereditary generalized topological space and let x ∈ X. Then the following are equivalent:

(a) {x} is π - H –open.
(b) {x} is b - H –open.
(c) {x} is strong β - H -open.

Proof:
(a) ⇒ (b) and **(b) ⇒ (c)** follows from proposition 2.2.

(c) ⇒ (a): Assume that {x} is strong β - H -open and {x} is not π - H -open. Then $\{x\} \not\subset i_\mu(c^*_\mu(\{x\}))$, that is, $\{x\} \cap i_\mu(c^*_\mu(\{x\})) = \emptyset$. We have $i_\mu(c^*_\mu(\{x\}))$ is μ -open, it follows from Lemma 2.15, $c^*_\mu(\{x\}) \cap i_\mu(c^*_\mu(\{x\})) = \emptyset$ and thus $i_\mu(c^*_\mu(\{x\})) = \emptyset$. Therefore $c^*_\mu(i_\mu(c^*_\mu(\{x\}))) = \emptyset$. But {x} is strong β - H -open, a contradiction. Hence {x} is π - H -open.

Proposition 2.17: Let (X, μ, H) be a hereditary generalized topological space and let x ∈ X. Then the following are equivalent:

(a) {x} is π - H –open.
(b) {x} is b - H –open.
(c) {x} is strong β - H -open.

Proof:
(a) ⇒ (b) and (b) ⇒ (c) follows from proposition 2.2.

(c) ⇒ (a): Assume that {x} is strong β - H -open and {x} is not π - H -open. Then $\{x\} \not\subset i_\mu(c^*_\mu(\{x\}))$, that is,
$\{x\} \cap i_\mu(c^*_\mu(\{x\})) = \emptyset$. We have $i_\mu(c^*_\mu(\{x\}))$ is μ -open, it follows from Lemma 2.15, $c^*_\mu(\{x\}) \cap i_\mu(c^*_\mu(\{x\})) = \emptyset$ and thus $i_\mu(c^*_\mu(\{x\})) = \emptyset$. Therefore $c^*_\mu(i_\mu(c^*_\mu(\{x\}))) = \emptyset$. But {x} is strong β - H -open, a contradiction. Hence {x} is π - H -open.

Proposition 2.18: Let (X, μ, H) be a hereditary generalized topological space and $A \subseteq X$ such that $(i_\mu(A^*))^* \subseteq i_\mu(A^*)$. Then the following are equivalent:

(a) $A \subseteq i_\mu(A^*)$.
(b) A is b - H -open and $A \subseteq A^*$.

Proof:
(a) ⇒ (b): If $A \subseteq i_\mu(A^*) \subseteq A^*$. Since $A \subseteq i_\mu(A^*) \subset i_\mu(A^*) \cup i_\mu(A) \subseteq i_\mu(A^* \cup A) = i_\mu(c^*_\mu(A)) \subseteq i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A))$. Then A is b - H -open.

(b) ⇒ (a): If A is a b - H -open and $A \subseteq A^*$, then $A \subseteq i_\mu(c^*_\mu(A)) \cup c^*_\mu(i_\mu(A)) = i_\mu(A \cup A^*) \cup (i_\mu(A) \cup (i_\mu(A))^*) \subseteq (i_\mu(A^*) \cup i_\mu(A)) \cup (i_\mu(A))^* = i_\mu(A^*) \cup (i_\mu(A))^* = i_\mu(A^*)$.

Definition 2.19: A subset A of a hereditary generalized topological space (X, μ, H) is said to be b-H-closed if its complement is b - H -open.

Theorem 2.20: Let (X, μ, H) be a hereditary generalized topological space and $A \subseteq X$. If A is b - H -closed, then $i_\mu(c^*_\mu(A)) \cap c^*_\mu(i_\mu(A)) \subseteq A$.

Proof: If A is b - H -closed, then X −A is b - H -open. We have
$X - A \subseteq c^*_\mu(i_\mu(X - A)) \cup i_\mu(c^*_\mu(X - A)) \subseteq c_\mu(i_\mu(X - A)) \cup i_\mu(c_\mu(X - A)) = (X - (i_\mu(c_\mu(A)))) \cup (X - (c_\mu(i_\mu(A)))) \subseteq (X - (i_\mu(c^*_\mu(A)))) \cup (X - (c^*_\mu(i_\mu(A)))) = X - ((i_\mu(c^*_\mu(A))) \cup c^*_\mu(i_\mu(A)))$. Hence $i_\mu(c^*_\mu) \cup c^*_\mu(i_\mu(A)) \subseteq A$.

Remark 2.21: The following example shows that the converse of theorem 2.20 need not be true.

Example 2.22: Let X = {a, b, c, d}, μ = {∅,{b},{b, c},{c, d},{b, c, d}} and H = {∅,{a},{b}}. Let A = {c}, then
$i_\mu(c^*_\mu(A)) \cap c^*_\mu(i_\mu(A)) \subseteq A$ but A is not b - H –closed.

Corollary 2.23: Let (X, μ, H) be a hereditary generalized topological space and $A \subseteq X$ such that $X - i_\mu(c^*_\mu(A)) = c^*_\mu(i_\mu(X - A))$ and $X - c^*_\mu(i_\mu(A)) = i_\mu(c^*_\mu(X - A))$. Then A is b - H -closed if and only if $i_\mu(c^*_\mu(A)) \cap c^*_\mu(i_\mu(A)) \subseteq A$.

Proof: By theorem 2.20, if A b - H -closed, then $i_\mu(c^*_\mu(A)) \cap c^*_\mu(i_\mu(A)) \subseteq A$. Conversely, if $i_\mu(c^*_\mu(A)) \cap c^*_\mu(i_\mu(A)) \subseteq A$, then $X - A \subseteq X - (i_\mu(c^*_\mu(A)) \cap c^*_\mu(i_\mu(A))) \subseteq (X - i_\mu(c^*_\mu(A))) \cup (X - c^*_\mu(i_\mu(A))) = c^*_\mu(i_\mu(X - A)) \cup i_\mu(c^*_\mu(X - A))$. Therefore, X − A is b - H -open and hence A is b - H -closed.

Definition 2.24: A subset A of a hereditary generalized topological space (X, μ, H) is said to be strong B_H -set if $A = U \cap V$, where $U \in \mu$ and V is a t - H -set and $i_\mu(c^*_\mu(V)) = c^*_\mu(i_\mu(V))$

Proposition 2.25: Let (X, μ, H) be a hereditary generalized topological space and $A \subseteq X$. If A is a strong B_H -set, then A is B_H -set.

Proof: Obvious.

Remark 2.26: The following example shows that the converse of proposition 2.25 need not be true.

Example 2.27: Let X = {a, b, c, d}, μ = {∅, {b}, {b, c}, {c, d}, {b, c, d}} and H = {∅, {a}, {b}}. A = {b} is B_H -set but not strong B_H -set.

Proposition 2.28: Let (X, μ, H) be a strong hereditary generalized topological space and $A \subseteq X$. Then the following are equivalent.

(a) A is μ -open.
(b) A is b - H -open and a strong B_H -set.

Proof:
(a) ⇒ (b): Clearly every μ -open set is b - H -open. Now every μ -open set is strong B_H -set, because X is t - H -set and $i_\mu(c^*_\mu(X)) = c^*_\mu(i_\mu(X))$.

(b) ⇒ (a): If A is b - H -open and strong B_H -set, then $A \subseteq i_\mu(c_\mu^*(A)) \cup c_\mu^*(i_\mu(A)) = i_\mu(c_\mu^*(U \cap V)) \cup c_\mu^*(i_\mu(U \cap V))$, where U is μ -open and V is t - H - set and $i_\mu(c^*_\mu(V)) = c^*_\mu(i_\mu(V))$. Hence $A \subseteq (i_\mu(c^*_\mu(U)) \cap i_\mu(c^*_\mu(V))) \cup (c^*_\mu(i_\mu(U)) \cap c^*_\mu(i_\mu(V))) \subseteq U \cap (i_\mu(c^*_\mu(V)) \cup c^*_\mu(i_\mu(V))) = U \cap i_\mu(c^*_\mu(V)) = U \cap i_\mu(V) = i_\mu(U \cap V) = i_\mu(A)$. Hence A is μ -open.

Remark 2.29: The following examples show that the notions of b - H -open and strong B_H -set are independent.

Example 2.30: Let X = {a, b, c, d}, μ = {∅, {b}, {b, c}, {c, d}, {b, c, d}} and H = {∅, {a}, {b}}.
1. A = {d} is strong B_H -set but not b - H -open.
2. A = {a, b} is b - H -open but not strong B_H -set.

3. DECOMPOSITION OF (μ, λ) -CONTINUITY AND (σ_H, λ) - CONTINUITY

Definition 3.1: A function f: (X, μ, H) → (Y, λ) is called (b_H, λ) -Continuous function, if the inverse image of each λ -open set in Y is b - H -open in X.

Proposition 3.2: If a function f: (X, μ, H) → (Y, λ) is either (σ_H, λ) -continuous or (π_H, λ) -continuous, then f is (b_H, λ) -continuous.

Proof: Obvious.

Remark 3.3: The following example shows that the converse of Proposition 3.2 need not be true.

Example 3.4: Let X = Y= {a, b, c, d}, μ = λ = {∅, {a}, {c}, {b, d}, {a, c}, {a, b, c}, X} and H = {∅, {a}, {d}, {a, d}}. Define f: (X, μ, H) →(Y, λ) by f (a) = a, f (b). Then f is (b_H, λ) -continuous but not strong B_H -continuous. In Example 3.8 f is strong B_H–continuous but not (b_H, λ) –continuous. Since f^{-1}({b, d}) = {b, c} is b - H -open but it is neither σ - H -open nor π - H –open.

Definition 3.5: A function f: (X, μ, H) → (Y, λ) is B_H -continuous (resp. strong B_H -continuous) if the inverse image of λ -open set in Y is B_H -set (resp. strong B_H -set) in X.

Theorem 3.6: If a function f: (X, μ, H) → (Y, λ) is strong B_H -continuous, then f is B_H -continuous.

Proof: It follows from Proposition 2.25.

Remark 3.7: The following example shows that the converse of theorem 3.6 need not be true.

Example 3.8: Let X = {a, b, c}, μ = {∅, {a, c}} and H= {∅, {a}, {b}}. Also, let Y = X and λ = {∅, {a {a, c}}.
Define f: (X, μ, H)→ (Y, λ) by f (a) = a, f (b) = c and f (c) = b. Then f is strong B_H -continuous but not BH -continuous.

Proposition 3.9: Let (X, μ, H) be a strong hereditary generalized topological space. For a function f: (X, μ, H)→ (Y, λ), the following are equivalent:
(a) f is (μ, λ) -continuous,
(b) f is (b_H, λ) -continuous and strong B_H -continuous.

Proof: This is an immediate consequence from Proposition 2.28.

Remark 3.10: The following examples show that the notions of (b_H, λ)-continuous and B_H -continuous are independent.

Example 3.11: Let X = {a, b, c, d}, μ = {∅, {c}, {a, b}, {a, b, c}, {b, c, d}, X} and H = {∅, {d}}. Also, let Y = {a, b, c} and λ = {∅, {a}, {b}, {a, b}}. Define a function f: (X, μ, H)→ (Y, λ) such that f (a) = f (b) = a, f (c) = f (d) = b. Then f is (b_H, λ) -continuous but not strong B_H -continuous. In Example 3.8 f is strong B_H–continuous but not (b_H, λ) –continuous.

Proposition 3.12: Let (X, μ, H) be a hereditary generalized topological space. For a function f: (X, μ, H) → (Y, λ), the following are equivalent:
(a) f is (σ_H, λ) –continuous.
(b) f is (b_H, λ) -continuous and (δ_H, λ) -continuous.

Proof: This is an immediate consequence from Proposition 2.12.

Remark 3.13: The notions of (δ_H, λ) -continuous and (b_H, λ) -continuous are independent as shown in the following examples.

Example 3.14: Let X = {a, b, c, d}, μ = {∅, {a, b}, {b, d}, {a, b, c}, X} and H = {∅, {a}, {b}, {a, b}}. Also let Y= X and λ = {∅, {a}, {c}, {b, d}, {a, c}, {a, b, c}, X}. Define function f: (X, μ, H) → (Y, λ) such that f (a) = a, f (b) = f (c) = b. Then f is (δ_H, λ) -continuous but neither (b_H, λ) –continuous nor (σ_H, λ) - continuous.

Example 3.15: Let X = {a, b, c, d}, μ={∅,{c},{b, d},{a, b, c},{b, c, d}, X} and H = {∅, {a}, {b}, {d}, {a, d}}. Let Y = {a, b, c}, λ = {∅, {a, c}, {b, c}, {a, b, c}}. Define the identity function f: X → Y is (b_H, λ) -continuous but it is neither (δ_H, λ) -continuous nor (σ_H, λ) -continuous.

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