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SUPER EDGE TRIMAGIC TOTAL LABELING OF SOME DISCONNECTED GRAPHS

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ABSTRACT

An edge trimagic total labeling of a (p, q) graph G is a bijection f: $V(G)\cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u)+f(uv)+f(v)$ is equal to either k_1 or k_2 or k_3. In this paper we prove that the disconnected graphs $(C_m \odot K_1)\cup P_n$, $(C_m \odot K_1)\cup C_n$ and $P_m \cup P_n \cup P_r$ admit edge trimagic total labeling and super edge trimagic total labeling.

Keywords: *Function, Bijection, Labeling, Magic, Trimagic.*

AMS Subject Classification: *05C78.*

1. INTRODUCTION

A Graph labeling is an assignment of integers to the elements of a graph, the vertices or edges or both subject to certain conditions. In 1967 Rosa introduced the concept of graph labeling. In 1970, Kotzig and Rosa[7] defined, the magic labeling of graph G is a bijection f: $V\cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ is a magic constant. W. D. Wallis [8] introduced this as edge magic total labeling. J. Baskar Babujee introduced the bimagic labeling of graphs in 2004[1]. In 2013, C. Jayasekaran, M. Regees and C. Davidraj introduced the edge trimagic total labeling of graphs [4]. M. Regees and C. Jayasekaran proved that some classes and families of graphs are edge trimagic total [5, 6]. Some definitions relevant to this paper are given below.

Definition 1.1: [4] An edge trimagic total labeling of a (p, q) graph G is a bijective function f: $V(G)\cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge $xy \in E(G)$, the value of $f(x)+f(xy)+f(y)$ is equal to any of the distinct constants k_1 or k_2 or k_3. A graph G is said to be edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling is called a super edge trimagic total labeling if G has the additional property that the vertices are labeled with smallest positive integers.

Definition 1.2: The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1\cup G_2$ with vertex set $V = V_1\cup V_2$ and the edge set $E = E_1\cup E_2$.

Definition 1.4: [4] If G is of order n, the corona of G with H, $G\odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with every vertex in the i^{th} copy of H.

The dynamic survey of graph labeling by J.A.Gallian[3] can be used for further references. The notations and terminology are taken from [2]. This paper prove that the graphs $(C_m\odot K_1)\cup P_n$, $(C_m\odot K_1)\cup C_n$ and $P_m\cup P_n\cup P_r$ are edge trimagic total and super edge trimagic total.

2. MAIN RESULTS

Theorem 2.1: $(C_m\odot K_1)\cup P_n$ admits an edge trimagic total labeling.

Proof: Let $u_1u_2\ldots u_mu_1$ be the cycle C_m and let v_i be the vertex which is joined to the vertex u_i of the cycle C_m, $1 \le i \le m$. The resultant graph is $C_m\odot K_1$. Let $w_1w_2\ldots w_n$ be the path P_n. Then $(C_m\odot K_1)\cup P_n$ is a disconnected graph with $2m + n$ vertices and $2m + n - 1$ edges.

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Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 4m+2n-1\}$ such that,

Case-1: n is odd.

$f(u_i) = i,\ 1 \le i \le m,\ f(v_i) = m+i,\ 1 \le i \le m,$

$$f(w_i) = \begin{cases} 2m + \frac{i+1}{2} & ,1 \le i \le n \text{ and i is odd} \\ 2m + \frac{n+i+1}{2} & ,1 \le i \le n \text{ and i is even,} \end{cases}$$

$f(u_iu_{i+1}) = 4m+2n-2i-1,\ 1 \le i \le m-1;\ f(u_iv_i) = 4m+2n-2i,\ 1 \le i \le m;$

$f(w_iw_{i+1}) = 2m+2n-i,\ 1 \le i \le n-1$ and $f(u_mu_1) = 4m+2n-1$.

Now we can verify that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ yields any of the trimagic constants $\lambda_1 = 4m+2n$, $\lambda_2 = 5m+2n$ and $\lambda_3 = \frac{12m+5n+3}{2}$.

Case-2: n is even.

$f(u_i) = i,\ 1 \le i \le m,\ f(v_i) = m+i,\ 1 \le i \le m,$

$$f(w_i) = \begin{cases} 2m + \frac{i+1}{2} & ,1 \le i \le n \text{ and i is odd} \\ 2m + \frac{n+i}{2} & ,1 \le i \le n \text{ and i is even,} \end{cases}$$

$f(u_iu_{i+1}) = 4m+2n-2i-1,\ 1 \le i \le m-1;\ f(u_iv_i) = 4m+2n-2i,\ 1 \le i \le m;$

$f(w_iw_{i+1}) = 2m+2n-i,\ 1 \le i \le n-1$ and $f(u_mu_1) = 4m+2n-1$.

Now we can verify that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ yields any of the trimagic constants $\lambda_1 = 4m+2n$, $\lambda_2 = 5m+2n$ and $\lambda_3 = \frac{12m+5n+2}{2}$.

By case 1 and case 2, the graph $(C_m \odot K_1) \cup P_n$ admits an edge trimagic total labeling for all m and n.

Corollary 2.2: The graph $(C_m \odot K_1) \cup P_n$ admits a super edge trimagic total labeling.

Proof: We proved that the graph $(C_m \odot K_1) \cup P_n$ admits an edge trimagic total labeling. The labeling given in the proof of Theorem 2.1, the vertices get labels 1, 2… 2m+n. Since the graph $(C_m \odot K_1) \cup P_n$ has 2m+n vertices and all the vertices are labeled with smallest positive integers, the graph $(C_m \odot K_1) \cup P_n$ admits a super edge trimagic total labeling.

Example 2.3: The super edge trimagic total labeling of $(C_6 \odot K_1) \cup P_7$ is given in figure 1.

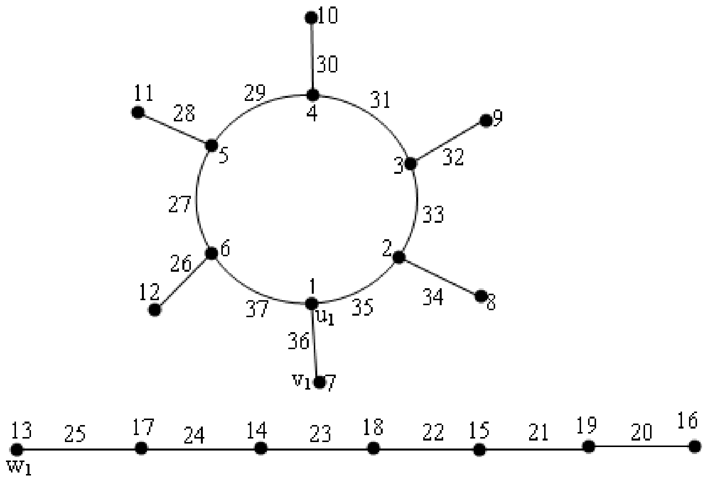

Figure-1: $(C_6 \odot K_1) \cup P_7$ with $\lambda_1 = 38$, $\lambda_2 = 44$ and $\lambda_3 = 55$.

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Theorem 2.4: $(C_m \odot K_1) \cup C_n$ admits an edge trimagic total labeling.

Proof: Let $u_1u_2\ldots u_mu_1$ be the cycle C_m and let v_i be the vertex which is joined to the vertex u_i of the cycle C_m, $1 \le i \le m$. The resultant graph is $C_m \odot K_1$. Let $w_1w_2\ldots w_nw_1$ be the cycle C_n. Then $G = (C_m \odot K_1) \cup C_n$ is a disconnected graph with 2m + n vertices and 2m + n edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 4m+2n\}$ such that,

Case-1: n is odd.

$f(u_i) = i, 1 \le i \le m$, $f(v_i) = m+ i, 1 \le i \le m$,

$$f(w_i) = \begin{cases} 2m + \frac{i+1}{2}, & 1 \le i \le n \text{ and i is odd} \\ 2m + \frac{n+i+1}{2}, & 1 \le i \le n \text{ and i is even,} \end{cases}$$

$f(u_iu_{i+1}) = 4m+2n-2i, 1 \le i \le m-1$; $f(u_iv_i) = 4m+2n-2i+1, 1 \le i \le m$;

$f(w_iw_{i+1}) = 2m+2n - i, 1 \le i \le n - 1$, $f(u_mu_1) = 4m+2n$ and $f(w_nw_1) = 2m+2n$.

Now we can verify that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ yields any of the trimagic constants $\lambda_1 = 4m+2n+1$, $\lambda_2 = 5m+2n+1$ and $\lambda_3 = \frac{12m+5n+3}{2}$.

Case-2: n is even.

$f(u_i) = i, 1 \le i \le m$, $f(v_i) = m+ i, 1 \le i \le m$,

$$f(w_i) = \begin{cases} 2m + \frac{i+1}{2}, & 1 \le i \le n \text{ and i is odd} \\ 2m + \frac{n+i}{2}, & 1 \le i \le n \text{ and i is even,} \end{cases}$$

$f(u_iu_{i+1}) = 4m+2n-2i, 1 \le i \le m-1$; $f(u_iv_i) = 4m+2n-2i+1, 1 \le i \le m$;

$f(w_iw_{i+1}) = 2m+2n - i, 1 \le i \le n - 1$, $f(u_mu_1) = 4m+2n$ and $f(w_nw_1) = 2m+2n$.

Now we can verify that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ yields any of the trimagic constants $\lambda_1 = 4m+2n+1$, $\lambda_2 = 5m+2n+1$ and $\lambda_3 = \frac{12m+5n+2}{2}$.

Example 2.5: The super edge trimagic total labeling of $(C_6 \odot K_1) \cup C_5$ is given in figure 2.

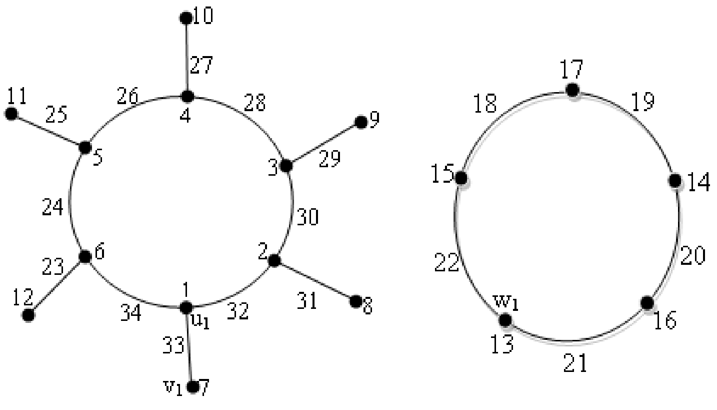

Figure-2: $(C_6 \odot K_1) \cup C_5$ with $\lambda_1 = 35$, $\lambda_2 = 41$ and $\lambda_3 = 50$.

Corollary 2.6: The graph $(C_m \odot K_1) \cup C_n$ admits a super edge trimagic total labeling.

Proof: We proved that the graph $(C_m \odot K_1) \cup C_n$ admits an edge trimagic total labeling. The labeling given in the proof of Theorem 2.4, the vertices get labels 1, 2, …, 2m+n. Since the graph $(C_m \odot K_1) \cup C_n$ has 2m+n vertices and all the vertices are labeled with smallest positive integers, the graph $(C_m \odot K_1) \cup C_n$ admits a super edge trimagic total labeling.

Theorem 2.7: The graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling.

Proof: Let $V = \{u_i \,/\, 1 \le i \le m\} \cup \{v_j \,/\, 1 \le j \le n\} \cup \{w_k \,/\, 1 \le k \le r\}$ be the vertex set and $E = \{u_iu_{i+1} \,/ 1 \le i \le m-1\} \cup \{v_jv_{j+1}/1 \le j \le n-1\} \cup \{w_kw_{k+1}/1 \le k \le r-1\}$ be the edge set of the graph $P_m \cup P_n \cup P_r$. The disconnected graph $P_m \cup P_n \cup P_r$ has m+n+r vertices and m+n+r–3 edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 2m+2n+2r-3\}$ such that,

For all cases the edge labels are $f(u_iu_{i+1}) = 2m+2n+2r-i-2$, $1 \le i \le m-1$,

$f(v_jv_{j+1}) = m+2n+2r-i-1, 1 \le j \le n-1$ and $f(w_kw_{k+1}) = m+n+2r-k, 1 \le k \le r-1$.

Case-1: m odd, n is even and r odd.

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is odd} \\ \frac{m+i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_j) = \begin{cases} m+\frac{j+1}{2}, & 1 \le j \le n \text{ and } j \text{ is odd} \\ m+\frac{n+j}{2}, & 1 \le j \le n \text{ and } j \text{ is even,} \end{cases}$$

$$f(w_k) = \begin{cases} m+n+\frac{k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is odd} \\ m+n+\frac{r+k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is even,} \end{cases}$$

It is easy to verify that for each edge $uv \in E$, the value of f(u)+f(uv)+f(v) yields any of the trimagic constants $\lambda_1 = \frac{5m+4n+4r-1}{2}$, $\lambda_2 = \frac{6m+5n+4r}{2}$ and $\lambda_3 = \frac{6m+6n+5r+3}{2}$. Therefore, the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for odd m, even n and odd r.

Case-2: m, n and r are odd.

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is odd} \\ \frac{m+i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_j) = \begin{cases} m+\frac{j+1}{2}, & 1 \le j \le n \text{ and } j \text{ is odd} \\ m+\frac{n+j+1}{2}, & 1 \le j \le n \text{ and } j \text{ is even,} \end{cases}$$

$$f(w_k) = \begin{cases} m+n+\frac{k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is odd} \\ m+n+\frac{r+k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is even,} \end{cases}$$

It is easy to verify that for each edge $uv \in E$, the value of f(u)+f(uv)+f(v) yields any of the trimagic constants $\lambda_1 = \frac{5m+4n+4r-1}{2}$, $\lambda_2 = \frac{6m+5n+4r+1}{2}$ and $\lambda_3 = \frac{6m+6n+5r+3}{2}$. Therefore, the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for m , n and r are odd.

Case-3: m, n odd and r even.

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is odd} \\ \frac{m+i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_j) = \begin{cases} m+\frac{j+1}{2}, & 1 \le j \le n \text{ and } j \text{ is odd} \\ m+\frac{n+j+1}{2}, & 1 \le j \le n \text{ and } j \text{ is even,} \end{cases}$$

$$f(w_k) = \begin{cases} m+n+\frac{k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is odd} \\ m+n+\frac{r+k}{2}, & 1 \le k \le r \text{ and } k \text{ is even,} \end{cases}$$

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It is easy to verify that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ yields any of the trimagic constants $\lambda_1 = \frac{5m+4n+4r-1}{2}$, $\lambda_2 = \frac{6m+5n+4r+1}{2}$ and $\lambda_3 = \frac{6m+6n+5r+2}{2}$. Therefore, the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for m, n odd and r even. .

Case-4: m odd and n, r even.

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is odd} \\ \frac{m+i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_j) = \begin{cases} m+\frac{j+1}{2}, & 1 \le j \le n \text{ and } j \text{ is odd} \\ m+\frac{n+j}{2}, & 1 \le j \le n \text{ and } j \text{ is even,} \end{cases}$$

$$f(w_k) = \begin{cases} m+n+\frac{k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is odd} \\ m+n+\frac{r+k}{2}, & 1 \le k \le r \text{ and } k \text{ is even,} \end{cases}$$

It is easy to verify that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ yields any of the trimagic constants $\lambda_1 = \frac{5m+4n+4r-1}{2}$, $\lambda_2 = \frac{6m+5n+4r-1}{2}$ and $\lambda_3 = \frac{6m+6n+5r+2}{2}$. Therefore, the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for m odd and n, r even.

Case-5: m, n and r are even.

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is odd} \\ \frac{m+i}{2}, & 1 \le i \le m \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_j) = \begin{cases} m+\frac{j+1}{2}, & 1 \le j \le n \text{ and } j \text{ is odd} \\ m+\frac{n+j}{2}, & 1 \le j \le n \text{ and } j \text{ is even,} \end{cases}$$

$$f(w_k) = \begin{cases} m+n+\frac{k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is odd} \\ m+n+\frac{r+k}{2}, & 1 \le k \le r \text{ and } k \text{ is even,} \end{cases}$$

It is easy to verify that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ yields any of the trimagic constants $\lambda_1 = \frac{5m+4n+4r-2}{2}$, $\lambda_2 = \frac{6m+5n+4r-1}{2}$ and $\lambda_3 = \frac{6m+6n+5r+2}{2}$. Therefore, the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for m, n and r are even.

Case-6: m, n even and r odd.

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le m \text{ and } i \text{ is odd} \\ \frac{m+i}{2}, & 1 \le i \le m \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_j) = \begin{cases} m+\frac{j+1}{2}, & 1 \le j \le n \text{ and } j \text{ is odd} \\ m+\frac{n+j}{2}, & 1 \le j \le n \text{ and } j \text{ is even,} \end{cases}$$

$$f(w_k) = \begin{cases} m+n+\frac{k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is odd} \\ m+n+\frac{r+k+1}{2}, & 1 \le k \le r \text{ and } k \text{ is even,} \end{cases}$$

It is easy to verify that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ yields any of the trimagic constants $\lambda_1 = \frac{5m+4n+4r-2}{2}$, $\lambda_2 = \frac{6m+5n+4r-1}{2}$ and $\lambda_3 = \frac{6m+6n+5r+3}{2}$. Therefore, the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for m, n even and r odd.

Case-7: m even and n, r odd.

$$f(u_i) = \begin{cases} \frac{i+1}{2}, 1 \le i \le m \text{ and i is odd} \\ \frac{m+i}{2}, 1 \le i \le m \text{ and i is even,} \end{cases}$$

$$f(v_j) = \begin{cases} m+\frac{j+1}{2}, 1 \le j \le n \text{ and j is odd} \\ m+\frac{n+j+1}{2}, 1 \le j \le n \text{ and j is even,} \end{cases}$$

$$f(w_k) = \begin{cases} m+n+\frac{k+1}{2}, 1 \le k \le r \text{ and k is odd} \\ m+n+\frac{r+k+1}{2}, 1 \le k \le r \text{ and k is even,} \end{cases}$$

It is easy to verify that for each edge uv∈E, the value of f(u)+f(uv)+f(v) yields any of the trimagic constants $\lambda_1 = \frac{5m+4n+4r-2}{2}$, $\lambda_2 = \frac{6m+5n+4r+1}{2}$ and $\lambda_3 = \frac{6m+6n+5r+3}{2}$. Therefore, the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for m even and n, r odd.

Case-8: m even, n odd and r even.

$$f(u_i) = \begin{cases} \frac{i+1}{2}, 1 \le i \le m \text{ and i is odd} \\ \frac{m+i}{2}, 1 \le i \le m \text{ and i is even,} \end{cases}$$

$$f(v_j) = \begin{cases} m+\frac{j+1}{2}, 1 \le j \le n \text{ and j is odd} \\ m+\frac{n+j+1}{2}, 1 \le j \le n \text{ and j is even,} \end{cases}$$

$$f(w_k) = \begin{cases} m+n+\frac{k+1}{2}, 1 \le k \le r \text{ and k is odd} \\ m+n+\frac{r+k}{2}, 1 \le k \le r \text{ and k is even,} \end{cases}$$

It is easy to verify that for each edge uv∈E, the value of f(u)+f(uv)+f(v) yields any of the trimagic constants $\lambda_1 = \frac{5m+4n+4r-2}{2}$, $\lambda_2 = \frac{6m+5n+4r+1}{2}$ and $\lambda_3 = \frac{6m+6n+5r+3}{2}$. Therefore, the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for m even and n, r odd.

The above cases prove that the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling for all m, n and r.

Corollary 2.8: The graph $P_m \cup P_n \cup P_r$ admits a super edge trimagic total labeling.

Proof: We proved that the graph $P_m \cup P_n \cup P_r$ admits an edge trimagic total labeling. The labeling given in the proof of Theorem 2.7, the vertices get labels 1, 2, …, m+n+r. Since the graph $P_m \cup P_n \cup P_r$ has m+n+r vertices and all the vertices are labeled with smallest positive integers, the graph $P_m \cup P_n \cup P_r$ admits a super edge trimagic total labeling.

Example 2.9: The super edge trimagic total labeling of $P_8 \cup P_7 \cup P_9$ is given in figure 3**.**

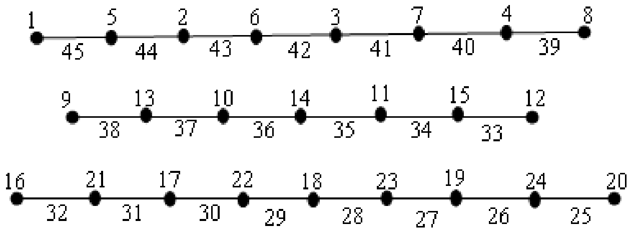

Figure-3: $P_8 \cup P_7 \cup P_9$ with $\lambda_1 = 51$, $\lambda_2 = 60$ and $\lambda_3 = 69$.

3. CONCLUSION

Here we presented some results concerning edge trimagic total labeling and super edge trimagic total labeling for disconnected graphs $(C_m \odot K_1) \cup P_n$, $(C_m \odot K_1) \cup C_n$ and $P_m \cup P_n \cup P_r$. However, there are many graphs which were not been studied. We believe that these results can be extended to vertex trimagic total labeling of graphs.

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