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EFFECT OF MAGNETIC FIELD ON FLOW AND HEAT TRANSFER OF A DUSTY FLUID OVER AN EXPONENTIALLY STRETCHING SURFACE

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ABSTRACT

The boundary layer flow and heat transfer towards an exponential stretching sheet in presence of a magnetic field is presented in this analysis. The velocity and temperature on the surface are assumed to vary with specific exponential forms. Similarity transformations are used to convert the partial differential equations corresponding to the momentum and energy equations into non-linear ordinary differential equations. Numerical solutions of these equations are obtained by Runge-Kutta-Fehlberg fourth-fifth order method. The obtained numerical solutions are compared with the exits results and it shows the influence of different physical parameters on velocity and temperature fields are discussed in detail and are shown graphically.

Keywords: *Magneto hydrodynamics, boundary layer flow, exponentially stretching sheet, dusty fluid, numerical solution.*

AMS Subject Classification (2010): *76T15, 80A20.*

1. INTRODUCTION

Boundary layer flow due to a stretching sheet is very significant due to its huge applications in many manufacturing processes. Such applications include polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass fiber and in metallurgical process like metal extrusion and metal spinning. Few examples of such technological processes are the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the boundary layer along a liquid film in condensation processes. So the study of two-dimensional boundary layer flow and heat transfer over a stretching sheet has gained much interest. In all these cases, the quality of the final product depends on the rate of heat transfer at the stretching surface.

The boundary layer behavior was initially investigated by Sakiadis [15, 16] and encouraged many researchers. Among them Crane [6] has obtained the exact solutions for boundary layer flow caused by stretching surface. The effect of temperature field in flow over a stretching sheet with uniform heat flux was analyzed by Dutta *et. al* [8]. Some of the authors made attempts with non-standard stretching, known as exponential stretching. A new model on exponentially stretching sheet was given by Magyari *et. al* [11] by considering the boundary layers on continuous surface with an exponential temperature distribution. The numerical analysis was investigated by Al-odat *et. al* [1] and they introduce a local similarity solution of an exponentially stretching surface. Bidin *et. al* [5] numerically solved the boundary layer flow problem over an exponentially stretching sheet by considering thermal radiation effect.

Recently Partha *et. al* [13] and Dulal Pal [7] investigated the effect of viscous dissipation and magnetic field on the mixed convection heat transfer in the boundary layers on an exponentially stretching surface respectively. On the other hand, Anuar Ishak [2] carried out a flow due to an exponentially stretching sheet problem under the effect of magnetic and radiation parameter. Later Bikash *et. al* [4] discussed the flow and heat transfer analysis of a third grade fluid past an exponentially stretching sheet with partial slip boundary condition.

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A numerical solutions are obtained by Reddy *et. al* [3] for the effect of thermal radiation on hydro-magnetic flow due to an exponentially stretching sheet. Recently, the radiation effects on hydromagnetic Newtonian liquid flow due to an exponential stretching sheet was discussed by Kameswaran *et. al* [10].

These investigations deals with the flow and heat transfer analysis only for pure fluids induced by stretching sheet. The study of the flow of dusty fluid has an important applications in the fields of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology and fluid droplets sprays. Initially Saffman [14] describes the fluid-particle system and obtained the motion of fluid equations carrying the dust particles. Vajravelu *et. al* [17] obtained the solution for the hydromagnetic flow of a dusty fluid over a stretching sheet. The heat transfer effects on dusty gas flow past a semi-infinite inclined plate was analyzed by Palani *et. al* [12]. Recently, Gireesha *et. al* [9] have discussed the flow and heat transfer analysis of a dusty fluid over a stretching sheet in presence of MHD and viscous dissipation.

The MHD boundary layer flow and heat transfer of a fluid over an exponentially stretching sheet with the effect of viscous dissipation has received little attention. Hence, the main objective of the present chapter is to study the effect of magnetic field and viscous dissipation on flow and heat transfer of a dusty fluid over an exponentially stretching sheet. The governing partial differential equations are converted into non-linear ordinary differential equations by applying similarity transformations. Obtained equations are solved numerically using Runge-Kutta-Fehlberg-45 method with the help of Maple. The governing parameters which are of physical and engineering interest are discussed graphically and tabularly.

2. MATHEMATICAL FORMULATION

Consider a steady two-dimensional laminar boundary layer flow and heat transfer of an incompressible viscous dusty fluid near an impermeable plane wall stretching sheet. It is assumed that an impermeable surface is stretched with exponential velocity $U_w = U_0 e^{x/L}$ in quiescent fluid and the surface is maintained at a temperature $T_\omega = T_\infty + (T_0 - T_\infty)e^{x/2L}$. The $x - axis$ is chosen along the sheet and y-axis normal to it. Two equal and opposite forces are applied along the sheet so that the wall is stretched exponentially. A uniform magnetic field B is assumed to be applied in the y −direction. The geometry of the flow configuration is as shown in the figure 1.

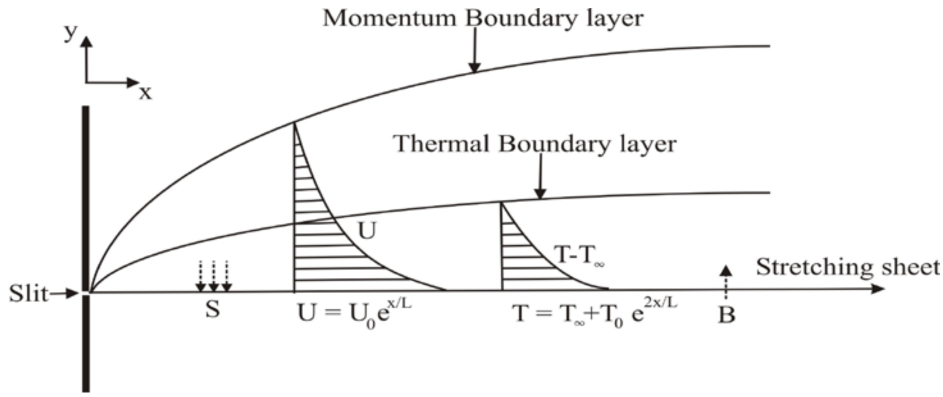

Figure-1: Schematic representation of boundary layer flow.

Under these assumptions, the two dimensional boundary layer equations can be written as,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho}(u_\rho - u) - \frac{\sigma B^2 u}{\rho}, \tag{2.2}$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0, \tag{2.3}$$

$$u_P\frac{\partial u_p}{\partial x} + v_p\frac{\partial v_p}{\partial y} = \frac{K}{m}(u - u_p), \tag{2.4}$$

Where x and y represents coordinate axes along the continuous surface in the direction of motion and perpendicular to it, respectively. (u, v) and (u_p, v_p) denotes the velocity components of the fluid and particle phase along the x and y directions respectively, v is the coefficient of viscosity of fluid, ρ is the density of the fluid phase, K is the Stoke's resistance, N is the number density of dust particles, m is the mass concentration of dust particles, $T_v = m/K$ is the relaxation time of particle phase and ρ is the electrical conductivity.

In order to solve the governing boundary layer equations consider the following appropriate boundary conditions on velocity:

$u = U_w(x), v = 0 \ at\ y = 0,$

$u \to 0, u_p \to 0,\ \ v_p \to v,\ \ as\ y \to \infty,$ (2.5)

where $U_\omega(x) = U_0\, e^{\frac{x}{L}}$ is the sheet velocity, U_0 is reference velocity and L is the reference length.

Equations (2.1) to (2.4) are subjected to boundary conditions (2.5), admits self-similar solutions in terms of the similarity function f and the similarity variable η as

$$u = U_0 e^{\frac{x}{L}} f'(\eta), \qquad v = -\sqrt{\frac{U_0 v}{2L}}\, e^{\frac{x}{2L}}[f(\eta) + \eta f'(\eta)],$$

$$u_p = U_o e^{\frac{x}{L}} F'(\eta), \qquad v_p = -\sqrt{\frac{U_0 v}{2L}}\, e^{\frac{x}{2L}}[F(\eta) + \eta F'(\eta)],$$

$$\eta = \sqrt{\frac{U_0 v}{2L}}\, e^{\frac{x}{2L}} y, \qquad B = B_0 e^{\frac{x}{2L}}, \tag{2.6}$$

Where B_0 is magnetic field flux density.

These equations identically satisfies the governing equation (2.1) and (2.3). Substitute (2.6)into (2.2) and (2.4)and on equating the co-efficient of $\left(\frac{x}{L}\right)^0$ on both sides then one can get

$$f'''(\eta) + f(\eta)f''(\eta) - 2f'(\eta)^2 + 2l\beta[F'(\eta) - f'(\eta)] - Mf'(\eta) = 0, \tag{2.7}$$

$$F(\eta)F''(\eta) - 2F'(\eta)^2 + 2\beta[f'(\eta) - F'(\eta)] = 0, \tag{2.8}$$

where prime denotes the differentiation with respect to η and $l = \frac{mN}{\rho}$ is the mass concentration, $\beta = \frac{L}{T_v U_0}$ is the fluid-particle interaction parameter for velocity and $M = \frac{2\sigma B_0^2 L}{\rho U_0}$ is the magnetic parameter.

Similarity boundary conditions (2.5) will become,

$f'(\eta) = 1,\ \ f(\eta) = 0 \ \ at \ \ \ \eta = 0,$

$f'(\eta) = 0,\ \ F'(\eta) = 0,\ \ F(\eta) = f(\eta + \eta f'(\eta) - \eta F'(\eta) \ \ as\ n \ \to \infty.$ (2.9)

3. HEAT TRANSFER ANALYSIS

The governing steady, dusty boundary layer heat transport equations with viscous dissipation are given by,

$$\rho c_p \left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2} + \frac{Nc_p}{T_T}(T_p - T) + \frac{N}{T_v}(u_p - u)^2 + \mu\left(\frac{\partial u}{\partial y}\right)^2, \tag{3.1}$$

$$Nc_m\left[u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right] = -\frac{Nc_p}{T_T}(T_p - T), \tag{3.2}$$

where T and T_p are the temperatures of the fluid and dust particle inside the boundary layer, c_p and cm are the specific heat of fluid and dust particles, T_T is the thermal equilibrium time i.e., it is time required by a dust cloud to adjust its temperature to the fluid, k is the thermal conductivity and T_v is the relaxation time of the dust particle i.e., the time required by adust particle to adjust its velocity relative to the fluid.

To solve the temperature equations (3.1) and (3.2) we employ the following temperature boundary conditions:

$T = T_\omega(x) \ \ \ at \ \ \ y = 0,$

$T \to T_\infty, \ \ \ T_p \to T_\infty \ \ as\ y \to \infty,$ (3.3)

Where $T_\omega = T_\infty + T_0 e^{2x/L}$ is the temperature distribution is in the stretching surface, T_0 is a reference temperature.

Introduces the dimensionless for the temperature $\theta(\eta)$ and $\theta_p(\eta)$ as follows:

$$\theta(\eta) = \frac{T - T_\infty}{T_\omega - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_\omega - T_\infty}, \tag{3.4}$$

Where $T - T_\infty = T_0 e^{\frac{2x}{L}}\theta(\eta)$.

Using the similarity variable η and (3.9) into (3.1) to (3.2) and on equating the co-efficient of $\left(\frac{X}{L}\right)^0$ on both sides, one can arrive the following system of equations:

$$\theta''(\eta) + \Pr[f(\eta)\theta'(\eta) - 4f'(\eta)\theta(\eta)] + \frac{N}{\rho}\beta_T Pr[\theta p(\eta) - \theta(\eta)] + \frac{N}{\rho}\beta PrEc[F'(\eta) - f'(\eta)]^2 + PrEc[f(\eta)'']^2 = 0, \tag{3.5}$$

$$4F'(\eta)\theta_p(\eta) - F(\eta)\theta_p'(\eta) + 2\gamma\beta_T[\theta_p(\eta) - \theta(\eta) = 0, \tag{3.6}$$

Where $Pr = \frac{\mu C_p}{k}$ Prandtl number, $Ec = \frac{U_0^2}{C_p T_0}$ Eckert number, $\gamma = \frac{C_p}{C_m}$ is the ratio of specific heat, $\beta = \frac{L}{\tau_v U0}$ and $\beta_T = \frac{L}{\tau_T U_0}$ are the fluid-particle interaction parameter for velocity and heat transfer respectively.

Corresponding thermal boundary conditions reduces to,

$\theta(\eta) = 1 \quad at \ \eta = 0,$

$$\theta(\eta) \to 0, \ \ \theta_p(\eta \to 0) \quad as \ \eta \to \infty, \tag{3.7}$$

The important physical parameters for the boundary layer flow are the skin-friction coefficient and heat transfer coefficient. The non-dimensional form of skin-friction at the stretching surface is given by,

$$C_f = -2(Re)^{-\frac{1}{2}} f''(0) \tag{3.8}$$

The rate of heat transfer coefficient in terms of the Nusselt number at the stretching surface is given by,

$$N_u = -(Re)^{-\frac{1}{2}} \theta'(0) \tag{3.9}$$

Where $Re = \frac{U_o L}{v}$ is the Reynolds number.

4. NUMERICAL SOLUTION

The non-linear differential equations (2.7) to (2.8) and (3.5) to (3.6) under the boundary conditions (2.9) and (3.7) have been solved numerically by applying a Runge-Kutta-Fehlberg 45 scheme with the help of Maple software. We have chosen suitable finite values of $\eta \to \infty$ say $\eta = 5$. Results of skin friction coefficient and local Nusselt number for various values of parameters which are tabulated in table 1.

Table-1: Values of skin friction – $f''(0)$ and the rate of heat transfer – $\theta'(0)$.

β	M	Pr	Ec	N	$-f''(0)$	$-\theta'(0)$
0.2					−1.73673	−1.47513
0.6	0.5	0.72	0.5	0.5	−1.7505	−1.46670
1.0					−1.75956	−1.46393
	0.5				−1.75075	−1.46670
0.6	1.0	0.72	0.5	0.5	−1.91137	−1.40026
	1.5				−2.05718	−1.34098
		0.72			−1.75075	−1.46670
0.6	0.5	1.0	0.5	0.5	−1.75075	−1.80740
		1.5			−1.75075	−2.32719
			0		−1.75075	−1.71692
0.6	0.5	0.72	0.5	0.5	−1.75075	−1.46670
			1.0		−1.75075	−1.21647
				0.5	−1.75075	−1.46670
0.6	0.5	0.72	0.5	1.0	−1.75075	−1.58495
				1.5	−1.75075	−1.69174

Table2 provides the values of the skin friction coefficient for different values of the magnetic parameter M. Comparison of our results of $f''(0)$ with those obtained by Kameswaran *et al* [10] in absence of fluid-interaction parameter and Number of dust particles. From the Table 2, one can notice that there is a close agreement with these approaches and thus verifies the accuracy of the method used.

Table-2: Comparison of the results of skin friction coefficient $-f''(0)$ for various values of M with $\beta = N = 0$.

M	Kameswaran et al	Present Study
0	−1.28181	−1.28268
1.0	−1.62918	−1.62919
2.0	−1.91262	−1.91262
3.0	−2.15874	−2.15873
4.0	−2.37937	−2.37936

Further, we study the effect of magnetic field on velocity and temperature profiles and are depicted graphically for different values of fluid-particle interaction parameter β, Magnetic parameter (M), Prandtl number (Pr) and Eckert number (Ec).

5. RESULTS AND DISCUSSION

Numerical calculations are performed for velocity and temperature profiles for various values of physical parameters such as fluid-particle interaction parameter β, magnetic parameter (M), Prandtl number (Pr) and Eckert number (Ec) and are depicted graphically (from figure 2 to 8).

Figure 2 illustrates the effects of fluid-particle interaction parameter β on the velocity profiles $f^{'}(\eta)$ and $F^{'}(\eta)$ with η. It is noticed from the figure that the velocity profiles decreases with increasing values of β for fluid phase and increases for dust phase in the boundary layer. The effect of increasing value of β is to reduce the velocity $f^{'}(\eta)$ and thereby increases the boundary layer thickness as clearly seen from figure *2*. The temperature profiles $\theta(\eta)$ and $\theta_p(\eta)$ versus η for different values of the fluid-particle interaction parameter β is presented in figure *3*. It shows that the temperature increases with increases in β.

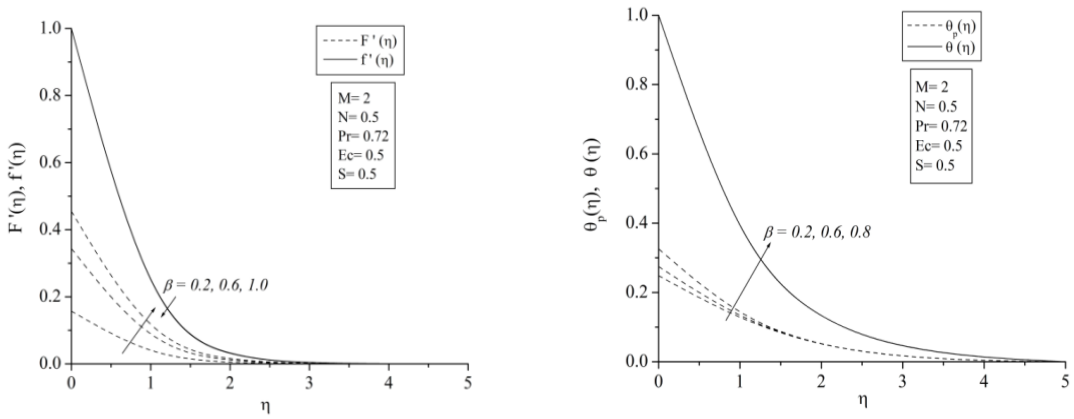

Figure-2 & 3: Effect of fluid-particle interaction parameter on velocity and temperature profiles.

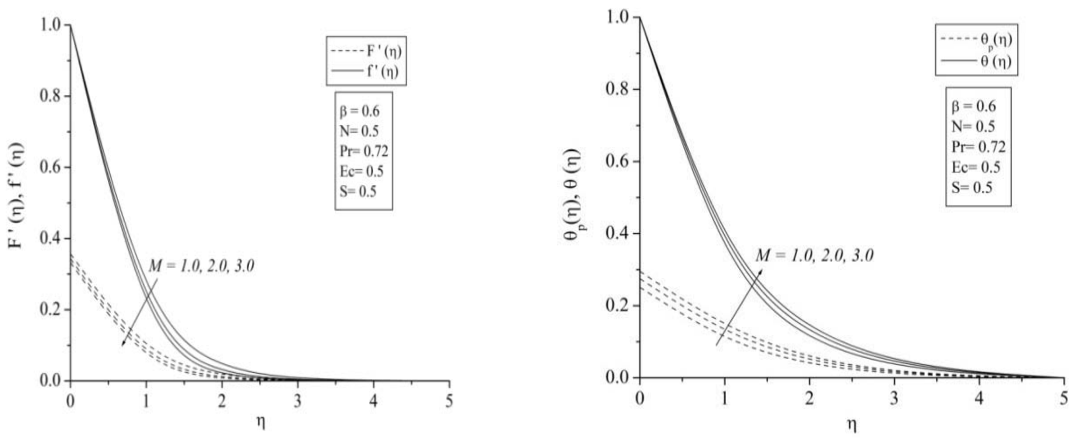

Figure-4 & 5: Effect of magnetic parameter on velocity and temperature profiles.

Further, from Figure *4*, can be seen that the horizontal velocity$f^{'}(\eta), F^{'}(\eta)$ decreases with an increase in the magnetic field parameter M. This is due to the fact that, the introduction of transverse magnetic field (normal to the flow direction) has a tendency to create a drag, known as the Lorentz force which tends to resist the flow. This behavior is even true in the case of increasing values of fluid-particle interaction parameter for fluid phase. Now we discussed the effects of magnetic field parameter M on the temperature profiles $\theta(\eta)$and $\theta_p(\eta)$ and are depicted in figure *5*. We infer from this figure that temperature increases with increases in magnetic field parameter M and also it indicates that both the fluid and dust particle temperature are parallel to each other.

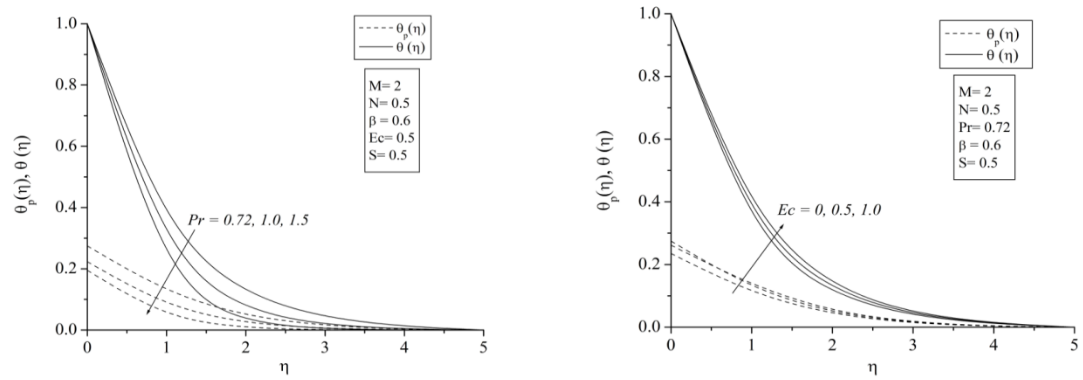

Figure-6 & 7: Effect of Prandtl number and Eckert number on temperature profiles.

The profiles in Figure *6* exhibit the role of Prandtl number on temperature profile $\theta^{'}(n)$ .The effect of increasing values of Pr results in decrease of the temperature distribution and hence thermal boundary layer thickness decreases as Pr increases. Figure 7 explains the effect of Eckert number Ec on temperature profiles with η. From this one can see that the temperature increases with increase in the value of Ec. This is due to the heat energy is stored in the liquid due to the frictional heating.

We have used throughout our thermal analysis the values of $\beta_T = 0.6,\ \rho = 1,\ l = 0.1$.

6. CONCLUSION

The present chapter deals with the effect of thermal radiation on MHD boundary layer flow and heat transfer of an incompressible viscous dusty fluid over an exponentially stretching surface. The similarity transformations are used to reduce the given problem into set of nonlinear ordinary differential equations.

The transformed equations are then solved numerically using Runge-Kutta-Fehlberg fourth-fifth order method. The effects of thermal radiation are carried out for two cases of heat transfer analysis. Some of the interesting observations of the study are listed as follows;

- Fluid phase temperature is higher than the dust phase temperature.
- Thermal boundary layer thickness reduces for fluid particle interaction parameter and increases for magnetic parameter.
- Prandtl number Pr stabilizes the boundary layer growth.
- The boundary layers are highly influenced by Eckert number Ec.
- It is found that the skin friction coefficient decreases and the local Nusselt number increases as the magnetic parameter increases.
- If $\beta \to 0$ and $N \to 0$ then our results coincide with the results of Kameswaran *et.al* [10] for different values of magnetic parameter.

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