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pg**- CLOSED SETS IN TOPOLOGICAL SPACES

PUNITHA THARANI
Associate Professor, St. Mary's College, Tuticorin.

PRISCILLA PACIFICA*
Assistant Professor, St. Mary's College, Tuticorin.

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ABSTRACT

*In this paper we introduce a new class of sets called pg**- closed sets in topological spaces which is properly placed in between the class of closed sets and gsp-closed sets. As an application, we introduce new spaces namely, ${}_pT^{**}_{1/2}$- space, ${}_{\alpha p}T^*_c$ -space, ${}^*_pT^*_{1/2}$ -space, ${}^{**}_pT_{1/2}$ -space and ${}_pT^*_c$ -space. Further, pg** -continuous, pg**-irresolute mappings are also introduced and investigated.*

Key words: *pg**-closed set, pg**-continuous map, pg**-irresolute map, ${}_pT^{**}_{1/2}$- space, ${}_{\alpha p}T^*_c$ -space, ${}^*_pT^*_{1/2}$ -space, ${}^{**}_pT_{1/2}$-space and ${}_pT^*_c$-spaces.*

1. INTRODUCTION

Levine [11]introduced the class of g-closed sets in 1970. Arya and Tour [3]defined gs-closed sets in 1990. Dontchev [9], Gnanambal [10] Palaniappan and Rao [17] introduced gsp-closed sets, gpr-closed sets and rg-closed sets respectively. Veerakumar [18]introduced g*-closed sets in 1991. Dontchev [8] introduced gsp-closed sets in 1995.P M Helen [20] introduced g**-closed sets. Levine [11] Devi [6,8] introduced $T_{1/2}$ -spaces, T_b spaces and ${}_\alpha T_b$ spaces respectively. The purpose of this paper is to introduce the concepts of pg**-closed set, pg**-continuous map, pg**-irresolute maps. ${}_pT^{**}_{1/2}$- space, ${}_{\alpha p}T^*_c$-space, ${}^*_pT^*_{1/2}$-space, ${}^{**}_pT_{1/2}$-space and ${}_pT^*_c$-space are introduced and investigated.

2. PRILIMINARIES

Throughout this paper (X,τ),(Y,σ) and (Z,η) represent non-empty topological spaces of which no separation axioms are assumed unless otherwise stated. For a subset A of a space (X,τ), cl(A) and int(A) denote the closure and the interior of A respectively. The class of all closed subsets of a space (X,τ) is denoted by C(X,τ). The smallest semi-closed (resp.pre-closed and α-closed) set containing a subset A of (X, τ) is called the semi-closure (resp.pre-closure and α-closure) of A and is denoted by scl(A) (resp.pcl(A) and αcl(A)).

Definition 2.1: A subset A of a topological space (X, τ) is called
1. a pre-open set [14] if A ⊆ int(cl(A) and a pre-closed set if cl(int(A))⊆ A.
2. a semi-open set [12] if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆ A.
3. a semi-preopen set [1] if A ⊆ cl(int(cl(A)) and a semi-preclosed set [1] if int(cl(int(A))) ⊆ A.
4. an α-open set [16] if A ⊆ int(cl(int(A))) and an α-closed set [16] if cl(int(cl(A)) ⊆ A.
5. a regular-open set [14] if int(cl(A) = A and regular-closed set [14] if A = int(cl(A).

Definition 2.2: A subset A of topological space (X,τ) is called
1. a generalized closed set (briefly g-closed) [1] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
2. generalized semi-closed set (briefly gs-closed) [3] if scl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
3. an α-generalized closed set (briefly αg-closed) [19] if αcl (A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
4. a generalized semi pre-closed set (briefly gsp-closed) [9] if sp cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
5. a regular generalized closed set (briefly rg-closed) [17]if cl(A) ⊆ U whenever A ⊆ U and U is regular open in (X,τ).

*Corresponding Author: Priscilla Pacifica**
Assistant Professor, St. Mary's College, Tuticorin.

(6) a generalized pre-closed set (briefly gp-closed) [13] if p cl (A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
(7) a generalized pre regular-closed set (briefly gpr-closed)[10] if p cl(A) ⊆ U whenever A ⊆ U and U is regular open in (X,τ).
(8) a g*-closed set [18] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X,τ).
(9) a wg-closed set [16] if cl(int(A) whenever A ⊆ U and U is open in (X,τ).
(10) a g**-closed set [20] if cl(A) ⊆ U whenever A ⊆ U and U is g*-open in (X,τ).

Definition 2.3: A function f: (X,τ) →(Y, σ) is called
(1) g-continuous [4] if f^{-1}(V) is a g-closed set of (X,τ) for every closed set V of (Y, σ).
(2) αg-continuous [10] if f^{-1}(V) is an αg-closed set of (X,τ) for every closed set V of (Y, σ).
(3) gs-continuous [7] if f^{-1}(V) is a gs-closed set of (X,τ) for every closed set V of (Y, σ).
(4) gsp-continuous [9] if f^{-1}(V) is a gsp-closed set of (X,τ) for every closed set V of (Y, σ).
(5) rg-continuous [17] if f^{-1}(V) is a rg-closed set of (X,τ) for every closed set V of (Y, σ).
(6) gp-continuous [2] if f^{-1}(V) is a gp-closed set of (X,τ) for every closed set V of (Y, σ).
(7) gpr-continuous [10] if f^{-1}(V) is a gpr-closed set of (X,τ) for every closed set V of (Y, σ).
(8) g*-continuous [18] if f^{-1}(V) is a g*-closed set of (X,τ) for every closed set V of (Y, σ).
(9) g*-irresolute[18] if f^{-1}(V) is a g*-closed set of (X,τ) for every g*-closed set V of (Y, σ).
(10) wg-continuous [16] if f^{-1}(V) is a wg-closed set of (X,τ) for every closed set V of (Y, σ).
(11) g**-continuous[20] if f^{-1}(V) is a g**-closed set of (X,τ) for every closed set V of (Y,σ).
(12) g**-irresolute[20] if f^{-1}(V) is a g**-closed set of (X,τ) for every g**-closed set V of (Y, σ).

Definition 2.4: A topological space (X,τ) is said to be
(1) a $T_{1/2}$-space [11] if every g-closed set in it is closed.
(2) a T_b space [6] if every gs-closed set in it is closed.
(3) a $_{\alpha}T_b$ -space [8] if every αg-closed set in it is closed.
(4) a $T_{1/2}^{*}$-space [18] if every g*-closed set in it is closed.
(5) a $T_{1/2}^{**}$-space [20] if every g**-closed set is closed.
(6) a $^{**}T_{1/2}$-space [20] if every g**-closed set is g*- closed.

3. Basic properties of pg- closed sets**

We introduce the following definition

Definition 3.1: A subset A of (X,τ) is said to be a pg**-closed set if pcl(A)⊆U whenever A ⊆U and U is g*-open in X.

The class of pg**- closed subset of (X,τ) is denoted by PG**C(X,τ).

Proposition 3.2: Every closed set is pg**- closed.

Proof follows from the definition.

The following example supports that a pg**- closed set need not be closed in general.

Proposition 3.3: Every pre closed set is pg**- closed.

Proof follows from the definition.

Proposition 3.4: Every g**-closed set is pg**- closed.

Proof follows from the definition.

Proposition 3.5: Every g*-closed set is pg**- closed.

Proof follows from the definition.

Proposition 3.6: Every g-closed set is pg**- closed.

Proof follows from the definition.

The converse of the above propositions need not be true in general.

Example 3.7: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a, c\}\}$. Let A = $\{a\}$ then A is a pg**- closed set but not a closed set and a g**-closed set of (X,τ). So the class of pg**- closed sets properly contains the class of closed sets and the class of g**-closed sets. Also A = $\{a\}$ is not a g-closed set.

Example 3.8: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}\}$. Let A = $\{a, b\}$ then A is a pg**- closed set but not a pre closed set and a g*-closed set of (X,τ). So the class of pg**- closed sets properly contains the class of pre closed sets and the class of g*-closed sets.

Proposition 3.9: Every pg**- closed set is (1) rg-closed (2) gpr-closed (3) gsp-closed.

Proof follows from the definition.

The converse of the above propositions need not be true in general as seen in the following examples.

Example 3.10: In example (3.8), let A = $\{a\}$ is gpr-closed and rg-closed but it is not pg**- closed. Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$. Let A = $\{a\}$ then A is a gsp-closed set but not a pg**- closed set of (X,τ). Therefore the class of pg**- closed sets is properly contained in the class of gpr-closed, rg-closed, gsp-closed sets.

Remark 3.11: pg**- closedness is independent from α-closedness, semi-closedness, sg-closedness, gα-closedness, gα*-closedness and semi-preclosedness.

Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{a, c\}\}$. Let A = $\{a, b\}$ then A is a pg**- closed set. A is neither α-closed nor semi-closed, in fact, it is not even a semi-preclosed set. Also it is not sg-closed, gα-closed and gα*-closed set.

Proposition 3.12: If A and B are pg**- closed sets, then A∪B is also a pg**- closed set.

Proof follows from the fact that pcl(A∪B) = pcl(A) ∪ pcl(B).

Proposition 3.13: If A is both g*-open and pg**- closed, then A is pre closed.

Proof follows from the definition of pg**- closed sets.

Proposition 3.14: A is a pg**- closed of (X,τ) if pcl(A)$\setminus$A does not contain any non-empty g*-closed set.

Proof: Let F be a g*-closed set of (X,τ) such that F ⊆ pcl(A)$\setminus$A. Then A ⊆ $X \setminus F$. Since A is pg**- closed and $X \setminus F$ is g*-open, pcl(A)⊆ $X \setminus F$. This implies F⊆ $X \setminus$ pcl(A). So, F⊆ ($X \setminus$ pcl(A)) ∩ (pcl(A)$\setminus$A) ⊆ ($X \setminus$ pcl(A)) ∩ (pcl(A) = ϕ. Therefore F = ϕ.

Proposition 3.15: If A is a pg**- closed set of (X,τ) such that $A \subseteq B \subseteq pcl(A)$, then B is also a pg**- closed set of (X,τ).

Proof: Let U be a g*-open set of (X,τ) such that B ⊆U. Then A ⊆U, since A is pg**- closed, then $pcl(A)$ ⊆U. Now $pcl(B) \subseteq pcl\big(pcl(A)\big) = pcl(A))$ ⊆U. Therefore B is also a pg**- closed set of (X,τ).

4. pg- continuous and pg**- irresolute maps.**

We introduce the following definitions.

Definition 4.1: A function $f : (X, \tau) \to (Y, \sigma)$ is calledpg**- continuous if $f^{-1}(V)$ is a pg**- closed set of (X, τ) for every closed set of (Y, σ).

Theorem 4.2: Every continuous map is pg**- continuous.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be continuous and let F be any closed set of Y, then $f^{-1}(V)$ is closed in X. Since every closed set is pg**- closed, $f^{-1}(V)$ is pg**- closed. Therefore f is pg**- continuous.

The following example shows that the converse of the above theorem need not be true in general.

Example 4.3: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}, \sigma = \{\phi. X, \{b\}\}$, $f : (X, \tau) \to (Y, \sigma)$ is defined as the identity map. The inverse image of all the closed sets of (Y, σ) are pg**- closed in (X, τ). Therefore f is pg**- continuous but not continuous.
Thus the class of all pg**- continuous maps properly contains the class of continuous maps.

Theorem 4.4: Every pg**- continuous map is rg- continuous, gpr- continuous and gsp-continuous maps.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a pg**- continuous map. Let V be a closed set of (Y,σ). Since f is pg**- continuous, then $f^{-1}(V)$ is pg**- closed set in (X,τ). By proposition (3.9) $f^{-1}(V)$ is rg-closed, gpr-closed and gsp-closed set of (X,τ).

The converse of the above theorem need not be true as seen in the following example.

Example 4.5: Let $X=Y=\{a,b,c\},\tau=\{\phi,X,\{a\}\},\sigma=\{\phi.X,\{b,c\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then $f^{-1}(\{a\})=\{a\}$ is not pg**- closed in(X,τ). But $\{a\}$ is rg-closed and gpr-closed. Therefore f is rg- continuous and gpr- continuous but f is not pg**- continuous.

Example 4.6: Let $X=Y=\{a,b,c\},\tau=\{\varphi,X,\{a\},\{b\},\{a,b\}\},\sigma=\{\phi.X,\{b,c\}\}$.

Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then $f^{-1}(\{a\})=\{a\}$ is not pg**- closed in(X,τ). But $\{a\}$ is gsp-closed. Therefore f is gsp - continuous but f is not pg**- continuous.

Thus the class of all pg**-continuous maps is properly contained in the classes of rg-continuous, gpr- continuous and gsp-continuous maps.

The following example shows that the compositions of two pg**- continuous maps need not be a pg**- continuous map.

Example 4.7: Let $X=Y=Z=\{a,b,c\}$ and let $f:(X,\tau)\to(Y,\sigma)$, $g:(Y,\sigma)\to(Z,\eta)$, be the identity maps. $\tau=\{\varphi,X,\{a\},\{a,c\}\},\sigma=\{\phi.X,\{a\}\}$, $\eta=\{\phi.X,\{b\}\}$. $(f\circ g)^{-1}(\{a,c\})=f^{-1}(g^{-1}(\{a,c\}))=f^{-1}(\{a,c\})=\{a,c\}$ is not pg**- closed in(X,τ). But f and$g$ are pg**- continuous maps.

Theorem 4.8: Every g*- continuous map is pg**- continuous map.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be g*- continuous and let V be a closed set of Y. Then $f^{-1}(V)$ is g*- closed and hence by proposition (3.5), it is pg**- closed. Hence f is pg**- continuous map.

The following example shows that the converse of the above theorem is not true in general.

Example 4.9: Let $X=Y=\{a,b,c\},\tau=\{\phi,X,\{a\}\},\sigma=\{\phi.X,\{b\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then $A=\{a,c\}$ is closed in (Y,σ) and is pg**- closed in (X,τ) but not g*- closed in (X,τ). Therefore is$f$ is pg**- continuous but not g*- continuous.

Theorem 4.10: Every g - continuous map is pg**- continuous map.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be g - continuous and let V be a closed set of Y. Then $f^{-1}(V)$ is g- closed and hence by proposition (3.6), it is pg**- closed. Hence f is pg**- continuous map.

The following example shows that the converse of the above theorem is not true in general.

Example 4.11: Let $X=Y=\{a,b,c\},\tau=\{\phi,X,\{a\},\{a,c\}\},\sigma=\{\phi.X,\{a,b\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then $A=\{c\}$ is closed in (Y,σ) and is pg**- closed in (X,τ) but not g - closed in (X,τ). Therefore is$f$ is pg**- continuous but not g - continuous.

Theorem 4.12: Every g**- continuous map is pg**- continuous map.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be g**- continuous and let V be a closed set of Y. Then $f^{-1}(V)$ is g**- closed and hence by proposition (3.4), it is pg**- closed. Hence f is pg**- continuous map.

The following example shows that the converse of the above theorem is not true in general.

Example 4.13: Let $X=Y=\{a,b,c\},\tau=\{\phi,X,\{a,c\}\},\sigma=\{\phi.X,\{b,c\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then $A=\{a\}$ is closed in (Y,σ) and is pg**- closed in (X,τ) but not g**- closed in (X,τ). Therefore is$f$ is pg**- continuous but not g**- continuous.

Definition 4.14: A function $f:(X,\tau)\to(Y,\sigma)$ is calledpg**- irresolute if $f^{-1}(V)$ is a pg**- closed set of (X,τ) for every pg**- closed set V of (Y,σ).

Definition 4.15: Let (X, τ) and (Y, σ) be two topological spaces and f : (X, τ) → (Y, σ) is said to be pg**- resolute if f(U) is pg**- open in Y whenever U is pg**- open in X.

Definition 4.16: A function f : (X, τ) → (Y, σ) is calledpg**-homeomorphism if
(i) f is one – one and onto.
(ii) f is pg**- irresolute and pg**- resolute.

Theorem 4.17: Every pg**- irresolute function is pg**- continuous.

Proof follows from the definition.

Theorem 4.18: Every g - irresolute function is pg**- continuous.

Proof follows from the definition.

Theorem 4.19: Every g*- irresolute function is pg**- continuous.

Proof follows from the definition.

Theorem 4.20: Every g**- irresolute function is pg**- continuous.

Proof follows from the definition.

Converse of the above theorems need not be true in general as seen in the following example.

Example 4.21: Let $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a, c\}\}, \sigma = \{\phi. X, \{a\}\}$. Let f : (X, τ) → (Y, σ) by f(a) = b, f(b) = a, f(c) = c, {b, c} is the only closed set of Y. $f^{-1}(\{b, c\}) = \{a, c\}$ is pg**- closed in (X, τ). Therefore f is pg**- continuous. {b, c} is g – closed, g*- closed and g**-closed set of Y but $f^{-1}(\{b, c\}) = \{a, c\}$ is not g – closed, g*-closed and g**-closed set in X.

Therefore f is not g – irresolute, g*- irresolute and g**- irresolute. Therefore f is pg**- continuous but not g – irresolute, g*- irresolute and g**- irresolute. Also $\{b, c\}$ is a pg**-closed set in Y but $f^{-1}(\{b, c\}) = \{a, c\}$ is not pg**- closed in (X, τ). Therefore f is not a pg**- irresolute. Hence f is pg**- continuous but not pg**- irresolute.

Theorem 4.22: Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) , be any two functions then,
(i) $g \circ f$ is pg**- continuous if g is continuous and f is pg**- continuous.
(ii) $g \circ f$ is pg**- irresolute if both f and g are pg**- irresolute.
(iii) $g \circ f$ is pg**- continuous if g is pg**- continuous and f is pg**- irresolute.

5. Applications of pg- closed sets**

As applications of pg**- closed sets, new spaces, namely, ${}_{p}T^{**}_{1/2}$- space, ${}_{\alpha p}T^{*}_{c}$-space, ${}^{*}_{p}T^{*}_{1/2}$-space, ${}^{**}_{p}T_{1/2}$-space and ${}_{p}T^{*}_{c}$-space are introduced.

We introduce the following definition.

Definition 5.1: A space (X,τ) is called a ${}_{p}T^{**}_{1/2}$space if every pg**- closed set is closed.

Theorem 5.2: Every ${}_{p}T^{**}_{1/2}$space is $T_{1/2}$ space.

Proof follows from the definition.

Theorem 5.3: Every ${}_{p}T^{**}_{1/2}$space is $T^{*}_{1/2}$ space.

Proof follows from the definition.

The converse need not be true in general as seen in the following example.

Example 5.4: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}\}$, G*C(X,$\tau$) $= \{\varphi, X, \{b, c\}\}$ = C(X,τ). Therefore (X,τ) is a $T^{*}_{1/2}$ space but not ${}_{p}T^{**}_{1/2}$space since $\{a, b\}$ is a pg**- closed set but not a closed set of (X,τ).

Theorem 5.5: Every T_b space is ${}_{\mathrm{p}}T^{**}_{1/2}$ space.

Proof follows from the definition.

The converse need not be true in general as seen in the following example.

Example 5.6: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$. (X,$\tau$) is a ${}_{\mathrm{p}}T^{**}_{1/2}$ space but not a T_b space since $\{a\}$ is gs-closed but not closed.

Remark 5.7: T_d-ness is independent of ${}_{\mathrm{p}}T^{**}_{1/2}$-ness as it can be seen from the following example.

Example 5.8: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$. (X,$\tau$) is a ${}_{\mathrm{p}}T^{**}_{1/2}$ space but not a T_d space since $\{a\}$ is gs-closed but not g-closed.

Example 5.9: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{b, c\}\}$. (X,$\tau$) is a T_d space but not a ${}_{\mathrm{p}}T^{**}_{1/2}$ space since $\{c\}$ is pg**- closed but not closed.

Theorem 5.10: The following conditions are equivalent in topological space (X,τ).
- (i) (X,τ) is a ${}_{\mathrm{p}}T^{**}_{1/2}$ space.
- (ii) Every singleton of X is either g*-closed or open.

Proof:
(i) ⟹ (ii): Let (X,τ) be a ${}_{\mathrm{p}}T^{**}_{1/2}$ space. Let $x \in X$ and suppose $\{x\}$ is not g*-closed. Then $X \setminus \{x\}$ is not g*-open. This implies that X is the only g*-open set containing$X \setminus \{x\}$. Therefore $X \setminus \{x\}$ is closed since (X,τ) is a ${}_{\mathrm{p}}T^{**}_{1/2}$ space. Therefore $\{x\}$ is open in (X,τ).

(ii) ⟹ (i): Let A be a pg**- closed set of (X,τ)$A \subseteq pcl(A) \subseteq cl(A)$ and let $x \in pcl(A)$ this implies $x \in cl(A)$. By (ii) $\{x\}$ isg*-closed or open.

Case-(i): Let $\{x\}$ be g*-closed. If $x \notin A$, then $pcl(A)) \setminus A$ contains a non-empty g*-closed set $\{x\}$. But it is not possible by proposition (3.14). Therefore $x \in A$.

Case-(ii): Let $\{x\}$ be open. Now $x \in cl(A)$, then $\{x\} \cap A \neq \phi$. Therefore $x \in A$ and so $cl(A) \subseteq A$ and hence A = $cl(A)$ or A is closed. Therefore (X,τ) is a ${}_{\mathrm{p}}T^{**}_{1/2}$ space.

We introduce the following definition.

Definition 5.11: A space (X,τ) is called an ${}_{\alpha p}T^{*}_{c}$-space if every αg-closed set of (X,τ) is pg**-closed.

Theorem 5.12: Every ${}_{\alpha}T_b$-space is an ${}_{\alpha p}T^{*}_{c}$-space but not conversely.

Example 5.13: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}\}$. (X,$\tau$) is an ${}_{\alpha p}T^{*}_{c}$-space but not ${}_{\alpha}T_b$-space since $\{a, c\}$ is αg-closed but not closed.

Definition 5.14: A subset A of a space (X,τ) is called a pg**-open set if its complement is a pg**- closed set of (X,τ).

Theorem 5.15: If (X,τ) is an ${}_{\alpha p}T^{*}_{c}$-space for each $x \in X$, $\{x\}$ is either αg-closed or pg**-open.

Proof: Let $x \in X$ suppose that $\{x\}$ is not an αg-closed set of (X,τ). Then $\{x\}$ is not a closed set since every closed set is an αg-closed set. Therefore $X \setminus \{x\}$ is not open. Therefore $X \setminus \{x\}$ is an αg-closed set since X is the only open set which contains $X \setminus \{x\}$. Since (X,τ) is an ${}_{\alpha p}T^{*}_{c}$-space, $X \setminus \{x\}$ is a pg**- closed set or $\{x\}$ is pg**-open.

Remark 5.16: The converse of the above theorem is not true as it can be seen from the following example.

Example 5.17: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a, c\}\}$. (X,$\tau$) is not a ${}_{\alpha p}T^{*}_{c}$-space but $\{b\}$$\alpha$g-closed and $\{a\}$ and $\{c\}$ are pg**-open.

We introduce the following definition.

Definition 5.18: A space (X,τ) is called a ${}^{**}_{p}T_{1/2}$-space if every pg**- closed set of (X,τ) is a g*- closed set.

Theorem 5.19: Every ${}_{p}T^{**}_{1/2}$ space is ${}^{**}_{p}T_{1/2}$-space.

Proof: Let (X,τ) be a ${}_{p}T^{**}_{1/2}$ space. Let A be a pg**- closed set of (X,τ). Since (X,τ) is a ${}_{p}T^{**}_{1/2}$-space, A is closed. But since every closed set is g*- closed, A is g*- closed. Therefore (X,τ) is a${}^{**}_{p}T_{1/2}$-space.

Theorem 5.20: Every T_b-space is a ${}^{**}_{p}T_{1/2}$-space.

Proof: Let (X,τ) be a T_b- space. Then by theorem (4.5), it is a ${}_{p}T^{**}_{1/2}$ space. Therefore by theorem (4.19), it is ${}^{**}_{p}T_{1/2}$-space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.21: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, (X,$\tau$) is a ${}^{**}_{p}T_{1/2}$-space but not a T_b- space since A = $\{a\}$ is gs-closed but not closed.

Theorem 5.22: Every${}^{**}_{p}T_{1/2}$-space is a ${}^{*}T_{1/2}$-space.

Proof: Let (X,τ) be a ${}^{**}_{p}T_{1/2}$- space. Let A be a g- closed set of (X,τ). Then by proposition (3.6), A is pg**- closed. Since (X,τ) is an ${}^{**}_{p}T_{1/2}$- space, A is g*- closed. Therefore it is a ${}^{*}T_{1/2}$-space .

The converse of the above theorem need not be true as seen in the following example.

Example 5.23: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{a, c\}\}$, (X,$\tau$) is a ${}^{*}T_{1/2}$-space but not a${}^{**}_{p}T_{1/2}$- space since A = $\{c\}$ is pg**- closed but not g*- closed.

Theorem 5.24: Every${}^{**}_{p}T_{1/2}$-space is a${}^{**}T_{1/2}$-space.

Proof: Let (X,τ) be a ${}^{**}_{p}T_{1/2}$- space. Let A be a g**- closed set of (X,τ). Then by proposition (3.4), A is pg**- closed. Since (X,τ) is a ${}^{**}_{p}T_{1/2}$- space, A is g*- closed. Therefore it is a${}^{**}T_{1/2}$-space .

The converse of the above theorem need not be true as seen in the following example.

Example 5.25: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a, c\}\}$, (X,$\tau$) is a ${}^{**}T_{1/2}$-space but not a${}^{**}_{p}T_{1/2}$- space since A = $\{c\}$ is pg**- closed but not g*- closed.

Theorem 5.26: If (X,τ) is a${}^{**}_{p}T_{1/2}$- space, then for each $x \in X$, $\{x\}$ is either closed or g*-open.

Proof: Suppose (X,τ) is a${}^{**}_{p}T_{1/2}$- space. Let $x \in X$ and let $\{x\}$ not be closed. Then $X \setminus \{x\}$ is not open set. Therefore $X \setminus \{x\}$ is a g-closed set since X is the only open set which contains $X \setminus \{x\}$. By theorem (3.6) $X \setminus \{x\}$ is a pg**- closed set. Since (X,τ) is a${}^{**}_{p}T_{1/2}$- space, $X \setminus \{x\}$ is g*-closed set. Therefore $\{x\}$ is g*-open.

Definition 5.27: A space (X,τ) is called an ${}^{*}_{p}T^{*}_{1/2}$-space if every pg**-closed set of (X,τ) is g-closed.

Theorem 5.28: Every ${}_{p}T^{**}_{1/2}$-space is a ${}^{*}_{p}T^{*}_{1/2}$-space.

Proof: Let (X,τ) be a ${}_{p}T^{**}_{1/2}$- space. Let A be a pg**- closed set of (X,τ).Then A is closed since (X,τ) is a ${}_{p}T^{**}_{1/2}$-space. But every closed set is g-closed set, Therefore A is g- closed. Therefore (X,τ) is a ${}^{*}_{p}T^{*}_{1/2}$-space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.29: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}\}$, (X,$\tau$) is a${}^{*}_{p}T^{*}_{1/2}$-space but not a ${}_{p}T^{**}_{1/2}$-space since A = $\{c\}$ is pg**- closed but not closed.

Theorem 5.30: The space (X,τ) is a${}_{p}T^{**}_{1/2}$-space if and only if it is a ${}^{*}_{p}T^{*}_{1/2}$-space and a $T_{1/2}$ space.

Proof: Necessity: Let (X,τ) be a ${}_{p}T^{**}_{1/2}$-space. Let A be a g-closed set of (X,τ). Then by theorem (3.6) A is pg**- closed. Also since (X,τ) is a ${}_{p}T^{**}_{1/2}$-space, A is a closed set. Therefore (X,τ) is a $T_{1/2}$ space. By theorem (4.24) (X,τ) is a ${}^{*}_{p}T^{*}_{1/2}$-space.

Sufficiency: Let (X,τ) be a $T_{1/2}$ space and a ${}^{*}_{p}T^{*}_{1/2}$-space. Let A be a pg**- closed set. Then A is g-closed since (X,τ) is a ${}^{*}_{p}T^{*}_{1/2}$-space. Also since (X,τ) is a $T_{1/2}$-space, A is a closed set. Therefore (X,τ) is a ${}_{p}T^{**}_{1/2}$-space.

Theorem 5.31: Every ${}_p^{**}T_{1/2}$-space is a ${}_p^{*}T_{1/2}^{*}$-space.

Let (X,τ) be a ${}_p^{**}T_{1/2}$-space. Let A be a pg**- closed set. Then A is g*-closed since (X,τ) is a ${}_p^{**}T_{1/2}$-space. But every g*-closed set is g-closed, and hence A is a g-closed set. Therefore (X,τ) is a ${}_p^{*}T_{1/2}^{*}$-space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.32: Let X = {a, b, c,d}, $\tau = \{\varphi, X, \{a\}\}$, (X,$\tau$) is a ${}_p^{*}T_{1/2}^{*}$-space but not a ${}_p^{**}T_{1/2}$- space since A = $\{c\}$ is pg**- closed but not g*- closed.

We introduce the following definition

Definition 5.33: A space (X,τ) is called a ${}_{\mathrm{p}}T_c^*$-space if every gs- closed set of (X,τ) is a pg**- closed set.

Theorem 5.34: Every T_c-space is a ${}_{\mathrm{p}}T_c^*$-space.

Proof: Let (X,τ) be a T_c- space. Let A be a gs- closed set of (X,τ).Then A isg*-closed since (X,τ) is a T_c-space. But by proposition (3.5) A is pg**- closed set. Therefore (X,τ) is a ${}_{\mathrm{p}}T_c^*$-space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.35: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{a, c\}\}$, (X,$\tau$) is a ${}_{\mathrm{p}}T_c^*$space but not a T_c- space since A = $\{c\}$ is gs - closed but not g*- closed.

Theorem 5.36: Every T_b-space is a ${}_{\mathrm{p}}T_c^*$-space.

Proof: Let (X,τ) be a T_b- space. Let A be a gs- closed set of (X,τ).Then A isclosed since (X,τ) is a T_b-space. But by proposition (3.2) A is pg**- closed set. Therefore (X,τ) is a ${}_{\mathrm{p}}T_c^*$-space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.37: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a\}, \{a, c\}\}$, (X,$\tau$) is a ${}_{\mathrm{p}}T_c^*$-space but not a T_b- space since A = $\{c\}$ is gs - closed but not aclosed set.

Theorem 5.38: If (X,τ) is a ${}_{\mathrm{p}}T_c^*$-space and a ${}_p^{*}T_{1/2}^{*}$ -space, then it is a ${}_{\alpha}T_d$ -space.

Proof: Let (X,τ) be a ${}_{\mathrm{p}}T_c^*$-space and a ${}_p^{*}T_{1/2}^{*}$ -space. Let A be a αg- closed set of (X,τ).Then A is also gs-closed. Since (X,τ) is a ${}_{\mathrm{p}}T_c^*$-space, A is pg**- closed set. Also since (X,τ) is a ${}_p^{*}T_{1/2}^{*}$ -space, A is a g-closed set. Therefore (X,τ) is a ${}_{\alpha}T_d$ -space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.39: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a, c\}\}$, (X,$\tau$) is a ${}_{\alpha}T_d$ -space but not a ${}_p^{*}T_{1/2}^{*}$- space since A = $\{c\}$ is pg**- closed but not ag-closed set.

Theorem 5.40: If (X,τ) is a ${}_{\mathrm{p}}T_c^*$-space and a ${}_{\mathrm{p}}T_{1/2}^{**}$ -space, then it is a ${}_{\alpha}T_b$ -space.

Proof: Let (X,τ) be a ${}_{\mathrm{p}}T_c^*$-space and a ${}_{\mathrm{p}}T_{1/2}^{**}$ -space. Let A be a αg- closed set of (X,τ).Then A is also gs-closed. Since (X,τ) is a ${}_{\mathrm{p}}T_c^*$-space, A is pg**- closed set. But every pg**- closed set is closed since (X,τ) is a ${}_{\mathrm{p}}T_{1/2}^{**}$-space, A is a closed set. Therefore (X,τ) is a ${}_{\alpha}T_b$ -space.

Theorem 5.41: If (X,τ) is a ${}_{\mathrm{p}}T_c^*$-space and a ${}_p^{*}T_{1/2}^{*}$ -space, then it is a T_d-space.

Proof: Let (X,τ) be a ${}_{\mathrm{p}}T_c^*$-space and a ${}_p^{*}T_{1/2}^{*}$ -space. Let A be a gs-closed set of (X,τ). Since (X,τ) is a ${}_{\mathrm{p}}T_c^*$-space, A is pg**- closed set. Also since (X,τ) is a${}_p^{*}T_{1/2}^{*}$ -space, A is a g-closed set. Therefore (X,τ) is a T_d-space.

The converse of the above theorem need not be true as seen in the following example.

Example 5.42: Let X = {a, b, c}, $\tau = \{\varphi, X, \{a, c\}\}$, (X,$\tau$) is a T_d-space but not a ${}_p^{*}T_{1/2}^{*}$- space since A = $\{c\}$ is pg**- closed but not ag-closed set.

Theorem 5.43: If (X,τ) is a $_{p}T_{c}^{*}$- space, then for each $x \in X$, $\{x\}$ is either semi-closed or pg**-open in (X,τ).

Proof: Suppose (X,τ) is a $_{p}T_{c}^{*}$- space. Let $x \in X$ and let $\{x\}$ not be semi-closed. Then $X \setminus \{x\}$ is sg-closed. Also $X \setminus \{x\}$ is gs-closed. Since (X,τ) is a $_{p}T_{c}^{*}$- space, $X \setminus \{x\}$ is pg**-closed set. Therefore $\{x\}$ is pg**-open.

Theorem 5.44: Let $f : (X, \tau) \to (Y, \sigma)$ be a pg**-continuous map. If (X,τ) is $_{p}T_{1/2}^{**}$-space then$f$ is continuous.

Theorem 5.45: Let $f : (X, \tau) \to (Y, \sigma)$ be a pg**-continuous map. If (X,τ) is $^{**}_{p}T_{1/2}$- space then$f$ is g*-continuous.

Theorem 5.46: Let $f : (X, \tau) \to (Y, \sigma)$ be a pg**-continuous map. If (X,τ) is $^{*}_{p}T_{1/2}^{*}$ -space then$f$ is g-continuous.

Theorem 5.47: Let $f : (X, \tau) \to (Y, \sigma)$ be a gs-continuous map. If (X,τ) is $_{p}T_{c}^{*}$- spacethen f is pg**-continuous.

Theorem 5.48: Let $f : (X, \tau) \to (Y, \sigma)$ be ag*- irresolute map and a pre-closed map. Then $f(A)$ is a pg**-closed set of (Y, σ) for every pg**-closed set A of (X, τ).

Proof: Let A be a pg**-closed set of (X, τ). Let U be a g*-open set of (Y, σ) such that $f(A) \subseteq U$. Since f is g*- irresolute, f^{-1}(U) is g*-open in (X, τ). Now f^{-1}(U) is g*-open and A is pg**-closed set of (X, τ), then $pcl(A) \subseteq f^{-1}$(U). Then $f\big(pcl(A)\big) = pcl(f\big(pcl(A)\big))$. Therefore $pcl[f(A)] \subseteq pcl[f(pcl(A))] = f\big(pcl(A)\big) \subseteq U$. Therefore $f(A)$ is a pg**-closed set of (Y, σ).

Theorem 5.49: Let $f : (X, \tau) \to (Y, \sigma)$ be onto, pg**- irresolute and closed. If $(X, \tau)$is $_{p}T_{1/2}^{**}$ then $(Y, \sigma)$is also a $_{p}T_{1/2}^{**}$-space.

Definition 5.50: A function $f : (X, \tau) \to (Y, \sigma)$ is called a pg**-closed map if $f(A)$ is a pg**-closed set of (Y, σ) for every pg**-closed set A of (X, τ).

Theorem 5.51: Let $f : (X, \tau) \to (Y, \sigma)$ be onto, pg**- irresolute and pre - g*- closed. If (X, τ) is $^{**}_{p}T_{1/2}$, then (Y, σ) is also a $^{**}_{p}T_{1/2}$ - space.

Theorem 5.52: Let $f : (X, \tau) \to (Y, \sigma)$ be onto, gs - irresolute and pg**-closed map. If (X, τ) is $_{p}T_{c}^{*}$, then (Y, σ) is also a $_{p}T_{c}^{*}$ - space.

Theorem 5.53: Let $f : (X, \tau) \to (Y, \sigma)$ be onto, pg** - irresolute and g-closed map. If (X, τ) is $^{*}_{p}T_{1/2}^{*}$, then (Y, σ) is also a $^{*}_{p}T_{1/2}^{*}$ - space.

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