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# ANTI Ω-FUZZY SUBBIGROUP OF A BIGROUP

**T. JUSTIN PRABU***
**Department of Mathematics,**
**Alagappa University Evening College, Paramakudi-623707, Tamilnadu, India.**

**K. ARJUNAN**
**Department of Mathematics,**
**H. H. The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.**

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***ABSTRACT***

*In this paper, we made an attempt to study the algebraic nature of anti Ω-fuzzy subbigroup of a bigroup.*

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***Keywords:*** *Bigroup, fuzzy subset, fuzzy subbigroup, Ω-fuzzy subbigroup, anti Ω-fuzzy subbigroup.*

## INTRODUCTION

In 1965, the fuzzy subset was introduced by L.A.Zadeh [10], after that several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups was introduced by Azriel Rosenfeld [2] and Gyu Ihn Chae, Young Sik Park and Chul Hwan Park [3] have introduced and defined a new algebraic structure called Ω-bifuzzy subsemigroup. After that A.Solairaju, R.Nagarajan [7, 8, 9] and K.Arjunan, Selvak Kumaraen [6] extend the theory to many algebraic structure. N.Palaniappan & K.Arjunan [4] defined a new algebraic structure called anti fuzzy ideal. In this paper, we introduce the some theorems in anti Ω-fuzzy subbigroup of a bigroup.

## 1. PRELIMINARIES

**1.1 Definition:** A set $(G, +, \bullet)$ with two binary operations + and $\bullet$ is called a bigroup if there exist two proper subsets $G_1$ and $G_2$ of G such that
- (i) $G = G_1 \cup G_2$
- (ii) $(G_1, +)$ is a group
- (iii) $(G_2, \bullet)$ is a group.

**1.2 Definition:** Let X be a non–empty set. A **fuzzy subset A** of X is a function $A: X \rightarrow [0, 1]$.

**1.3 Definition:** Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup. Then a fuzzy set A of G is said to be a fuzzy subbigroup of G if there exist two fuzzy subsets $A_1$ of $G_1$ and $A_2$ of $G_2$ such that
- (i) $A = A_1 \cup A_2$
- (ii) $A_1$ is a fuzzy subgroup of $(G_1, +)$
- (iii) $A_2$ is a fuzzy subgroup of $(G_2, \bullet)$.

**1.4 Definition:** Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup and Ω be a nonempty set. The fuzzy subset $A: G \times \Omega \rightarrow [0, 1]$ of G is said to be a Ω-fuzzy subbigroup of G if there exist two fuzzy subsets $A_1: G_1 \times \Omega \rightarrow [0, 1]$ of $G_1$ and $A_2 : G_2 \times \Omega \rightarrow [0, 1]$ of $G_2$ such that
(i) $A = A_1 \cup A_2$
(ii) $A_1$ is a Ω-fuzzy subgroup of $(G_1, +)$
(iii) $A_2$ is a Ω-fuzzy subgroup of $(G_2, \bullet)$.

*Corresponding Author: T. Justin Prabu*, Department of Mathematics,*
*Alagappa University Evening College, Paramakudi-623707, Tamilnadu, India.*

**1.5 Definition:** Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup and $\Omega$ be a nonempty set. The fuzzy subset $A: G\times\Omega\to[0, 1]$ of G is said to be a anti $\Omega$-fuzzy subbigroup of G if there exist two fuzzy subsets $A_1: G_1\times\Omega\to[0, 1]$ of $G_1$ and $A_2 : G_2\times\Omega\to [0, 1]$ of $G_2$ such that

(i) $A = A_1 \cup A_2$
(ii) $A_1$ is an anti $\Omega$-fuzzy subgroup of $(G_1, +)$
(iii) $A_2$ is an anti $\Omega$-fuzzy subgroup of $(G_2, \bullet)$.

## 2. PROPERTIES

**2.1 Theorem:** If $A = M\cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E\cup F$, then $\mu_M(-x, q) = \mu_M(x, q)$, $\mu_M(x, q) \geq \mu_M(e, q)$, $\mu_N(x^{-1}, q) = \mu_N(x, q)$, $\mu_N(x, q) \geq \mu_N(e', q)$ for all x, e in E and x, e' in F and q in $\Omega$.

**Proof:** Let x, e in E and x, e' in F and q in $\Omega$. Now $\mu_M(x, q) = \mu_M( (-(-x)), q) \leq \mu_M(-x, q) \leq \mu_M(x, q)$. Therefore $\mu_M(-x, q) = \mu_M(x, q)$ for all x in E and q in $\Omega$. And $\mu_M(e, q) = \mu_M( x-x, q) \leq \max \{\mu_M(x, q), \mu_M(x, q)\} = \mu_M(x, q)$. Therefore $\mu_M(e, q) \leq \mu_M(x, q)$ for all x, e in E and q in $\Omega$. Also $\mu_N(x, q) = \mu_N( (x^{-1} )^{-1}, q) \leq \mu_N(x^{-1}, q) \leq \mu_N(x, q)$. Therefore $\mu_N(x^{-1}, q) = \mu_N(x, q)$ for all x in F and q in $\Omega$. And $\mu_N(e', q) = \mu_N(xx^{-1}, q) \leq \max\{\mu_N(x, q), \mu_N(x^{-1}, q)\}= \mu_N(x, q)$. Therefore $\mu_N(e', q) \leq \mu_N(x, q)$ for all x, e' in F and q in $\Omega$.

**2.2 Theorem:** If $A = M\cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E\cup F$, then

(i) $\mu_M(x-y, q) = \mu_M(e, q)$ gives $\mu_M(x, q) = \mu_M(y, q)$ for all x, y and e in E and q in $\Omega$
(ii) $\mu_N(xy^{-1}, q) = \mu_N(e', q)$ gives $\mu_N(x, q) = \mu_N(y, q)$ for all x, y and e' in F and q in $\Omega$.

**Proof:**

(i) Let x, y and e in E and q in $\Omega$. Then $\mu_M(x, q) = \mu_M(x-y+y, q) \leq \max \{\mu_M(x-y, q), \mu_M(y, q)\} = \max \{\mu_M(e, q), \mu_M(y, q)\}= \mu_M(y, q) = \mu_M(y-x+x, q) \leq \max \{\mu_M(y-x, q), \mu_M(x, q)\}= \max \{\mu_M(e, q), \mu_M(x, q)\}= \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in $\Omega$.
(ii) Let x, y and e' in F and q in $\Omega$. Then $\mu_N(x, q) = \mu_N(xy^{-1}y, q) \leq \max \{\mu_N(xy^{-1}, q), \mu_N(y, q)\} = \max \{\mu_N(e', q), \mu_N(y, q)\}= \mu_N(y, q) = \mu_N(yx^{-1}x, q) \leq \max \{\mu_N(yx^{-1}, q), \mu_N(x, q)\}= \max \{\mu_N(e', q), \mu_N(x, q)\}= \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in $\Omega$.

**2.3 Theorem:** If $A= M\cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E\cup F$, then

(i) $H_1= \{x / x\in E \text{ and } \mu_M(x, q) = 0\}$ is either empty or a subgroup of E.
(ii) $H_2 = \{x / x\in F \text{ and } \mu_N(x, q) = 0\}$ is either empty or a subgroup of F.
(iii) $K = H_1 \cup H_2$ is either empty or a subbigroup of G.

**Proof:** If no element satisfies this condition, then $H_1$ and $H_2$ are empty. Also $K = H_1\cup H_2$ is empty.

(i) If x and y in $H_1$, then $\mu_M(x-y, q) \leq \max \{\mu_M(x, q), \mu_M(y, q) \}\leq \max \{0, 0\}= 0$. Therefore $\mu_M(x-y, q) = 0$. We get x–y in $H_1$. Hence $H_1$ is a subgroup of $G_1$.
(ii) If x and y in $H_2$, then $\mu_N(xy^{-1}, q) \leq \max \{\mu_N(x, q), \mu_N(y, q)\} = \max \{0, 0\} = 0$. Therefore $\mu_N( xy^{-1}, q) = 0$. We get $xy^{-1}$ in $H_2$. Hence $H_2$ is a subgroup of $G_2$.
(iii) From (i) and (ii) we get $K = H_1 \cup H_2$ is a subbigroup of G.

**2.4 Theorem:** If $A = M\cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E\cup F$, then

(i) $H_1= \{x / x\in E \text{ and } \mu_M(x, q) = \mu_M(e, q)\}$ is a subgroup of E
(ii) $H_2 = \{x / x\in F \text{ and } \mu_N(x, q) = \mu_N(e', q)\}$ is a subgroup of F
(iii) $K = H_1\cup H_2$ is a subbigroup of G.

**Proof:**

(i) Clearly e in $H_1$ so $H_1$ is a non empty. Let x and y be in $H_1$. Then $\mu_M(x-y, q) \leq \max \{\mu_M(x, q), \mu_M(y, q)\} = \max \{\mu_M(e, q), \mu_M(e, q)\}= \mu_M(e, q)$. Therefore $\mu_M(x-y, q) \leq \mu_M(e, q)$ for all x and y in $H_1$ and q in $\Omega$. We get $\mu_M( x-y, q) = \mu_M(e, q)$ for all x and y in $H_1$ and q in $\Omega$. Therefore x–y in $H_1$. Hence $H_1$ is a subgroup of E.
(ii) Clearly e' in $H_2$ so $H_2$ is a non empty. Let x and y be in $H_2$. Then $\mu_N(xy^{-1}, q) \leq \max \{\mu_N(x, q), \mu_N(y, q)\} = \max \{\mu_N(e', q), \mu_N(e', q)\} = \mu_N(e', q)$. Therefore $\mu_N(xy^{-1}, q) \leq \mu_N(e', q)$ for all x and y in $H_2$ and q in $\Omega$. We get $\mu_N( xy^{-1}, q) = \mu_N(e', q)$ for all x and y in $H_2$ and q in $\Omega$. Therefore $xy^{-1}$ in $H_2$. Hence $H_2$ is a subgroup of F.
(iii) From (i) and (ii) we get $K = H_1 \cup H_2$ is a subbigroup of G.

**2.5 Theorem:** Let $A= M\cup N$ be an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E\cup F$.

(i) If $\mu_M(x-y, q) = 0$, then $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in $\Omega$.
(ii) If $\mu_N(xy^{-1}, q) = 0$, then $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in $\Omega$.

**Proof:**

(i) Let x and y belongs to E and q in $\Omega$. Then $\mu_M(x, q) = \mu_M(x-y+y, q) \le \max\{\mu_M(x-y, q), \mu_M(y, q)\} = \max\{0, \mu_M(y, q)\} = \mu_M(y, q) = \mu_M(-y, q) = \mu_M(-x+x-y, q) \le \max\{\mu_M(-x, q), \mu_M(x-y, q)\} = \max\{\mu_M(-x, q), 0\} = \mu_M(-x, q) = \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in $\Omega$.

(ii) Let x and y belongs to F and q in $\Omega$. Then $\mu_N(x, q) = \mu_N(xy^{-1}y, q) \le \max\{\mu_N(xy^{-1}, q), \mu_N(y, q)\} = \max\{0, \mu_N(y, q)\} = \mu_N(y, q) = \mu_N(y^{-1}, q) = \mu_N(x^{-1}xy^{-1}, q) \le \max\{\mu_N(x^{-1}, q), \mu_N(xy^{-1}, q)\} = \max\{\mu_N(x^{-1}, q), 0\} = \mu_N(x^{-1}, q) = \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in $\Omega$.

**2.6 Theorem:** If $A = M \cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E \cup F$, then

(i) $\mu_M(x+y, q) = \max\{\mu_M(x, q), \mu_M(y, q)\}$ for each x and y in E and q in $\Omega$ with $\mu_M(x, q) \ne \mu_M(y, q)$

(ii) $\mu_N(xy, q) = \max\{\mu_N(x, q), \mu_N(y, q)\}$ for each x and y in F and q in $\Omega$ with $\mu_N(x, q) \ne \mu_N(y, q)$.

**Proof:**

(i) Let x and y belongs to E and q in $\Omega$. Assume that $\mu_M(x, q) < \mu_M(y, q)$, then $\mu_M(y, q) = \mu_M(-x+x+y, q) \le \max\{\mu_M(-x, q), \mu_M(x+y, q)\} \le \max\{\mu_M(x, q), \mu_M(x+y, q)\} = \mu_M(x+y, q) \le \max\{\mu_M(x, q), \mu_M(y, q)\} = \mu_M(y, q)$. Therefore $\mu_M(x+y, q) = \mu_M(y, q) = \max\{\mu_M(x, q), \mu_M(y, q)\}$ for x and y in E and q in $\Omega$.

(ii) Let x and y belongs to F and q in $\Omega$. Assume that $\mu_N(x, q) < \mu_N(y, q)$, then $\mu_N(y, q) = \mu_N(x^{-1}xy, q) \le \max\{\mu_N(x^{-1}, q), \mu_N(xy, q)\} \le \max\{\mu_N(x, q), \mu_N(xy, q)\} = \mu_N(xy, q) \le \max\{\mu_N(x, q), \mu_N(y, q)\} = \mu_N(y, q)$. Therefore $\mu_N(xy, q) = \mu_N(y, q) = \max\{\mu_N(x, q), \mu_N(y, q)\}$ for x and y in F and q in $\Omega$.

**2.7 Theorem:** If $A = M \cup N$ and $B = O \cup P$ are two anti $\Omega$-fuzzy subbigroups of a bigroup $G = E \cup F$, then their union $A \cup B$ is an anti $\Omega$-fuzzy subbigroup of G.

**Proof:** Let $A = M \cup N = \{\langle (x, q), \mu_A(x, q)\rangle / x \in G \text{ and } q \in \Omega\}$ where $M = \{\langle (x, q), \mu_M(x, q)\rangle / x \in E \text{ and } q \in \Omega\}$ and $N = \{\langle (x, q), \mu_N(x, q)\rangle / x \in F \text{ and } q \in \Omega\}$ and $B = O \cup P = \{\langle (x, q), \mu_B(x, q)\rangle / x \in G \text{ and } q \in \Omega\}$ where $O = \{\langle (x, q), \mu_O(x, q)\rangle / x \in E \text{ and } q \in \Omega\}$ and $P = \{\langle (x, q), \mu_P(x, q)\rangle / x \in F \text{ and } q \in \Omega\}$. Let $C = A \cup B = R \cup S$ where $C = \{\langle (x, q), \mu_C(x, q)\rangle / x \in G \text{ and } q \in \Omega\}$, $R = M \cup O = \{\langle (x, q), \mu_R(x, q)\rangle / x \in E \text{ and } q \in \Omega\}$ and $S = N \cup P = \{\langle (x, q), \mu_S(x, q)\rangle / x \in F \text{ and } q \in \Omega\}$. Let x and y belong to E and q in $\Omega$. Then $\mu_R(x-y, q) = \max\{\mu_M(x-y, q), \mu_O(x-y, q)\} \le \max\{\max\{\mu_M(x, q), \mu_M(y, q)\}, \max\{\mu_O(x, q), \mu_O(y, q)\}\} \le \max\{\max\{\mu_M(x, q), \mu_O(x, q)\}, \max\{\mu_M(y, q), \mu_O(y, q)\}\} = \max\{\mu_R(x, q), \mu_R(y, q)\}$. Therefore $\mu_R(x-y, q) \le \max\{\mu_R(x, q), \mu_R(y, q)\}$ for all x and y in E and q in $\Omega$. Let x and y belong to F and q in $\Omega$. Then $\mu_S(xy^{-1}, q) = \max\{\mu_N(xy^{-1}, q), \mu_P(xy^{-1}, q)\} \le \max\{\max\{\mu_N(x, q), \mu_N(y, q)\}, \max\{\mu_P(x, q), \mu_P(y, q)\}\} \le \max\{\max\{\mu_N(x, q), \mu_P(x, q)\}, \max\{\mu_N(y, q), \mu_P(y, q)\} = \max\{\mu_S(x, q), \mu_S(y, q)\}$. Therefore $\mu_S(xy^{-1}, q) \le \max\{\mu_S(x, q), \mu_S(y, q)\}$ for all x and y in F and q in $\Omega$. Hence $A \cup B$ is an anti $\Omega$-fuzzy subbigroup of G.

**2.8 Theorem:** The union of a family of anti $\Omega$-fuzzy subbigroups of a bigroup G is an anti $\Omega$-fuzzy subbigroup of G.

**Proof:** It is trivial.

**2.9 Theorem:** If $A = M \cup N$ is an anti $\Omega$-fuzzy subbigroup of a bigroup $G = E \cup F$, then

(i) $\mu_M(x+y, q) = \mu_M(y+x, q)$ if and only if $\mu_M(x, q) = \mu_M(-y+x+y, q)$ for all x and y in E and q in $\Omega$

(ii) $\mu_N(xy, q) = \mu_N(yx, q)$ if and only if $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$ for all x and y in F and q in $\Omega$.

**Proof:**

(i) Let x and y be in E and q in $\Omega$. Assume that $\mu_M(x+y, q) = \mu_M(y+x, q)$, then $\mu_M(-y+x+y, q) = \mu_M(-y+y+x, q) = \mu_M(e_1+x, q) = \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(-y+x+y, q)$ for all x and y in E and q in $\Omega$. Conversely assume that $\mu_M(x, q) = \mu_M(-y+x+y, q)$, then $\mu_M(x+y, q) = \mu_M(x+y-x+x, q) = \mu_M(y+x, q)$. Therefore $\mu_M(x+y, q) = \mu_M(y+x, q)$ for all x and y in E and q in $\Omega$.

(ii) Let x and y be in F and q in $\Omega$. Assume that $\mu_N(x+y, q) = \mu_N(y+x, q)$, then $\mu_N(y^{-1}xy, q) = \mu_N(y^{-1}yx, q) = \mu_N(e_2x, q) = \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$ for all x and y in F and q in $\Omega$. Conversely, assume that $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$, then $\mu_N(xy, q) = \mu_N(xyxx^{-1}, q) = \mu_N(yx, q)$. Therefore $\mu_N(xy, q) = \mu_N(yx, q)$ for all x and y in F and q in $\Omega$.

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