International Journal of Mathematical Archive-6(7), 2015, 102-105
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ANTI Ω-FUZZY SUBBIGROUP OF A BIGROUP

T. JUSTIN PRABU*
Department of Mathematics,
Alagappa University Evening College, Paramakudi-623707, Tamilnadu, India.

K. ARJUNAN
Department of Mathematics,
H. H. The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.

(Received On: 07-07-15; Revised & Accepted On: 22-07-15)

ABSTRACT

In this paper, we made an attempt to study the algebraic nature of anti Ω-fuzzy subbigroup of a bigroup.

2000 AMS Subject classification: *03F55, 06D72, 08A72.*

Keywords: *Bigroup, fuzzy subset, fuzzy subbigroup, Ω-fuzzy subbigroup, anti Ω-fuzzy subbigroup.*

INTRODUCTION

In 1965, the fuzzy subset was introduced by L.A.Zadeh [10], after that several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups was introduced by Azriel Rosenfeld [2] and Gyu Ihn Chae, Young Sik Park and Chul Hwan Park [3] have introduced and defined a new algebraic structure called Ω-bifuzzy subsemigroup. After that A.Solairaju, R.Nagarajan [7, 8, 9] and K.Arjunan, Selvak Kumaraen [6] extend the theory to many algebraic structure. N.Palaniappan & K.Arjunan [4] defined a new algebraic structure called anti fuzzy ideal. In this paper, we introduce the some theorems in anti Ω-fuzzy subbigroup of a bigroup.

1. PRELIMINARIES

1.1 Definition: A set $(G, +, \bullet)$ with two binary operations $+$ and $\bullet$ is called a bigroup if there exist two proper subsets G_1 and G_2 of G such that

(i) $G = G_1 \cup G_2$
(ii) $(G_1, +)$ is a group
(iii) $(G_2, \bullet)$ is a group.

1.2 Definition: Let X be a non–empty set. A **fuzzy subset A** of X is a function $A: X \to [0, 1]$.

1.3 Definition: Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup. Then a fuzzy set A of G is said to be a fuzzy subbigroup of G if there exist two fuzzy subsets A_1 of G_1 and A_2 of G_2 such that

(i) $A = A_1 \cup A_2$
(ii) A_1 is a fuzzy subgroup of $(G_1, +)$
(iii) A_2 is a fuzzy subgroup of $(G_2, \bullet)$.

1.4 Definition: Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup and Ω be a nonempty set. The fuzzy subset $A: G \times \Omega \to [0, 1]$ of G is said to be a Ω-fuzzy subbigroup of G if there exist two fuzzy subsets $A_1: G_1 \times \Omega \to [0, 1]$ of G_1 and $A_2 : G_2 \times \Omega \to [0, 1]$ of G_2 such that

(i) $A = A_1 \cup A_2$
(ii) A_1 is a Ω-fuzzy subgroup of $(G_1, +)$
(iii) A_2 is a Ω-fuzzy subgroup of $(G_2, \bullet)$.

Corresponding Author: T. Justin Prabu*, Department of Mathematics,
Alagappa University Evening College, Paramakudi-623707, Tamilnadu, India.

1.5 Definition: Let G = ($G_1 \cup G_2$, +, •) be a bigroup and Ω be a nonempty set. The fuzzy subset A: G×Ω→[0, 1] of G is said to be a anti Ω-fuzzy subbigroup of G if there exist two fuzzy subsets A_1: G_1×Ω→[0, 1] of G_1 and A_2 : G_2×Ω→ [0, 1] of G_2 such that

(i) A = $A_1 \cup A_2$
(ii) A_1 is an anti Ω-fuzzy subgroup of (G_1, +)
(iii) A_2 is an anti Ω-fuzzy subgroup of (G_2, •).

2. PROPERTIES

2.1 Theorem: If A = M∪N is an anti Ω-fuzzy subbigroup of a bigroup G = E∪F, then $\mu_M(-x, q) = \mu_M(x, q)$, $\mu_M(x, q) \geq \mu_M(e, q)$, $\mu_N(x^{-1}, q) = \mu_N(x, q)$, $\mu_N(x, q) \geq \mu_N(e', q)$ for all x, e in E and x, e' in F and q in Ω.

Proof: Let x, e in E and x, e' in F and q in Ω. Now $\mu_M(x, q) = \mu_M((-(-x)), q) \leq \mu_M(-x, q) \leq \mu_M(x, q)$. Therefore $\mu_M(-x, q) = \mu_M(x, q)$ for all x in E and q in Ω. And $\mu_M(e, q) = \mu_M(x-x, q) \leq \max \{\mu_M(x, q), \mu_M(x, q)\} = \mu_M(x, q)$. Therefore $\mu_M(e, q) \leq \mu_M(x, q)$ for all x, e in E and q in Ω. Also $\mu_N(x, q) = \mu_N((x^{-1})^{-1}, q) \leq \mu_N(x^{-1}, q) \leq \mu_N(x, q)$. Therefore $\mu_N(x^{-1}, q) = \mu_N(x, q)$ for all x in F and q in Ω. And $\mu_N(e', q) = \mu_N(xx^{-1}, q) \leq \max\{\mu_N(x, q), \mu_N(x^{-1}, q)\} = \mu_N(x, q)$. Therefore $\mu_N(e', q) \leq \mu_N(x, q)$ for all x, e' in F and q in Ω.

2.2 Theorem: If A = M∪N is an anti Ω-fuzzy subbigroup of a bigroup G = E∪F, then

(i) $\mu_M(x-y, q) = \mu_M(e, q)$ gives $\mu_M(x, q) = \mu_M(y, q)$ for all x, y and e in E and q in Ω
(ii) $\mu_N(xy^{-1}, q) = \mu_N(e', q)$ gives $\mu_N(x, q) = \mu_N(y, q)$ for all x, y and e' in F and q in Ω.

Proof:

(i) Let x, y and e in E and q in Ω. Then $\mu_M(x, q) = \mu_M(x-y+y, q) \leq \max \{\mu_M(x-y, q), \mu_M(y, q)\} = \max \{\mu_M(e, q), \mu_M(y, q)\} = \mu_M(y, q) = \mu_M(y-x+x, q) \leq \max \{\mu_M(y-x, q), \mu_M(x, q)\} = \max \{\mu_M(e, q), \mu_M(x, q)\} = \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in Ω.
(ii) Let x, y and e' in F and q in Ω. Then $\mu_N(x, q) = \mu_N(xy^{-1}y, q) \leq \max \{\mu_N(xy^{-1}, q), \mu_N(y, q)\} = \max \{\mu_N(e', q), \mu_N(y, q)\} = \mu_N(y, q) = \mu_N(yx^{-1}x, q) \leq \max \{\mu_N(yx^{-1}, q), \mu_N(x, q)\} = \max \{\mu_N(e', q), \mu_N(x, q)\} = \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in Ω.

2.3 Theorem: If A= M∪N is an anti Ω-fuzzy subbigroup of a bigroup G = E∪F, then

(i) $H_1 = \{x / x \in E \text{ and } \mu_M(x, q) = 0\}$ is either empty or a subgroup of E.
(ii) $H_2 = \{x / x \in F \text{ and } \mu_N(x, q) = 0\}$ is either empty or a subgroup of F.
(iii) K = $H_1 \cup H_2$ is either empty or a subbigroup of G.

Proof: If no element satisfies this condition, then H_1 and H_2 are empty. Also K = $H_1 \cup H_2$ is empty.

(i) If x and y in H_1, then $\mu_M(x-y, q) \leq \max \{\mu_M(x, q), \mu_M(y, q) \} \leq \max \{0, 0\} = 0$. Therefore $\mu_M(x-y, q) = 0$. We get x–y in H_1. Hence H_1 is a subgroup of G_1.
(ii) If x and y in H_2, then $\mu_N(xy^{-1}, q) \leq \max \{\mu_N(x, q), \mu_N(y, q)\} = \max \{0, 0\} = 0$. Therefore $\mu_N(xy^{-1}, q) = 0$. We get xy^{-1} in H_2. Hence H_2 is a subgroup of G_2.
(iii) From (i) and (ii) we get K = $H_1 \cup H_2$ is a subbigroup of G.

2.4 Theorem: If A = M∪N is an anti Ω-fuzzy subbigroup of a bigroup G = E∪F, then

(i) $H_1 = \{x / x \in E \text{ and } \mu_M(x, q) = \mu_M(e, q)\}$ is a subgroup of E
(ii) $H_2 = \{x / x \in F \text{ and } \mu_N(x, q) = \mu_N(e', q)\}$ is a subgroup of F
(iii) K = $H_1 \cup H_2$ is a subbigroup of G.

Proof:

(i) Clearly e in H_1 so H_1 is a non empty. Let x and y be in H_1. Then $\mu_M(x-y, q) \leq \max \{\mu_M(x, q), \mu_M(y, q)\} = \max \{\mu_M(e, q), \mu_M(e, q)\} = \mu_M(e, q)$. Therefore $\mu_M(x-y, q) \leq \mu_M(e, q)$ for all x and y in H_1 and q in Ω. We get $\mu_M(x-y, q) = \mu_M(e, q)$ for all x and y in H_1 and q in Ω. Therefore x–y in H_1. Hence H_1 is a subgroup of E.
(ii) Clearly e' in H_2 so H_2 is a non empty. Let x and y be in H_2. Then $\mu_N(xy^{-1}, q) \leq \max \{\mu_N(x, q), \mu_N(y, q)\} = \max \{\mu_N(e', q), \mu_N(e', q)\} = \mu_N(e', q)$. Therefore $\mu_N(xy^{-1}, q) \leq \mu_N(e', q)$ for all x and y in H_2 and q in Ω. We get $\mu_N(xy^{-1}, q) = \mu_N(e', q)$ for all x and y in H_2 and q in Ω. Therefore xy^{-1} in H_2. Hence H_2 is a subgroup of F.
(iii) From (i) and (ii) we get K = $H_1 \cup H_2$ is a subbigroup of G.

2.5 Theorem: Let A= M∪N be an anti Ω-fuzzy subbigroup of a bigroup G = E∪F.

(i) If $\mu_M(x-y, q) = 0$, then $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in Ω.
(ii) If $\mu_N(xy^{-1}, q) = 0$, then $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in Ω.

Proof:

(i) Let x and y belongs to E and q in Ω. Then $\mu_M(x, q) = \mu_M(x-y+y, q) \leq \max\{\mu_M(x-y, q), \mu_M(y, q)\} = \max\{0, \mu_M(y, q)\} = \mu_M(y, q) = \mu_M(-y, q) = \mu_M(-x+x-y, q) \leq \max\{\mu_M(-x, q), \mu_M(x-y, q)\} = \max\{\mu_M(-x, q), 0\} = \mu_M(-x, q) = \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in Ω.

(ii) Let x and y belongs to F and q in Ω. Then $\mu_N(x, q) = \mu_N(xy^{-1}y, q) \leq \max\{\mu_N(xy^{-1}, q), \mu_N(y, q)\} = \max\{0, \mu_N(y, q)\} = \mu_N(y, q) = \mu_N(y^{-1}, q) = \mu_N(x^{-1}xy^{-1}, q) \leq \max\{\mu_N(x^{-1}, q), \mu_N(xy^{-1}, q)\} = \max\{\mu_N(x^{-1}, q), 0\} = \mu_N(x^{-1}, q) = \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in Ω.

2.6 Theorem: If $A = M\cup N$ is an anti Ω-fuzzy subbigroup of a bigroup $G = E\cup F$, then

(i) $\mu_M(x+y, q) = \max\{\mu_M(x, q), \mu_M(y, q)\}$ for each x and y in E and q in Ω with $\mu_M(x, q) \neq \mu_M(y, q)$

(ii) $\mu_N(xy, q) = \max\{\mu_N(x, q), \mu_N(y, q)\}$ for each x and y in F and q in Ω with $\mu_N(x, q) \neq \mu_N(y, q)$.

Proof:

(i) Let x and y belongs to E and q in Ω. Assume that $\mu_M(x, q) < \mu_M(y, q)$, then $\mu_M(y, q) = \mu_M(-x+x+y, q) \leq \max\{\mu_M(-x, q), \mu_M(x+y, q)\} \leq \max\{\mu_M(x, q), \mu_M(x+y, q)\} = \mu_M(x+y, q) \leq \max\{\mu_M(x, q), \mu_M(y, q)\} = \mu_M(y, q)$. Therefore $\mu_M(x+y, q) = \mu_M(y, q) = \max\{\mu_M(x, q), \mu_M(y, q)\}$ for x and y in E and q in Ω.

(ii) Let x and y belongs to F and q in Ω. Assume that $\mu_N(x, q) < \mu_N(y, q)$, then $\mu_N(y, q) = \mu_N(x^{-1}xy, q) \leq \max\{\mu_N(x^{-1}, q), \mu_N(xy, q)\} \leq \max\{\mu_N(x, q), \mu_N(xy, q)\} = \mu_N(xy, q) \leq \max\{\mu_N(x, q), \mu_N(y, q)\} = \mu_N(y, q)$.
Therefore $\mu_N(xy, q) = \mu_N(y, q) = \max\{\mu_N(x, q), \mu_N(y, q)\}$ for x and y in F and q in Ω.

2.7 Theorem: If $A = M\cup N$ and $B = O\cup P$ are two anti Ω-fuzzy subbigroups of a bigroup $G = E\cup F$, then their union $A\cup B$ is an anti Ω-fuzzy subbigroup of G.

Proof: Let $A = M\cup N = \{\langle (x, q), \mu_A(x, q)\rangle / x\in G \text{ and } q\in\Omega\}$ where $M = \{\langle (x, q), \mu_M(x, q)\rangle / x\in E \text{ and } q\in\Omega\}$ and $N = \{\langle (x, q), \mu_N(x, q)\rangle / x\in F \text{ and } q\in\Omega\}$ and $B = O\cup P = \{\langle (x, q), \mu_B(x, q)\rangle / x\in G \text{ and } q\in\Omega\}$ where $O = \{\langle (x, q), \mu_O(x, q)\rangle / x\in E \text{ and } q\in\Omega\}$ and $P = \{\langle (x, q), \mu_P(x, q)\rangle / x\in F \text{ and } q\in\Omega\}$. Let $C = A\cup B = R\cup S$ where $C = \{\langle (x, q), \mu_C(x, q)\rangle / x\in G \text{ and } q\in\Omega\}$, $R = M\cup O = \{\langle (x, q), \mu_R(x, q)\rangle / x\in E \text{ and } q\in\Omega\}$ and $S = N\cup P = \{\langle (x, q), \mu_S(x, q)\rangle / x\in F \text{ and } q\in\Omega\}$. Let x and y belong to E and q in Ω. Then $\mu_R(x-y, q) = \max\{\mu_M(x-y, q), \mu_O(x-y, q)\} \leq \max\{\max\{\mu_M(x, q), \mu_M(y, q)\}, \max\{\mu_O(x, q), \mu_O(y, q)\}\} \leq \max\{\max\{\mu_M(x, q), \mu_O(x, q)\}, \max\{\mu_M(y, q), \mu_O(y, q)\}\} = \max\{\mu_R(x, q), \mu_R(y, q)\}$. Therefore $\mu_R(x-y, q) \leq \max\{\mu_R(x, q), \mu_R(y, q)\}$ for all x and y in E and q in Ω. Let x and y belong to F and q in Ω. Then $\mu_S(xy^{-1}, q) = \max\{\mu_N(xy^{-1}, q), \mu_P(xy^{-1}, q)\} \leq \max\{\max\{\mu_N(x, q), \mu_N(y, q)\}, \max\{\mu_P(x, q), \mu_P(y, q)\}\} \leq \max\{\max\{\mu_N(x, q), \mu_P(x, q)\}, \max\{\mu_N(y, q), \mu_P(y, q)\} = \max\{\mu_S(x, q), \mu_S(y, q)\}$. Therefore $\mu_S(xy^{-1}, q) \leq \max\{\mu_S(x, q), \mu_S(y, q)\}$ for all x and y in F and q in Ω. Hence $A\cup B$ is an anti Ω-fuzzy subbigroup of G.

2.8 Theorem: The union of a family of anti Ω-fuzzy subbigroups of a bigroup G is an anti Ω-fuzzy subbigroup of G.

Proof: It is trivial.

2.9 Theorem: If $A = M\cup N$ is an anti Ω-fuzzy subbigroup of a bigroup $G = E\cup F$, then

(i) $\mu_M(x+y, q) = \mu_M(y+x, q)$ if and only if $\mu_M(x, q) = \mu_M(-y+x+y, q)$ for all x and y in E and q in Ω

(ii) $\mu_N(xy, q) = \mu_N(yx, q)$ if and only if $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$ for all x and y in F and q in Ω.

Proof:

(i) Let x and y be in E and q in Ω. Assume that $\mu_M(x+y, q) = \mu_M(y+x, q)$, then $\mu_M(-y+x+y, q) = \mu_M(-y+y+x, q) = \mu_M(e_1+x, q) = \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(-y+x+y, q)$ for all x and y in E and q in Ω. Conversely assume that $\mu_M(x, q) = \mu_M(-y+x+y, q)$, then $\mu_M(x+y, q) = \mu_M(x+y-x+x, q) = \mu_M(y+x, q)$. Therefore $\mu_M(x+y, q) = \mu_M(y+x, q)$ for all x and y in E and q in Ω.

(ii) Let x and y be in F and q in Ω. Assume that $\mu_N(x+y, q) = \mu_N(y+x, q)$, then $\mu_N(y^{-1}xy, q) = \mu_N(y^{-1}yx, q) = \mu_N(e_2x, q) = \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$ for all x and y in F and q in Ω. Conversely, assume that $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$, then $\mu_N(xy, q) = \mu_N(xyxx^{-1}, q) = \mu_N(yx, q)$. Therefore $\mu_N(xy, q) = \mu_N(yx, q)$ for all x and y in F and q in Ω.

REFERENCE

1. Anthony.J.M. and Sherwood.H, Fuzzy groups Redefined, Journal of mathematical analysis and applications, 69,124 -130 (1979)
2. Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications, 35, 512-517 (1971).
3. Gyu Ihn Chae, Young Sik Park and Chul Hwan Park, Bifuzzy bi-ideals with operators in semigroups, International Mathematical Forum, 2, no. 59, 2927– 2935 (2007).
4. Palaniappan. N & K. Arjunan, 2007. Operation on fuzzy and anti fuzzy ideals, Antartica J. Math., 4(1): 59-64.

5. Rajesh Kumar, Fuzzy Algebra, Volume 1, University of Delhi Publication Division, July -1993.
6. Selvak Kumaraen.N and K. Arjunan, A study on anti (Q, L)-fuzzy subhemiring of a hemiring, International journal of computational and applied mathematics, Vol. 8, Num. 3, 267-274 (2013).
7. Solairaju.A and Nagarajan.R, A New Structure and Construction of Q-Fuzzy Groups, Advances in fuzzy mathematics, Volume 4, Number 1 (2009) pp. 23-29**.**
8. Solairaju.A and Nagarajan.R, Lattice Valued Q-fuzzy left R-submodules of near rings with respect to T-norms, Advances in fuzzy mathematics,Vol 4, Num. 2, 137-145(2009).
9. Solairaju.A and Nagarajan.R, “Q-Fuzzy left R-subgroups of near rings with respect to t-norms”. Antarctica Journal of Mathematics, 5(2008) 1-2, 59-63.
10. Zadeh.L.A, Fuzzy sets, Information and control, Vol.8, 338-353 (1965).

Source of support: Nil, Conflict of interest: None Declared