

**(gsp)\*\* - CLOSED SETS IN TOPOLOGICAL SPACES**

**PUNITHA THARANI**

Associate Professor, St. Mary's College, Tuticorin.

**PRISCILLA PACIFICA\***

Assistant Professor, St. Mary's College, Tuticorin.

(Received On: 23-06-15; Revised & Accepted On: 28-07-15)

**ABSTRACT**

*In this paper we introduce a new class of sets called (gsp)\*\*-closed sets in topological spaces which is properly placed in between the class of closed sets and gsp-closed sets. As an application, we introduce two new spaces namely,  $T_{gsp}^{**}$  space,  ${}_{\alpha}T_{gsp}^{**}$  space. Further, (gsp)\*\*-continuous, (gsp)\*\*-irresolute mappings are also introduced and investigated.*

**Key words:** (gsp)\*\*-closed set, (gsp)\*\*-continuous map, (gsp)\*\*-irresolute map,  $T_{gsp}^{**}$ ,  ${}_{\alpha}T_{gsp}^{**}$  -spaces.

**1. INTRODUCTION**

Levine [11] introduced the class of g-closed sets in 1970. Arya and Tour [3] defined gs-closed sets in 1990. Dontchev [9], Gnanambal [10] Palaniappan and Rao [17] introduced gsp-closed sets, gpr-closed sets and rg-closed sets respectively. Veerakumar [18] introduced g\*-closed sets in 1991. Dontchev [8] introduced gsp-closed sets in 1995. Levine [11] Devi [6,8] introduced  $T_{1/2}$ -spaces,  $T_b$  spaces and  ${}_{\alpha}T_b$  spaces respectively. PaulineMHelen[20] introduced (gsp)\* sets. The purpose of this paper is to introduce the concepts of (gsp)\*\*-closed set, (gsp)\*\*-continuous map, (gsp)\*\*-irresolute maps.  $T_{gsp}^{**}$ -space,  ${}_{\alpha}T_{gsp}^{**}$ -space are introduced and investigated.

**2. PRILIMINARIES**

Throughout this paper  $(X, \tau)$ ,  $(Y, \sigma)$  represent non-empty topological spaces of which no separation axioms are assumed unless otherwise stated. For a subset A of a space  $(X, \tau)$ ,  $cl(A)$  and  $int(A)$  denote the closure and the interior of A respectively. The class of all closed subsets of a space  $(X, \tau)$  is denoted by  $C(X, \tau)$ . The smallest semi-closed (resp. pre-closed and  $\alpha$ -closed) set containing a subset A of  $(X, \tau)$  is called the semi-closure (resp. pre-closure and  $\alpha$ -closure) of A and is denoted by  $scl(A)$  (resp.  $pcl(A)$  and  $\alpha cl(A)$ ).

**Definition 2.1:** A subset A of a topological space  $(X, \tau)$  is called

- (1) a pre-open set [14] if  $A \subseteq int(cl(A))$  and a pre-closed set if  $cl(int(A)) \subseteq A$ .
- (2) a semi-open set [12] if  $A \subseteq cl(int(A))$  and a semi-closed set if  $int(cl(A)) \subseteq A$ .
- (3) a semi-preopen set [1] if  $A \subseteq cl(int(cl(A)))$  and a semi-preclosed set [1] if  $int(cl(int(A))) \subseteq A$ .
- (4) an  $\alpha$ -open set [16] if  $A \subseteq int(cl(int(A)))$  and an  $\alpha$ -closed set [16] if  $cl(int(cl(A))) \subseteq A$ .
- (5) a regular-open set [14] if  $int(cl(A)) = A$  and regular-closed set [14] if  $A = int(cl(A))$ .

**Definition 2.2:** A subset A of topological space  $(X, \tau)$  is called

- (1) a generalized closed set (briefly g-closed) [1] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open in  $(X, \tau)$ .
- (2) generalized semi-closed set (briefly gs-closed) [3] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open in  $(X, \tau)$ .
- (3) an  $\alpha$ -generalized closed set (briefly  $\alpha g$ -closed) [19] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open in  $(X, \tau)$ .
- (4) a generalized semi pre-closed set (briefly gsp-closed) [9] if  $sp cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open in  $(X, \tau)$ .
- (5) a regular generalized closed set (briefly rg-closed) [17] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is regular open in  $(X, \tau)$ .

**Corresponding Author: Priscilla Pacifica\***  
 Assistant Professor, St. Mary's College, Tuticorin.

- (6) a generalized pre-closed set (briefly gp-closed) [13] if  $p\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- (7) a generalized pre regular-closed set (briefly gpr-closed)[10] if  $p\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $(X, \tau)$ .
- (8) a  $g^*$ -closed set [18] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ .
- (9) a  $wg$ -closed set [16] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- (10) a  $(gsp)^*$ -closed set [20] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $gsp$ -open in  $(X, \tau)$ .

**Definition 2.3:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called

- (1)  $g$ -continuous [4] if  $f^{-1}(V)$  is a  $g$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (2)  $\alpha g$ -continuous [10] if  $f^{-1}(V)$  is an  $\alpha g$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (3)  $gs$ -continuous [7] if  $f^{-1}(V)$  is a  $gs$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (4)  $gsp$ -continuous [9] if  $f^{-1}(V)$  is a  $gsp$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (5)  $rg$ -continuous [17] if  $f^{-1}(V)$  is a  $rg$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (6)  $gp$ -continuous [2] if  $f^{-1}(V)$  is a  $gp$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (7)  $gpr$ -continuous [10] if  $f^{-1}(V)$  is a  $gpr$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (8)  $g^*$ -continuous [18] if  $f^{-1}(V)$  is a  $g$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (9)  $wg$ -continuous [16] if  $f^{-1}(V)$  is a  $wg$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (10)  $(gsp)^*$ -continuous [20] if  $f^{-1}(V)$  is an  $(gsp)^*$ -closed set of  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

**Definition 2.4:** A topological space  $(X, \tau)$  is said to be

- (1) a  $T_{1/2}$ -space [11] if every  $g$ -closed set in it is closed.
- (2) a  $T_b$  space [6] if every  $gs$ -closed set in it is closed.
- (3) a  ${}_{\alpha}T_b$  -space [8] if every  $\alpha g$ -closed set in it is closed.
- (4) a  $T_{1/2}^*$ -space [18] if every  $g^*$ -closed set in it is closed.
- (5) a  $T_{gsp}^*$ -space [20] if every  $(gsp)^*$ -closed set is closed.
- (6) a  $gT_{gsp}^*$ -space [20] if every  $g$ -closed set is  $(gsp)^*$  closed.

### 3. Basic properties of $(gsp)^*$ - closed sets

We introduce the following definition

**Definition 3.1:** A subset  $A$  of  $(X, \tau)$  is said to be a  $(gsp)^*$ -closed set if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $(gsp)^*$ -open in  $X$ .

**Proposition 3.2:** Every closed set is  $(gsp)^*$ -closed.

**Proof:** Let  $A$  be closed set, Then  $\text{cl}(A) = A$ .

Let  $A \subseteq U$  and  $U$  be  $(gsp)^*$ -open.

Then  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $(gsp)^*$ -open.

Therefore  $A$  is  $(gsp)^*$ -closed.

**Proposition 3.3:** Every  $(gsp)^*$ -closed set is  $gs$ -closed.

**Proof:** Let  $A$  be a  $(gsp)^*$ -closed set. Let  $A \subseteq U$  and  $U$  be open. Then  $\text{cl}(A) \subseteq U$  since  $U$  is  $(gsp)^*$ -open and  $A$  is  $(gsp)^*$ -closed.  $\text{scl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is  $gs$ -closed.

The converse of the above proposition need not be true in general as seen in the following example.

**Example 3.4:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ . Then  $A = \{b\}$  is  $gs$ -closed but not  $(gsp)^*$ -closed in  $(X, \tau)$ .

**Proposition 3.5:** Every  $(gsp)^*$ -closed set is  $\alpha g$ -closed, but not conversely.

**Proof:** Let  $A$  be a  $(gsp)^*$ -closed set.  $\text{cl}(A) \subseteq U$  since  $U$  is  $(gsp)^*$ -open and  $A$  is  $(gsp)^*$ -closed. But  $\alpha\text{cl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is  $\alpha g$ -closed.

**Example 3.6:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ . Then  $A = \{b\}$  is  $\alpha g$ -closed but not  $(gsp)^*$ -closed in  $(X, \tau)$ .

**Proposition 3.7:** Every (gsp)\*\*-closed set is gsp-closed, but not conversely.

**Proof:** Let  $A$  be a (gsp)\*\*-closed set. Let  $A \subseteq U$  and  $U$  be open. Then  $\text{cl}(A) \subseteq U$  since  $U$  is (gsp)\*-open and  $A$  is (gsp)\*\*-closed.  $\text{spcl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is gsp-closed.

**Example 3.8:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$ . Then  $A = \{b\}$  is gsp-closed but not (gsp)\*\*-closed in  $(X, \tau)$ .

**Proposition 3.9:** Every (gsp)\*\*-closed set is rg-closed.

**Proof:** Let  $A$  be a (gsp)\*\*-closed set. Let  $A \subseteq U$  and  $U$  be regular open. Then  $A \subseteq U$  and  $U$  is (gsp)\*-open and  $\text{cl}(A) \subseteq U$ , since  $A$  is (gsp)\*\*-closed. Hence  $A$  is rg-closed.

The converse of the above proposition need not be true in general as seen in the following example.

**Example 3.10:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\varphi, X, \{b\}, \{a, b\}\}$ . Then  $A = \{a\}$  is rg-closed but not (gsp)\*\*-closed in  $(X, \tau)$ .

**Proposition 3.11:** Every (gsp)\*\*-closed set is gp-closed, but not conversely.

**Proof:** Let  $A$  be a (gsp)\*\*-closed set. Let  $A \subseteq U$  and  $U$  be open. Then  $\text{cl}(A) \subseteq U$  since  $U$  is (gsp)\*-open and  $A$  is (gsp)\*\*-closed.  $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is gp-closed.

The converse of the above proposition need not be true in general as seen in the following example.

**Example 3.12:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$ . Then  $A = \{b\}$  is gp-closed but not (gsp)\*\*-closed in  $(X, \tau)$ .

**Proposition 3.13:** Every (gsp)\*\*-closed set is gpr-closed, but not conversely.

**Proof:** Let  $A$  be a (gsp)\*\*-closed set. Let  $A \subseteq U$  and  $U$  be regular open. Then  $A \subseteq U$  and  $U$  is (gsp)\*-open and  $\text{cl}(A) \subseteq U$ , since  $A$  is (gsp)\*\*-closed. Then  $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is gpr-closed.

**Example 3.14:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$ . Then  $A = \{a\}$  is gpr-closed but not (gsp)\*\*-closed in  $(X, \tau)$ .

**Proposition 3.15:** Every (gsp)\*\*-closed set is wg-closed, but not conversely.

**Proof:** Let  $A$  be a (gsp)\*\*-closed set. Let  $A \subseteq U$  and  $U$  be open. Then  $U$  is (gsp)\*-open and  $\text{cl}(A) \subseteq U$ , since  $A$  is (gsp)\*\*-closed.  $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is wg-closed.

**Example 3.16:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\varphi, X, \{a\}, \{a, b\}\}$ . Then  $A = \{b\}$  is wg-closed but not (gsp)\*\*-closed in  $(X, \tau)$ .

**Proposition 3.17:** If  $A$  and  $B$  are (gsp)\*\*-closed sets then  $A \cup B$  is also (gsp)\*\*-closed.

**Proof:** follows from the fact that  $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B)$ .

**Proposition 3.18:** If  $A$  is (gsp)\*\*-closed set of  $(X, \tau)$  such that  $A \subseteq B \subseteq \text{cl}(A)$ , then  $B$  is also a (gsp)\*\*-closed set of  $(X, \tau)$ .

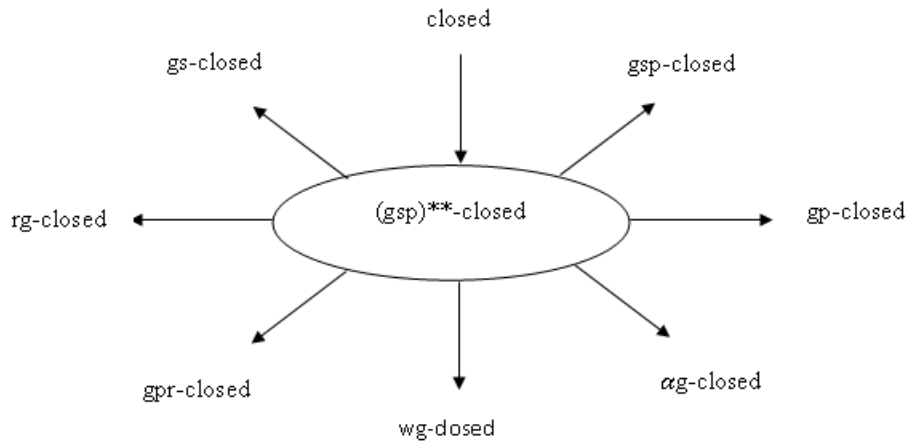
**Proof:** Let  $U$  be the (gsp)\*-open set of  $(X, \tau)$  such that  $B \subseteq U$ . Then  $A \subseteq U$  where  $U$  is (gsp)\*-open. Since  $A$  is (gsp)\*\*-closed,  $\text{cl}(A) \subseteq U$ . Then  $\text{cl}(B) \subseteq U$ , Hence  $B$  is (gsp)\*\*-closed.

**Proposition 3.19:** If  $A$  is (gsp)\*\*-closed set of  $(X, \tau)$ , then  $\text{cl}(A) \setminus A$  does not contain any non-empty (gsp)\*-closed set.

**Proof:** Let  $F$  be (gsp)\*-closed set of  $(X, \tau)$  such that  $F \subseteq \text{cl}(A) \setminus A$ . Then  $A \subseteq X \setminus F$ . Since  $A$  is (gsp)\*\*-closed  $\text{cl}(A) \subseteq X \setminus F$ . This implies  $F \subseteq X \setminus \text{cl}(A)$ . Hence  $F \subseteq (X \setminus \text{cl}(A)) \cap (\text{cl}(A) \setminus A) = \emptyset$ . Hence  $\text{cl}(A) \setminus A$  does not contain any non-empty (gsp)\*-closed set.

**Proposition 3.20:** If  $A$  is both (gsp)\*-open and (gsp)\*\*-closed then  $A$  is closed.

The above results can be represented in the following figure.



Where  $A \rightarrow B$  represents  $A$  implies  $B$  and  $B$  need not imply  $A$ .

#### 4. (gsp)\*\*-continuous and (gsp)\*\*-irresolute maps

We introduce the following definitions:

**Definition 4.1:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called (gsp)\*\*-continuous if  $f^{-1}(V)$  is a (gsp)\*\*-closed set in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

**Definition 4.2:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called (gsp)\*\*-irresolute if  $f^{-1}(V)$  is a (gsp)\*\*-closed set in  $(X, \tau)$  for every (gsp)\*\*-closed set  $V$  of  $(Y, \sigma)$ .

**Theorem 4.3:** Every continuous map is (gsp)\*\*-continuous.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a continuous map. Let  $F$  be a closed set in  $(Y, \sigma)$  since  $f$  is continuous  $f^{-1}(F)$  is closed in  $(X, \tau)$  and hence  $f^{-1}(F)$  is (gsp)\*\*-closed. Therefore  $f$  is (gsp)\*\*-continuous.

**Theorem 4.4:** Every (gsp)\*\*-continuous map is (1) gs-continuous (2)  $\alpha$ g-continuous (3) gsp-continuous (4) rg-continuous (5) gp-continuous (6) gpr-continuous and (7) wg-continuous but not conversely.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be (gsp)\*\*-continuous and let  $F$  be a closed set of  $(Y, \sigma)$ . Since  $f$  is (gsp)\*\*-continuous  $f^{-1}(F)$  is (gsp)\*\*-closed in  $(X, \tau)$ . Then  $f^{-1}(F)$  is gs- closed,  $\alpha$ g- closed, gsp- closed, rg- closed, gp- closed, gpr- closed and wg- closed. Hence  $f$  is gs-continuous,  $\alpha$ g-continuous, gsp-continuous, rg-continuous, gp-continuous, gpr-continuous and wg-continuous.

**Example 4.5:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ ,  $\sigma = \{\emptyset, Y, \{a, c\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity map. The closed sets of  $Y$  are  $\emptyset, Y, \{b\}$ .  $f^{-1}(b) = b$  is not (gsp)\*\*-closed in  $(X, \tau)$ . Hence  $f$  is not (gsp)\*\*-continuous.  $f^{-1}(b) = b$  is gs- closed,  $\alpha$ g- closed, gsp- closed, gp- closed and wg-closed. Hence  $f$  is gs-continuous,  $\alpha$ g-continuous, gsp-continuous, gp-continuous and wg-continuous.

**Example 4.6:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ ,  $\sigma = \{\emptyset, Y, \{b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be defined as  $f(a) = c$ ,  $f(b) = a$ ,  $f(c) = b$ .  $f^{-1}\{a, c\} = \{a, b\}$  is gpr- closed in  $(X, \tau)$ , but not (gsp)\*\*-closed in  $(X, \tau)$ . Hence  $f$  is gpr-continuous but not (gsp)\*\*-continuous.

**Example 4.7:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\emptyset, X, \{b\}, \{a, b\}\}$ ,  $\sigma = \{\emptyset, Y, \{c\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity map. The closed sets of  $Y$  are  $\emptyset, Y$  and  $\{a, b\}$ .  $f^{-1}\{a, b\} = \{a, b\}$  is rg- closed but not (gsp)\*\*-closed and hence  $f$  is rg-continuous but not (gsp)\*\*- continuous.

**Theorem 4.8:** Every (gsp)\*\*-irresolute is (gsp)\*\*- continuous.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a (gsp)\*\*- irresolute. Let  $V$  be a closed set of  $(Y, \sigma)$ . Then  $V$  is (gsp)\*\*-closed and  $f^{-1}(V)$  is (gsp)\*\*-closed since  $f$  is a (gsp)\*\*-irresolute. Hence  $f$  is (gsp)\*\*-continuous.

**Theorem 4.9:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a (gsp)\*\*-irresolute then  $f$  is

- (1) gs-continuous
- (2)  $\alpha$ g-continuous
- (3) gsp- continuous
- (4) rg-continuous
- (5) gp-continuous
- (6) gpr-continuous and
- (7) wg-continuous but not conversely.

**Proof:** Since every (gsp)\*\*-irresolute is (gsp)\*\*- continuous,  $f$  is (gsp)\*\*- continuous. Then by theorem 4.4 the result follows.

**Example 4.10:** Let  $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a\}, \{a, b\}\}, \sigma = \{\varphi, Y, \{a, b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be defined as  $f(a) = a, f(b) = c, f(c) = b. f^{-1}\{c\} = \{b\}$  is gs- closed in  $(X, \tau)$  and hence  $f$  is gs- continuous. (gsp)\*\*-closed sets of  $(Y, \sigma)$  are  $\varphi, Y, \{c\}, \{a, c\}$  and  $\{b, c\}. f^{-1}\{c\} = \{b\}$  is not (gsp)\*\*-closed set in  $(X, \tau)$ . Hence  $f$  is not a (gsp)\*\*-irresolute.

**Example 4.11:** Let  $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a\}, \{a, b\}\}, \sigma = \{\varphi, Y, \{a, b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be defined as  $f(a) = a, f(b) = c, f(c) = b. f^{-1}\{c\} = \{b\}$  is gp- closed in  $(X, \tau)$  and hence  $f$  is gp- continuous. (gsp)\*\*-closed sets of  $(Y, \sigma)$  are  $\varphi, Y, \{c\}, \{a, c\}$  and  $\{b, c\}. f^{-1}\{c\} = \{b\}$  is not (gsp)\*\*-closed set in  $(X, \tau)$ . Hence  $f$  is not a (gsp)\*\*-irresolute.

**Example 4.12:** Let  $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{b\}, \{a, b\}\}, \sigma = \{\varphi, Y, \{a, b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be defined by  $f(a) = b, f(b) = c, f(c) = a. f^{-1}\{c\} = \{b\}$  is rg- closed in  $(X, \tau)$  and hence  $f$  is rg- continuous. (gsp)\*\*-closed sets of  $(Y, \sigma)$  are  $\varphi, Y, \{c\}, \{a, c\}$  and  $\{b, c\}. f^{-1}\{c\} = \{b\}$  is not (gsp)\*\*-closed set in  $(X, \tau)$ . Hence  $f$  is not a (gsp)\*\*-irresolute.

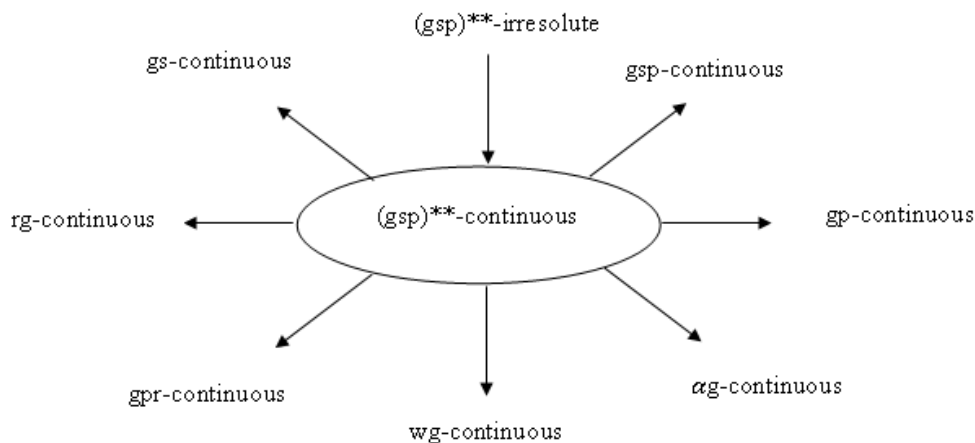
**Example 4.13:** Let  $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{b, c\}\}, \sigma = \{\varphi, Y, \{a, b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity map.  $f^{-1}\{c\} = \{c\}$  is gsp- closed in  $(X, \tau)$  and hence  $f$  is gsp- continuous. (gsp)\*\*-closed sets of  $(Y, \sigma)$  are  $\varphi, Y, \{c\}, \{a, c\}$  and  $\{b, c\}. f^{-1}\{c\} = \{c\}$  is not (gsp)\*\*-closed set in  $(X, \tau)$ . Hence  $f$  is not a (gsp)\*\*-irresolute.

**Example 4.14:** Let  $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a\}, \{a, b\}\}, \sigma = \{\varphi, Y, \{a, b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be defined as  $f(a) = a, f(b) = c, f(c) = b. f^{-1}\{c\} = \{b\}$  is wg- closed in  $(X, \tau)$  and hence  $f$  is wg- continuous. (gsp)\*\*-closed sets of  $(Y, \sigma)$  are  $\varphi, Y, \{c\}, \{a, c\}$  and  $\{b, c\}. f^{-1}\{c\} = \{b\}$  is not (gsp)\*\*-closed set in  $(X, \tau)$ . Hence  $f$  is not a (gsp)\*\*-irresolute.

**Example 4.15:** Let  $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a\}, \{a, b\}\}, \sigma = \{\varphi, Y, \{a, b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be defined as  $f(a) = b, f(b) = c, f(c) = a. f^{-1}\{c\} = \{b\}$  is  $\alpha$ g- closed in  $(X, \tau)$  and hence  $f$  is  $\alpha$ g- continuous. (gsp)\*\*-closed sets of  $(Y, \sigma)$  are  $\varphi, Y, \{c\}, \{a, c\}$  and  $\{b, c\}. f^{-1}\{c\} = \{b\}$  is not (gsp)\*\*-closed set in  $(X, \tau)$ . Hence  $f$  is not a (gsp)\*\*-irresolute.

**Example 4.16:** Let  $X = \{a, b, c\} = Y, \tau = \{\varphi, X, \{a\}, \{a, b\}\}, \sigma = \{\varphi, Y, \{a, b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be defined as  $f(a) = b, f(b) = c, f(c) = a. f^{-1}\{c\} = \{b\}$  is gpr- closed in  $(X, \tau)$  and hence  $f$  is gpr- continuous. (gsp)\*\*-closed sets of  $(Y, \sigma)$  are  $\varphi, Y, \{c\}, \{a, c\}$  and  $\{b, c\}. f^{-1}\{c\} = \{b\}$  is not (gsp)\*\*-closed set in  $(X, \tau)$ . Hence  $f$  is not a (gsp)\*\*-irresolute.

The above results can be represented in the following figure.



Where  $A \rightarrow B$  represents  $A$  implies  $B$  and  $B$  need not imply  $A$ .

## 5. APPLICATIONS OF (gsp)\*\*-CLOSED SETS

**Definition 5.1:** A space  $(X, \tau)$  is called a  $T_{gsp}^{**}$ -space if every (gsp)\*\*-closed set is closed.

**Definition 5.2:** A space  $(X, \tau)$  is called a  ${}_{\alpha}T_{gsp}^{**}$ -space if every  $\alpha g$ -closed set is (gsp)\*\*-closed.

**Theorem 5.3:** Every  $T_b$ -space is  $T_{gsp}^{**}$ -space but not conversely.

**Proof:** Let  $(X, \tau)$  be a  $T_b$ -space. Let  $A$  be a (gsp)\*\*-closed set. Since every (gsp)\*\*-closed set is  $g_s$ -closed and hence  $A$  is  $g_s$ -closed. Since  $(X, \tau)$  is a  $T_b$ -space,  $A$  is closed. Hence  $(X, \tau)$  is a  $T_{gsp}^{**}$ -space.

**Example 5.4:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ .  $(X, \tau)$  is a  $T_{gsp}^{**}$ -space  $A = \{a\}$  is  $g_s$ -closed, but it is not closed, and hence it is not a  $T_b$ -space. Hence a  $T_{gsp}^{**}$ -space need not be a  $T_b$ -space.

**Theorem 5.5:** Every  $T_b$ -space is a  ${}_{\alpha}T_{gsp}^{**}$ -space.

**Proof:** Let  $(X, \tau)$  be a  $T_b$ -space. Let  $A$  be  $\alpha g$ -closed. Then  $A$  is  $g_s$ -closed. Since the space is  $T_b$ -space,  $A$  is closed and hence  $A$  is (gsp)\*\*-closed. Therefore the space  $(X, \tau)$  is a  ${}_{\alpha}T_{gsp}^{**}$ -space.

**Example 5.6:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ . Here (gsp)\*\*-closed sets are  $\{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$ ,  $\alpha g$ -closed sets are  $\{\emptyset, X, \{c\}, \{b, c\}, \{a, c\}\}$  and the  $g_s$ -closed sets are  $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ . Since every  $\alpha g$ -closed set is (gsp)\*\*-closed the space  $(X, \tau)$  is a  ${}_{\alpha}T_{gsp}^{**}$ -space.  $A = \{a\}$  is  $g_s$ -closed, but it is not closed, and hence it is not a  $T_b$ -space.

**Theorem 5.7:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be (gsp)\*\*-continuous map and let  $(X, \tau)$  be a  $T_{gsp}^{**}$ -space then  $f$  is continuous.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be (gsp)\*\*-continuous map. Let  $F$  be a closed set of  $(Y, \sigma)$ . Since  $f$  is (gsp)\*\*-continuous,  $f^{-1}(F)$  is (gsp)\*\*-closed set in  $(X, \tau)$ . Since  $(X, \tau)$  is a  $T_{gsp}^{**}$ -space,  $f^{-1}(F)$  is closed in  $(X, \tau)$ . Therefore  $f$  is continuous.

**Theorem 5.8:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be  $\alpha g$ -continuous map where  $(X, \tau)$  is a  ${}_{\alpha}T_{gsp}^{**}$ -space. Then  $f$  is (gsp)\*\*-continuous.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $\alpha g$ -continuous map. Let  $F$  be a closed set of  $(Y, \sigma)$ . Since  $f$  is  $\alpha g$ -continuous,  $f^{-1}(F)$  is  $\alpha g$ -closed set in  $(X, \tau)$ . Since  $(X, \tau)$  is a  ${}_{\alpha}T_{gsp}^{**}$ -space,  $f^{-1}(F)$  is (gsp)\*\*-closed in  $(X, \tau)$ . Therefore  $f$  is (gsp)\*\*-continuous.

## REFERENCES

1. D.Andrijevic, Semi-preopen sets, Mat. Vesnik, 38(1) (1986), 24-32.
2. I.Arokiarani, K.Balachandran and J.Dontchev, Some characterizations of  $g_p$ -irresolute and  $g_p$ -continuous maps between topological spaces, Mem. Fac. Sci. Kochi Univ.Ser.A. Math., 20(1999), 93-104.
3. S.P.Arya and T.Nour, Characterizations of  $s$ -normal spaces, Indian J. Pure. Appl. Math., 21(8) (1990), 717-719.
4. K.Balachandran, P.Sundram and H.Maki, On generalized continuous maps in topological spaces, Mem. Fac. Kochi Univ.Ser.A. Math., 12(1991), 5-13.
5. P.Bhattacharya and B.K.Lahiri, Semi-generalized closed sets in topology, Indian J.Math., 29(3)1987), 375-382.
6. R.Devi. H.Maki and K.Balachandran, Semi-generalized closed maps and generalized closed maps, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., 14(1993), 41-54.
7. R.Devi. H.Maki and K.Balachandran, Semi-generalized homomorphisms and generalized semi-homeomorphism in topological spaces, Indian J. Pure. Appl. Math., 26(3) (1995), 271-284.
8. R.Devi, K.Balachandran and H.Maki, Generalized  $\alpha$ -closed maps and  $\alpha$ -generalized closed maps, Indian J. Pure. Appl. Math., 29(1)(1988), 37-49.
9. J. Dontchev, On generalizing semi-preopen sets, Mem.Fac.Sci.Kochi Ser.A, Math., 16(1995), 35-48.
10. Y.Gnanambal, On generalized preregular closed sets in topological spaces, Indian J.Pure. Appl. Math., 28(3) (1997), 351-360.
11. N.Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2)(1970), 89-96.
12. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
13. H.Maki, J.Umehara and T.Noiri, Every topological space in pre- $T_{1/2}$ , Mem. Fac. Sci. Kochi Univ. Ser.A, Math., 17(1996), 33-42.

14. A.S.Mashhour, M.E.Abd El-Monsef and S.N.E1-Deeb, On pre-continuous and weak pre-continuous mappings, Proc. Math. And Phys. Soc. Egypt, 53(1982), 47-53.
15. N.Nagaveni, studies on Generalizations of Homeomorphisms in Topological spaces, Ph.D, thesis, Bharathiar University, Coimbatore, 1999.
16. O.Njastad, On some classes of nearly open sets, Pacific J.Math., 15(1965), 961-970.
17. N. Palaniappaan and K.C.Rao, Regular generalized closed sets, Kyungpook Math.J., 33(2)(1993), 211-219.
18. M.K.R.S. Veerakumar, Between closed sets and g-closed sets, Mem. Fac. Sci. Koch. Univ. Ser.A, Math., 17(1996), 33-42.
19. H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized  $\alpha$ -closed sets and  $\alpha$ -generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser.A, Math., 15(1994), 51-63.
20. Pauline Mary Helen M, (gsp)\*-closed sets in Topological spaces International Journal of Mathematics Trends and Technology 6(1) 2014; 75–86.

**Source of support: Nil, Conflict of interest: None Declared**

***[Copy right © 2015. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]***