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ON RIGHT AND LEFT $2_{\otimes}$-ENGEL ELEMENTS OF DERIVATIVE OF GROUPS

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ABSTRACT

In this paper we study some properties of $2_{\otimes}$-Engel elements in derivative of groups. In particular, we prove that if G' is a derivative of the group G then $R_2^{\otimes}(G)$ is a characteristic subgroup of G.

Keyword and phrases: *$2_{\otimes}$-Engel Elements in Groups, Tensor Product of Groups, derivative of group.*

1. INTRODUCTION

For any group G, the nonabelian tensor square $G \otimes G$ is a group generated by the symbols g $\otimes$ h, subject to the relations:

$gg \otimes h = (g^g \otimes h^g)(g \otimes h)$ and $g \otimes hh = (g \otimes h)(g^h \otimes h^h)$, where g, g ,h, h∈G and $g^h = h^{-1}gh$.

The more general concept of nonabelian tensor product of groups acting on each other in certain compatible way was introduced by R. Brown and J.-L. Loday in [5], following the ideas of R. K. Dennis [6]. Also, tensor analogues of right n-Engel elements have been defined. Recall that the set of right n-Engel elements of a group G is defined by $R_n(G) = \{a \in G: [a, nx] = 1, for\ all\ x \in G\}$. Here $[a, nx]$ stands for the commutator $[\cdots [[a, x], x], \cdots]$ with n copies of x. It is well-known that $R_1(G) = Z(G)$ and that $R_2(G)$ is a subgroup of G . The set of right $n_{\otimes}$-Engel elements of a group G is then defined as

$$R_n{}^{\otimes}(G) = \{a \in G: [a, n-1x] \otimes x = 1_{\otimes}\ for\ all\ x \in G\}.$$

And for a group G we define the sets of right (left) $2_{\otimes}$-Engel elements of G by

$$R_2{}^{\otimes}(G) = \left\{a \in G: [a, x] \otimes x = 1_{\otimes}\ for\ all\ x \in G\right\}$$

$$L_2{}^{\otimes}(G) = \{a \in G: [x, a] \otimes a = 1_{\otimes} for\ all\ x \in G\},$$

One of the results of D P. Biddel and L.C. Kappe shows that $R_2{}^{\otimes}(G)$ is always a characteristic subgroup of G containing $Z(G)$ and contained in $R_2(G)$. H. Khosravi,H. Golmakani and H. M. Mohammadinezhd proved that in any derivative of group G the inverse of right 2-Engle element is a left 2-Engle element. Thus $R_2(G) \subseteq L_2(G)$ and $R_2(G)$ is a characteristic subgroup of G .

2. RESULTS

Lemma 1: ([5]) *Let g, g', h, h' ∈G. The following relations hold in G⊗G*

a) $(g^{-1} \otimes h)^g = (g \otimes h)^{-1} = (g \otimes h^{-1})^h$.
b) $(g' \otimes h')^{g \otimes h} = (g' \otimes h')^{[g,h]}$.
c) $[g, h] \otimes g' = (g \otimes h)^{-1}(g \otimes h)^{g'}$.
d) $g' \otimes [g, h] = (g \otimes h)^{-g'}(g \otimes h)$.
e) $[g, h] \otimes [g', h'] = [g \otimes h, g' \otimes h']$.

Note here that G acts on $G \otimes G$ by$(g \otimes h)^{g'} = g^{g'} \otimes h^{g'}$. The next result is crucial in studying the analogy between commutators and tensors.

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Proposition1: ([4]) *For a given group G there exists a homomorphism $k: G \otimes G \to G$ such that*

$$k: g \otimes h \to [g,h]$$

Moreover, $ker\ k \leq Z(G \otimes G)$ and G acts trivially on ker k.

Lemma 2: ([10]) Let G be any group.

(a) ${R_2}^{\otimes}(G) \subseteq R_2(G)$, ${L_2}^{\otimes}(G) \subseteq L_2(G)$.

(b) Every right $2_\otimes$-Engel element of G also belongs to ${L_2}^{\otimes}(G)$.

(c) $L^{2^{\otimes}}(G) = \{a \in G: a^x \otimes a^y = a \otimes a\ for\ all\ x, y \in G\}$.

Theorem 1: In any derivative of group G the inverse of right 2⊗-Engel element is a left 2⊗-Engel element. Thus, ${R_2}^{\otimes}(G') \subseteq {L_2}^{\otimes}(G')$.

Proof: Let $[x,y] \in {R_2}^{\otimes}(G')$, using the definition we have:

$[[x,y],[z,t]] \otimes [z,t] = 1_\otimes; [z,t] \in G$

$= [[x,y] \otimes [z,t], z \otimes t]$ by using (e) in lemma 1

$= [[x \otimes y, z \otimes t], z \otimes t]$ by using (e) and (c) in lemma 1

$= [x \otimes y, z \otimes t, (x \otimes y)(z \otimes t)]$

$= [[x \otimes y, z \otimes t], (x \otimes y)(z \otimes t)]$

$= [x \otimes y, z \otimes t]^{-1}((x \otimes y)(z \otimes t))^{-1}[x \otimes y, z \otimes t](x \otimes y)(z \otimes t)$

$= [x \otimes y, z \otimes t]^{-1}(z \otimes t)^{-1}(x \otimes y)^{-1}[x \otimes y, z \otimes t](x \otimes y)(z \otimes t)$

$= (z \otimes t)^{-1}[x \otimes y, z \otimes t]^{-1}(x \otimes y)^{-1}[x \otimes y, z \otimes t](x \otimes y)(z \otimes t)$

$= (z \otimes t)^{-1}[[x \otimes y, z \otimes t], x \otimes y](z \otimes t)$

$= [x \otimes y, z \otimes t, x \otimes y]^{(z \otimes t)}$

Since $[x \otimes y, z \otimes t, x \otimes y]^{(z \otimes t)} = 1_\otimes$

So, $[x \otimes y, z \otimes t, x \otimes y] = 1_\otimes = [z \otimes t, x \otimes y, x \otimes y] = [[z,t],[x,y]] \otimes [x,y]$

Thus $[x,y] \in {L_2}^{\otimes}(G')$.⊡

Theorem 2: *Let G be a group and G' be a derivative of G. The set of elements of ${R_2}^{\otimes}(G')$ is a characteristic subgroup of G'.*

Proof: Let $a \in Aut(G)$ be an arbitrary automorphism and $[x,y] \in {R_2}^{\otimes}(G')$. By definition we have:

For any $[z,t] \in G'$, we have $[[x,y],[z,t]] \otimes [z,t] = 1_\otimes$

So $a([[x,y],[z,t]] \otimes [z,t]) = 1_\otimes$

$= [a([x,y]), a([z,t])] \otimes a([z,t]) = 1_\otimes$

$= [[a(x), a(y)], [a(z), a(t)]] \otimes [a(z), a(t)]$

Since $a \in Aut(G)$,for any z, t there exists z', t' such that $a(z) = z'$ and $a(t) = t'$

Therefore $[a([x,y]), [z', t']] \otimes [z', t'] = 1_\otimes$

Thus $a([x,y]) \in {R_2}^{\otimes}(G')$.⊡

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