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SPLIT FUZZY SUPPORT STRONG DOMINATION

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ABSTRACT

Let G= (σ, μ) be a fuzzy graph. Let u be an element of V. Let N(u)={ $v \in V : \mu(uv) = \sigma(u) \wedge \sigma(v)$ }. The fuzzy concept of u is defined as the sum of the neighbourhood degrees of the elements in N(u). Then the split fuzzy support strong dominating set is the induced subgraph <V-D> is disconnected.

In this research work we introduce the concept of split fuzzy support strong domination in fuzzy graphs. The split fuzzy support of a graph is defined and domination based on the fuzzy support is considered. Several results involving this new fuzzy domination parameter are established. We also obtain the split fuzzy support strong domination number $\gamma^{s}{}_{S-f(\sup p)}$ for several classes of fuzzy graphs.

Key words: *Split fuzzy support strong dominating set, Split fuzzy support strong domination number.*

INTRODUCTION

Fuzzy concept is introduced in Graph Theory. To work on domination in Fuzzy Graphs, it is necessary to have a sound knowledge of fuzzy sets, Graph Theory and Domination Theory. Formally, a fuzzy graph G= (V, σ, μ) is a nonempty set V together with a pair of functions σ:V->[0,1] and $\mu : V \times V \rightarrow [0,1]$ such that μ(x, y)≤ σ(x) ∧ σ(y) for all x, y in V. σ is called the fuzzy vertex set of G and μ is called the fuzzy edge set of G.

PRELIMINARIES

Definition 1.1: A fuzzy graph G_f = (σ,μ) is a set with two functions σ: V→[0,1] and μ:E→[0,1] such that μ(x, y) ≤σ(x) Λσ(y) for all x, y ∈ V.

Definition 1.2: Let G= (σ, μ) be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(x) = \sigma(x)$ for all $x \in V_1$ and μ_1 on the collection E of two element subsets of V_1 by $\mu_1(xy)=\mu(xy)$ for all x, y ∈ V_1 . Then (σ_1, μ_1) is called the fuzzy sub graph of G induced by V_1 and is denoted by $\langle V_1 \rangle$.

Definition 1.3: The order p and size q of a fuzzy graph G = (σ, μ) are defined to be

$$p = \sum_{x \in V} \sigma(x) \quad and \quad q = \sum_{xy \in E} \mu(xy)$$

Definition1.4: The effective degree of a vertex u is defined to be the sum of the of the effective edges incident at u and is denoted by dE(u).

Definition 1.5: Let G= (V, E) be a graph. A subset S of V is called a dominating set in G if every vertex in V\S is adjacent to some vertex in S.

Remark: The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by $\gamma(G)$ *or* γ.

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Definition 1.6: Let u be an element of V. Let N(u)= { $v \in V : \mu(uv) = \sigma(u) \wedge \sigma(v)$ }. The **fuzzy support** of u is defined as the sum of the neighbourhood degrees of the elements in N(u) .
That is fuzzy supp (v) = $\sum_{u \in N(v)} \sigma(u)$.

Definition 1.7: Fuzzy support strong Domination
Let $u, v \in V(G)$. u is said to fuzzy support strong dominate v if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ and fuzzy supp(u)≥ fuzzy supp(v). A subset D of V(G) is called a fuzzy support dominating set if for every v in V-D there exists u in D such that u fuzzy support strong dominates v.

Definitions 1.8: A fuzzy support strong dominating set D of a fuzzy graph G=(V,E) is a split fuzzy support strong dominating set if the induced sub graph $\langle V - D \rangle$ is disconnected. The cardinality of a minimum split fuzzy support strong domination number of G is denoted by $\gamma^{s}{}_{S-f(\sup p)}(G)$

Proposition 1.9: Let G be a connected fuzzy graph which is not complete. Then there exists a split fuzzy support strong dominating set in G.

Proof: Since D is not complete, there exists vertices u,v which are not adjacent.

Case-(1): N[u] and N[v] are complete. Then for all x in N(u), x fuzzy support strong dominates u and for all y in N(v), y fuzzy support strong dominates v. Therefore D=V-{u,v} is a split fuzzy support strong dominating set of G.

Case-(2): N[u] is not complete. Then there exists x_1, x_2 are not adjacent.

Subcase-(2a): fuzzy supp(u)≥ fuzzy supp(x_1) and fuzzy supp(u)≥ fuzzy supp(x_2). Take D=V-{x_1,x_2} then D is a split fuzzy support strong dominates set of G, (Since u in D , fuzzy support strong dominates x_1, x_2)

Subcase (2b): fuzzy supp(u) < fuzzy supp(x_1)(similar proof if fuzzy supp(u) < fuzzy supp(x_2)). Then x_1 fuzzy support strong dominates u.

Subcase-(2b-1): N[v] is complete. Then D=V(G)-{u,v} is a split fuzzy support strong dominating set of G.

Subcase-(2b-2): N[v] is not complete. Then there exists y_1 and y_2 in N(v) such that y_1 and y_2 are not adjacent.

Subcase-(2b-2-i): If fuzzy supp(v) ≥ fuzzy supp(y_1) and fuzzy supp(v) ≥ fuzzy supp(y_2), then D=V(G)-{y_1, y_2} is a split fuzzy support strong dominating set of G,(since v fuzzy support strong dominating y_1 and y_2)

Subcase-(2b-2-ii): If fuzzy supp(v)<fuzzy supp(y_1) or fuzzy supp(v)< fuzzy supp(y_2) then y_1 or y_2 fuzzy support strong dominates v. Already x_1 fuzzy support strong dominates u. Therefore D-{u,v} is a split fuzzy support strong dominating set of G. Thus G has a split fuzzy support strong dominating set.

Theorem 2.0: Suppose G is a disconnected fuzzy graph. If G has either two non trivial components or a nontrivial component which is not complete, then G has a split fuzzy support strong dominating set of G.

Proof:
Case-(i): G has at least two non trivial components. Let G_1 and G_2 be two components of G which are not trivial. Then there exists x_1, x_2 in V(G_1) such that x_1 and x_2 are adjacent in G and y_1,y_2 in v(G_2) such that y_1 and y_2 are adjacent in G .

Without loss of generality, let fuzzy supp(x_1) ≥ fuzzy supp(x_2) and fuzzy supp(y_1) ≥ fuzzy supp(y_2)

Then D=V(G)-{x_2,y_2} is a split fuzzy support strong dominating set of G.

Case-(ii): G has a non trivial component which is not complete. Let G_1 be a nontrivial component of G, which is not complete. Then from the above proposition there exists a subset D of V(G_1), such that D is a split fuzzy support strong dominating set of G,

Let D_1=DU(V(G)-V(G_1)). Then D_1 is a split fuzzy support strong dominating set of G_1.

Theorem 2.1: A dominating set D of G is a split fuzzy support strong dominating set if and only if there exist two vertices w_1 and w_2 in V-D such that every w_1-w_2 path contains a vertex of D.

Proof: Let D be a split fuzzy support strong dominating set of V(G).

Let w_1 and w_2 be two vertices of V(G). Then there exists a path w_1 to w_2 in V(G). Since D is a dominating set, suppose a path contains some vertices in D. Then $\langle V-D \rangle$ is a disconnected graph. Hence there exist two vertices w_1 and w_2 in V-D , such that every w_1 - w_2 path contains a vertex of D.

Conversely, Suppose there exists two vertices w_1 and w_2 in V-D, such that every $w_1$to w_2 path contains a vertex of D. Then $\langle V-D \rangle$ is a disconnected graph. Hence D is a split fuzzy support strong dominating set.

Theorem 2.2: For any fuzzy graph G,

(i) $\gamma_f(G) \le \gamma^s{}_{f(S-\sup p)}(G).$

(ii) $\kappa(G) \le \gamma^s{}_{f(S-\sup p)}(G).$

Proof:

(i) Let D be a split fuzzy support strong dominating set of G. Then D is a dominating set of G. Therefore $\gamma_f(G) \le |D| = \gamma^s{}_{f(S-\sup p)}(G).$ *Therefore* $\gamma_f(G) \le \gamma^s{}_{f(S-\sup p)}(G).$

(ii) Let D be a $\gamma^s{}_{f(S-\sup p)}$ dominating set of G. Then $\langle V-D \rangle$ is disconnected. $\kappa(G) \le |D| = \gamma^s{}_{f(S-\sup p)}(G).$*Then* $\kappa(G) \le \gamma^s{}_{f(S-\sup p)}(G).$

Theorem 2.3: A Split fuzzy support strong dominating set D of G is minimal if and only if for each vertex v in D, one of the following conditions is satisfied.

(i) $u \in V-D,$ *such that* $N^S{}_{f(\sup p)}(u) \cap D = \{v\}.$ There exists a vertex V is a fuzzy support

(ii) strong isolated vertex in $\langle D \rangle$

(iii) $\langle (V-D) \cup \{v\} \rangle$ is connected.

Proof: Suppose D is minimal and there exists a vertex v in D such that v does not satisfy any of the above conditions (i),(ii) and (iii). Then D'=D-{v} is a fuzzy support strong dominating set of G. Also by (iii), $\langle V-D \rangle$is disconnected. This implies that D' is a split fuzzy support strong dominating set of G, a contradiction. Hence there exist a vertex v in D such that v satisfy the above three conditions.

Conversely, Suppose there exist a vertex v in D such that v satisfies the above three conditions. By condition (iii) $\langle (V-D) \cup \{v\} \rangle$ is connected. Therefore $\langle V-D \rangle$ is disconnected. Hence D is a split fuzzy support strong dominating set.

Theorem 2.4: For any fuzzy graph G, $\gamma^s{}_{f(S-\sup p)} \le \dfrac{n\Delta(G)}{\Delta(G)+1}$.

Proof: Let D be a split fuzzy support strong dominating set of G.

Case-(i): Let G have no fuzzy support strong isolated vertices. Let v in D. suppose v is a fuzzy support strong isolated vertex in $\langle D \rangle$. Then there exist u in V-D such that u Є N(v), fuzzy supp(u) fuzzy supp(v). Therefore v is fuzzy support strong dominated by u ЄV-D. suppose there exists a vertex $u \in V-D,$ such that $N^S{}_{f(\sup p)}(u) \cap D = \{v\}.$

Therefore v fuzzy support strong dominates u and hence V-D is a fuzzy dominating set of G.

$\gamma_f(G) \le |V-D| = |V| - |D| = |V| - \gamma^s{}_{f(S-\sup p)}(G)$. That is $\gamma^s{}_{f(S-\sup p)}(G) \le |V| - \gamma_f(G).$

Heance,

$$\gamma_f(G) \ge \frac{n}{\Delta(G)+1}.$$

Therefore,

$$\gamma^s{}_{f(S-\sup p)}(G) \le n - \frac{n}{\Delta(G)+1} = \frac{n\Delta(G)}{\Delta(G)+1}.$$

Case-(ii): Let u be a fuzzy support strong isolated vertex of G. If u has a neighbour v ЄV-D , then v dominates u. If u has no neighbour in V-D, then all the neighbours of u lie in D and they have fuzzy support strong private neighbours in V-D. Therefore there are deg(u) vertices in V-D and 2deg(u)+1 vertices in D. For ordinary domination deg(u) vertices are needed to dominate 2deg(u)+1 vertices. Thus, if there exist a fuzzy support strong isolated vertex with all neighbour in D, then at least $\frac{n}{\Delta(G)+1}+1$ vertices are required to dominate G. If there are k fuzzy support strong isolated vertices all of whose neighbours are in D, at least $\frac{n}{\Delta(G)+1}+k$ vertices required to dominate G.

Therefore $\gamma_f(G) \geq \frac{n}{\Delta(G)+1}+k.$ $|V-D|+k$ is a dominating set of G.

Therefore $\gamma_f(G) \leq |V-D|+k = n-\gamma^s{}_{f(S-\sup p)}(G)+k.$

Therefore

$$\begin{aligned}\gamma^s{}_{f(S-\sup p)}(G) &\leq n-\gamma_f(G)+k \\ &\leq n-\frac{n}{\Delta(G)+1}-k+k \\ &= n-\frac{n}{\Delta(G)+1} = \frac{n\Delta(G)}{\Delta(G)+1}\end{aligned}$$

Therefore $\gamma^s{}_{f(S-\sup p)}(G) \leq \frac{n\Delta(G)}{\Delta(G)+1}.$

APPLICATIONS

There are many origins to the domination theory. The earliest ideas of dominating sets date back, to the origin of game of chess in India. In this game, one studies of chess pieces which cover various opposing pieces or various squares of the board. Besides this paper also contained application to surveillance networks and game theory. In society as well as in administration, the influence of the individual depends on the strength that he derives from his supporter. In times of made the individual has to depend more on his supporter, than on himself.

In the fuzzy graph model, as influence function may be defined on the vertex set which gives a measure of the influence of the vertices. The fuzzy support of a vertex then, will be given by sum of the influences of the neighbors of the vertex. Domination using the fuzzy support strength may be defined by adjacency and superiority of the fuzzy support strength.

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