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THE STUDY OF DOUBLE DIFFUSIVE NATURAL CONVECTION IN ANISOTROPIC POROUS RECTANGULAR CHANNEL USING A LOCAL THERMAL NON-EQUILIBRIUM MODEL

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ABSTRACT

The effect of local thermal non-equilibrium on double diffusive convection in a rectangular channel filled with anisotropic porous media is considered, when the fluid and solid phases are not in local thermal equilibrium. Walls of the channels are non-uniformly heated to establish a linear temperature gradient and they are assumed to be impermeable and perfectly conducting. Darcy model with anisotropy permeability is used to describe the flow and a two field model is used for energy equation each representing fluid and solid phase separately. The critical Rayleigh number for the onset of convection using linear stability analysis obtained numerically as a function of mechanical anisotropy parameters, interphase heat transfer coefficient, solutal Rayleigh number, aspect ratio and results are investigated.

1.1 INTRODUCTION

The problem of double diffusive convection in porous media has attracted considerable interest because of its wide range of applications, from the solidification of binary mixture to the migration of solutes in water-saturated soils. Nilsen and Storesletton [1] presented an analytical study of two dimensional natural convection in horizontal rectangular channels filled with an isotropic porous medium. Rees and Pop [2] have investigated vertical free convection boundary layer flow in a porous medium using a thermal non-equilibrium model. Banu and Rees [3] have discussed thermal non-equilibrium effect on free convective flows in a porous medium. Free convection in a square porous cavity using a thermal non equilibrium model is studied by Baytas and Pop [4]. The problem of two-dimensional steady mixed convection in a vertical porous layer using thermal non-equilibrium model is investigated numerically by Saeid [5]. Straughan [6] has considered a problem of thermal convection in a fluid saturated porous layer using a global nonlinear stability analysis with a thermal non-equilibrium model. Postelnicu and Rees [7] have studied the onset of Darcy-Brinkmann convection in a porous layer using a thermal non-equilibrium model. Malashetty *et al.*, [8, 9] have studied the effect of thermal non-equilibrium on the onset of convection in a porous layer using the Lapwood-Brinkman model and also including anisotropy in permeability and thermal diffusivity in a densely packed porous layer. Balagondar and Pranesha Setty [10] have investigated natural convection in anisotropic porous rectangular channels using a thermal non-equilibrium model.

In this paper we study the local thermal non-equilibrium on double diffusive convection in a rectangular channel filled with anisotropic porous media, when the fluid and solid phases are not in local thermal equilibrium. Walls of the channels are non-uniformly heated to establish a linear temperature gradient and they are assumed to be impermeable and perfectly conducting. Darcy model with anisotropy permeability is used to describe the flow and a two field model is used for energy equation each representing fluid and solid phase separately. The critical Rayleigh number for the onset of convection using linear stability analysis obtained numerically as a function of mechanical anisotropy parameters, interphase heat transfer coefficient, solutal Rayleigh number, aspect ratio and results are discussed.

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1.2 MATHEMATICAL FORMULATION

We consider two-dimensional free convection in a horizontal porous media heated from below. The lower surface is held at temperature T_l while upper surface $T_u \left(\Delta T = T_l - T_u\right)$ and concentration gradient $\Delta S = S_l - S_u$ where $T_l > T_u$ and $S_l > S_u$ are maintained between the lower and upper surfaces. We assume that the solid and fluid phases of the medium are not in local thermal equilibrium and use a two field model for temperatures with anisotropy in porous media. The channel is rectangular with height'd' and width 'a', we choose a cartesian co-ordinate system with z-axis is in the vertical direction and x-axis is the horizontal direction perpendicular to the channel axis. The horizontal channel walls are z = 0 and z = d and the vertical walls at $x = -\frac{a}{2}$ and $x = \frac{a}{2}$. On assuming that the Prandtl-Darcy number is large, so that inertia term may be neglected and invoking Boussinesq approximation, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{1.2.1}$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\nu}{k_x} u = 0, \tag{1.2.2}$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial z} + \frac{\nu}{k_z} w + \frac{\rho}{\rho_0} g = 0, \tag{1.2.3}$$

$$\varepsilon(\rho c)_f \cdot \frac{\partial T_f}{\partial t} + (\rho c)_f \; (\vec{q}.\nabla) T_f = \varepsilon K_f \nabla^2 T_f + h(T_s - T_f), \tag{1.2.4}$$

$$(1-\varepsilon)\,(\rho c)_s \frac{\partial T_s}{\partial t} = (1-\varepsilon)\, k_s . \nabla^2 T_s - h(T_s - T_f), \tag{1.2.5}$$

$$\frac{\partial S}{\partial t} + \frac{1}{\varepsilon}(\vec{q}\cdot\nabla) S = D\nabla^2 S, \tag{1.2.6}$$

$$\rho = \rho_0 (1 - \beta_T \left(T_f - T_l\right) + \beta_S \left(S - S_l\right)). \tag{1.2.7}$$

Since the flow is two-dimensional, we introduce the stream function ψ as:

$$u = \frac{\partial \psi}{\partial z}, \; w = -\frac{\partial \psi}{\partial x}. \tag{1.2.8}$$

We also define non dimensional variables by

$$x = a\, x^*, \; z = d\, z^*, \; u = \frac{\varepsilon k_{fz} a}{(\rho c)_f d^2} u^*, \; w = \frac{\varepsilon k_{fz}}{(\rho c)_f d} w^*,$$

$$T_s = \left\{T_0^* + \Delta T\left(1 - \frac{z}{d}\right)\right\} + \varphi^*, \; S = \left\{S_0^* + \Delta S\left(1 - \frac{z}{d}\right)\right\} + S^*$$

$$\theta = (\Delta T)\theta^*, \; \phi = (\Delta T)\phi^*, \; S = (\Delta S) S^*, \; T_0 = (\Delta T) T_0^*, \; \psi = \frac{\varepsilon k_{fz} a}{(\rho c)_f d}\psi^* \tag{1.2.9}$$

Using the equation (1.2.8) and (1.2.9) into equations (1.2.1)-(1.2.7) the non-dimensional equations after dropping the * we obtained in the form:

$$\frac{\partial p}{\partial x} + \varepsilon \frac{a^2}{d^2}\frac{\partial \psi}{\partial z} = 0, \tag{1.2.10}$$

$$\frac{\partial p}{\partial z} - Ra_c\,\theta + R_s\, S - \varepsilon \frac{k_x}{k_z}\frac{\partial \psi}{\partial x} = 0, \tag{1.2.11}$$

$$\eta_f \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2} - \frac{\partial \psi}{\partial x} = \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial z}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial z} + H(\theta - \phi), \tag{1.2.12}$$

$$\eta_s \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=\alpha\frac{\partial\phi}{\partial t} -\gamma H(\theta-\phi). \tag{1.2.13}$$

$$\frac{1}{Le}\left(\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial z^2}\right)-\frac{\partial\psi}{\partial x}=\frac{\partial S}{\partial t}+\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z} \tag{1.2.14}$$

Eliminating pressure gradient from the equations (1.2.10) and (1.2.11), we get

$$\xi\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}+\xi Ra_c\frac{\partial\theta}{\partial x}-\xi R_s\frac{\partial S}{\partial x}=0, \tag{1.2.15}$$

where

$$\xi=\frac{k_x}{k_z}\left(\frac{d}{a}\right)^2,\ \eta_f=\frac{k_{fx}}{k_{fz}}\left(\frac{d}{a}\right)^2,\ \eta_s=\frac{k_{sx}}{k_{sz}}\left(\frac{d}{a}\right)^2,\ \alpha=\frac{(\rho c)_s}{(\rho c)_f}\frac{k_{fz}}{k_{sz}}$$

$$\gamma=\frac{\varepsilon k_{fz}}{(1-\varepsilon)k_{sz}},\ H=\frac{hd^2}{\varepsilon k_{fz}},\ Ra_c=\frac{\rho_0 g\beta_T\,\Delta T\,k_z d}{\varepsilon\mu k_{fz}},\ R_S=\frac{\rho_0 g\beta_S\,\Delta S\,k_z d}{\varepsilon\mu k_{fz}}$$

The asterisks have been dropped for simplicity

1.3 LINEAR STABILITY ANALYSIS AND NUMERICAL SOLUTION

The linearised forms of the governing equations are

$$\xi\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}+\xi Ra_c\frac{\partial\theta}{\partial x}-\xi R_s\frac{\partial S}{\partial x}=0. \tag{1.3.1}$$

$$\eta_f\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial z^2}-\frac{\partial\psi}{\partial x}=\frac{\partial\theta}{\partial t}+H(\theta-\varphi) \tag{1.3.2}$$

$$\eta_s\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}=\alpha\frac{\partial\varphi}{\partial t}-\gamma H(\theta-\varphi). \tag{1.3.3}$$

$$\frac{1}{Le}\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}\right)+\frac{\partial\psi}{\partial x}=\frac{\partial S}{\partial t} \tag{1.3.4}$$

The boundary conditions used are

$$\psi=\theta=\phi=S=0 \ \text{at} \ \begin{cases} x=-\frac{1}{2},\ x=\frac{1}{2},\ 0<z<1 \\ z=0,\ z=1,\ -\frac{1}{2}<x<\frac{1}{2} \end{cases}. \tag{1.3.5}$$

The onset of stationary convection is described by the linear version of equations (1.3.1)-(1.3.4) and the solution for ψ, θ , ϕand S is now taken as single mode component as

$$\psi=D(x)\sin\pi z,\ \theta=G(x)\sin\pi z,\ \phi=I(x)\sin\pi z,\ S=Q(x)\sin\pi z \tag{1.3.6}$$

In terms of D, G, I and Q the boundary conditions are

$$D\left(\pm\frac{1}{2}\right)=0,\ G\left(\pm\frac{1}{2}\right)=0,\ I\left(\pm\frac{1}{2}\right)=0,\ Q\left(\pm\frac{1}{2}\right)=0$$

Using equation (1.3.6) in (1.3.1) to (1.3.4), we get

$$\left(\xi\frac{d^2}{dx^2}-\pi^2\right)D(x)+\xi Ra_s G(x)-\xi R_s Q(x)=0, \tag{1.3.7}$$

$$\left(\eta_f\frac{d^2}{dx^2}-\pi^2\right)G(x)-\frac{dD}{dx}=H(G-I), \tag{1.3.8}$$

$$\left(\eta_s \frac{d^2}{d\,x^2} - \pi^2\right) I(x) \;=\; \gamma\, H\,(G - I)\,. \tag{1.3.9}$$

$$\frac{1}{Le}\left(\frac{d^2}{d\,x^2} - \pi^2\right) Q(x) \;= \frac{d\,D}{d\,x} \tag{1.3.10}$$

1.4 Anisotropic case: $\xi \neq \eta_f \neq \eta_s$

By eliminating $D(x), I(x)$ and $Q(x)$ between (1.3.7)-(1.3.10), we get a eighth order differential equation in the form:

$$\begin{aligned}
&[\left(\eta_f\, \eta_s\, \xi \frac{1}{Le}\right) D^8 - \frac{1}{Le}\Big(\left(H + \pi^2 - Ra_c\right) \eta_s\, \xi + \eta_f\,\Big(\left(Le\, R_s \eta_s + H\,\gamma\right)\xi + \pi^2\left(\eta_s + \xi + \eta_s\,\xi\right)\Big)\Big)\, D^6 \\
&\quad + \frac{1}{Le}\Big(\, \pi^2\left(-\left(Ra_c + Ra_c\;\eta_s - Le\;R_s\;\;\eta_s\right)\xi + \pi^2\left(\,\eta_s + \xi + \eta_s\,\xi\,\right)\right) \\
&\quad + H\left(\left(Le\, R_s\, \eta_s - Ra_c \gamma\,\right)\xi + \pi^2\left(\eta_s + \xi + \eta_s\,\xi + \gamma\xi\,\right)\right) \\
&\quad + \eta_f\,\left(H\; Le\, R_s\; \gamma\xi + \;\pi^4\left(1 + \eta_s + \xi\right) + \pi^2\left(Le\, Rs\,\xi + H\,\gamma\left(1 + \xi\right)\,\right)\right)\Big)\, D^4 \\
&\quad - \frac{1}{Le}(\pi^2\,(\pi^2\left(-Ra_c\,\xi + Le\;\,R_s\,\xi + \pi^2\left(1 + \eta_f + \eta_s + \xi\right)\right) \\
&\quad + H\left(\left(-Ra_c\,\gamma + Le\, R_s\left(1 + \gamma\right)\right)\xi + \pi^2\left(1 + \eta_s + \xi + \gamma\left(1 + \eta_f + \xi\right)\right)\right)))\; D^2 \\
&\quad + \frac{1}{Le}\left(\pi^6\left(H + \pi^2 + H\,\gamma\right)\right)]\, G(x) \,=\, 0 \qquad \text{where } D = \frac{d}{dx}
\end{aligned} \tag{1.4.1}$$

with boundary conditions

$$G\left(\pm\frac{1}{2}\right) = G^{/}\left(\pm\frac{1}{2}\right) = G^{//}\left(\pm\frac{1}{2}\right) = G^{///}\left(\pm\frac{1}{2}\right) = 0. \tag{1.4.2}$$

The general solution of equation (1.4.1) is

$$G(x) = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + c_4 e^{m_4 x} + c_5 e^{m_5 x} + c_6 e^{m_6 x} + c_7 e^{m_7 x} + c_8 e^{m_8 x} \tag{1.4.3}$$

where c_i's are arbitrary constants and m_i's are roots of the auxiliary equation of (1.4.3). Since the auxiliary equation involves fourth order in D^2, we can put

$$m_2 = -m_1,\; m_4 = -m_3,\; m_6 = -m_5,\; m_8 = -m_7 \tag{1.4.4}$$

where

$$m_1 = \sqrt{((\frac{1}{36\,\delta_1^2})(9\delta_2^2 - 3\sqrt{3}\,\delta_1^2\,\sqrt{K_4} - 3\sqrt{6}\,\delta_1^2\,\sqrt{((-\frac{1}{\delta_1^6}) - 3\delta_1^2\delta_2^4 + 8\delta_1^4\,\delta_3^2 + K_5\; + 2^{\frac{2}{3}}\;\;\delta_1^4\,K_3 + 3\sqrt{3}\;(\delta_2^6 - 4\delta_1^2\delta_2^2\delta_3^2 + 8\,\delta_1^4\delta_4^2)\;\;/\;\;\sqrt{K_4}\;)))))} \tag{1.4.5}$$

$$m_3 = \sqrt{((\frac{1}{36\,\delta_1^2})(9\delta_2^2 - 3\sqrt{3}\,\delta_1^2\,\sqrt{K_4} + 3\sqrt{6}\,\delta_1^2\,\sqrt{((-\frac{1}{\delta_1^6}) - 3\delta_1^2\delta_2^4 + 8\delta_1^4\,\delta_3^2 + K_5\; + 2^{\frac{2}{3}}\;\;\delta_1^4\,K_3 + 3\sqrt{3}\;(\delta_2^6 - 4\delta_1^2\delta_2^2\delta_3^2 + 8\,\delta_1^4\delta_4^2)\;\;/\;\;\sqrt{K_4}\;)))))} \tag{1.4.6}$$

$$m_5 = \sqrt{((\frac{1}{36\,\delta_1^2})(9\delta_2^2 + 3\sqrt{3}\,\delta_1^2\,\sqrt{K_4} - 3\sqrt{6}\,\delta_1^2\,\sqrt{((-\frac{1}{\delta_1^6}) - 3\delta_1^2\delta_2^4 + 8\delta_1^4\,\delta_3^2 + K_5\; + 2^{\frac{2}{3}}\;\;\delta_1^4\,K_3 - 3\sqrt{3}\;(\delta_2^6 - 4\delta_1^2\delta_2^2\delta_3^2 + 8\,\delta_1^4\delta_4^2)\;/\;\sqrt{K_4}\;)))))} \tag{1.4.7}$$

$$m_7 = \sqrt{\left(\left(\frac{1}{36\,\delta_1^2}\right)(9\delta_2^2 + 3\sqrt{3}\,\delta_1^2\sqrt{K_4}\right.} \tag{1.4.8}$$

$$\left. + 3\sqrt{6}\,\delta_1^2\sqrt{\left(\left(-\frac{1}{\delta_1^6}\right) - 3\delta_1^2\delta_2^4 + 8\delta_1^4\delta_3^2 + K_5 + 2^{\frac{2}{3}}\,\delta_1^4 K_3 - 3\sqrt{3}\,(\delta_2^6 - 4\delta_1^2\delta_2^2\delta_3^2 + 8\delta_1^4\delta_4^2)/\sqrt{K_4}\,\right)))))}\right.$$

$$\delta_1^2 = \frac{1}{Le}\,\eta_f\,\eta_s\,\xi$$

$$\delta_2^2 = \frac{1}{Le}\Big(\big(H + \pi^2 - Ra_c\big)\,\eta_s\,\xi + \eta_f\,\big(\big(Le\,R_s\eta_s + H\,\gamma\big)\xi + \pi^2\big(\eta_s + \xi + \eta_s\,\xi\big)\big)\Big)$$

$$\delta_3^2 = \frac{1}{Le}\Big(\,\pi^2\big(-\big(Ra_c + Ra_c\;\eta_s - Le\;R_s\;\;\eta_s\big)\xi + \pi^2\big(\,\eta_s + \xi + \eta_s\,\xi\,\big)\big)$$
$$+ H\big(\big(Le\,R_s\,\eta_s - Ra_c\gamma\,\big)\xi + \pi^2\big(\eta_s + \xi + \eta_s\,\xi + \gamma\xi\,\big)\big)$$
$$+ \eta_f\,\big(H\;Le\,R_s\;\gamma\xi + \;\pi^4\big(1 + \eta_s + \xi\big)\pi^2\big(Le\,Ra_c\big)\big)$$
$$+ \eta_f\,\big(H\;Le\,R_s\;\gamma\xi + \;\pi^4\big(1 + \eta_s + \xi\big) + \pi^2\big(Le\,R_s\,\xi + H\,\gamma\big(1 + \xi\big)\big)\big)\Big)$$

$$\delta_4^2 = \frac{1}{Le}\Big(\pi^2\,\Big(\pi^2\big(-Ra_c\,\xi + Le\;R_s\,\xi + \pi^2\big(1 + \eta_f + \eta_s + \xi\big)\big)$$
$$+ H\,\big(\big(-Ra_c\,\gamma + Le\,R_s\,(1 + \gamma)\big)\xi + \pi^2\big(1 + \eta_s + \xi + \gamma\big(1 + \eta_f + \xi\big)\big)\big)\Big)\Big)$$

$$\delta_5^2 = \frac{1}{Le}\Big(\pi^6\big(H + \pi^2 + H\,\gamma\big)\Big)\;K_1 = 3\,\delta_2^4 - 8\,\delta_1^2\,\delta_3^2\;,\;\;K_2 = 2^{\frac{1}{3}}\;\delta_1^2(\,\delta_3^4 - 3\delta_2^2\delta_4^2 + 12\,\delta_1^2\delta_5^2)$$

$$K_3 = (2\,\delta_3^6 + 27\;\delta_1^2\delta_4^4 + 27\,\delta_2^4\,\delta_5^2 - 9\,\delta_3^2(\delta_2^2\delta_4^2 + 8\delta_1^2\delta_5^2\;)$$
$$+ \sqrt{(-4(\delta_3^4 - 3\,\delta_2^2\delta_4^2 + 12\delta_1^2\delta_5^2\,)^3 + (2\,\delta_3^6 - 9\delta_3^2(\delta_2^2\delta_4^2 + 8\delta_1^2\delta_5^4) + 27\,(\,\delta_1^2\delta_4^4 + \delta_2^4\,\delta_5^2))^2}\;)^{\frac{1}{3}}$$

$$K_4 = \frac{1}{\delta_1^4}((K_1 + (4K_2)/3) + 2\times 2^{\frac{1}{3}}\,\delta_1^2 K_3)\,,\;\; K_5 = ((2\delta_1^2\,K_2)/K_3) \tag{1.4.9}$$

For a non-trivial solution of the system of equations (1.4.1), (1.4.2) and (1.4.3) we require:

$$\begin{vmatrix}
e^{0.5m_1} & e^{-0.5m_1} & e^{0.5m_3} & e^{-0.5m_3} & e^{0.5m_5} & e^{-0.5m_5} & e^{0.5m_7} & e^{-0.5m_7} \\
e^{-0.5m_1} & e^{0.5m_1} & e^{-0.5m_3} & e^{0.5m_3} & e^{-0.5m_5} & e^{0.5m_5} & e^{-0.5m_7} & e^{0.5m_7} \\
m_1e^{0.5m_1} & -m_1e^{-0.5m_1} & m_3e^{0.5m_3} & -m_3e^{-0.5m_3} & m_5e^{0.5m_5} & -m_5e^{-0.5m_5} & m_7e^{0.5m_7} & -m_7e^{-0.5m_7} \\
m_1e^{-0.5m_1} & -m_1e^{0.5m_1} & m_3e^{-0.5m_3} & -m_3e^{0.5m_3} & m_5e^{-0.5m_5} & -m_5e^{0.5m_5} & m_7e^{-0.5m_7} & -m_7e^{0.5m_7} \\
(m_1)^2e^{0.5m_1} & (m_1)^2e^{-0.5m_1} & (m_3)^2e^{0.5m_3} & (m_3)^2e^{-0.5m_3} & (m_5)^2e^{0.5m_5} & (m_5)^2e^{-0.5m_5} & (m_7)^2e^{0.5m_7} & (m_7)^2e^{-0.5m_7} \\
(m_1)^2e^{-0.5m_1} & (m_1)^2e^{0.5m_1} & (m_3)^2e^{-0.5m_3} & (m_3)^2e^{0.5m_3} & (m_5)^2e^{-0.5m_5} & (m_5)^2e^{0.5m_5} & (m_7)^2e^{-0.5m_7} & (m_7)^2e^{0.5m_7} \\
(m_1)^3e^{0.5m_1} & -(m_1)^3e^{-0.5m_1} & (m_3)^3e^{0.5m_3} & -(m_3)^3e^{-0.5m_3} & (m_5)^3e^{0.5m_5} & -(m_5)^3e^{-0.5m_5} & (m_7)^3e^{0.5m_7} & -(m_7)^3e^{-0.5m_7} \\
(m_1)^3e^{-0.5m_1} & -(m_1)^3e^{0.5m_1} & (m_3)^3e^{-0.5m_3} & -(m_3)^3e^{0.5m_3} & (m_5)^3e^{-0.5m_5} & -(m_5)^3e^{0.5m_5} & (m_7)^3e^{-0.5m_7} & -(m_7)^3e^{0.5m_7}
\end{vmatrix} = 0 \tag{1.4.10}$$

The left hand side of (1.4.10) may be viewed as a function of Ra_c, say $f\,(Ra_c)$, with R_a depending on $H, \eta_f, \eta_s, \gamma, \xi, Le$ and Rs hence equation (1.4.10) can be written as $f\,(Ra_c) = 0$. Using Newton-Raphson method for various values of $\xi, \eta_f, \eta_s, H, \gamma, Le$ and R_S and Ra_c can be now calculated numerically using the formula.

$$\left[Ra_C\right]_{k+1} = \left(Ra_C\right)_k - \frac{f\left(Ra_C\right)_k}{f'\left(Ra_C\right)_k},$$

where

$$f'\left(\left(Ra_c\right)_k\right) = \underset{\delta Ra_c \to 0}{Lt}\left[\frac{f\left(\left(Ra_c\right)_k + \delta Ra_c\right) - f\left(\left(Ra_c\right)_k\right)}{\delta Ra_c}\right] \qquad (1.4.11)$$

RESULTS AND DISCUSSION

Double diffusive convection in a horizontal fluid-saturated porous medium is carried out by considering a thermal non-equilibrium model. The expression for critical Rayleigh number is derived numerically by using Newton-Raphson method. The effect of solute diffusion and thermal non equilibrium on the stability of the system is investigated. It is observed that for very small and large values of H the stability criterion is found to be independent of H. However, the effect of H on the stability of the system is significant only for intermediate values of H. The physical reason for this is that when H→0 there is almost no transfer of heat between the fluid and solid phases and the prosperities of solid phase have no significant influence on the onset of criterion. When H→∞ the fluid and solid phase have almost equal temperatures and therefore may be treated as single phase (i.e. LTE model). Between these two extremes H gives rise to a strong non-equilibrium effect.

In figure 1 we show the effect of mechanical anisotropy parameter ξ on Rayleigh number Ra_c for a fixed value of other parameters. From this figure it is evident that increase in the value of ξ decreases Ra_c and thus advances the onset of convection.

The variation of critical Rayleigh number with inter-phase heat transfer coefficient H for different parameter values is shown in figs 2 - 6. These figures indicate that the critical Rayleigh number increases from the LTE values when H is small to an LTNE values when H is large. Thus, the inter-phase heat transfer coefficient makes the system more stable for its intermediate values. Fig 2 indicates the effect of solute Rayleigh number on the critical Rayleigh number. For small values of solute Rayleigh number ($R_s \leq 10$) the convection sets is stationary mode. As the value of Ra_c is increased further the motion become significant and the convection sets. The critical Rayleigh number for stationary convection is found to increase with the solute Rayleigh number, indicating that the presence of additional diffusing component stabilizes the system towards the stationary convection.

Figure 3 displays the variation of the critical Rayleigh number with H for different values of the Lewis number. This figure indicates that the increasing values of Le, the critical Rayleigh number increases. The Lewis number has very negligible effect on the convection.

The variation of the critical Rayleigh number with H for different values of porosity modified conductivity ratio γ is shown in figure 4 when all other parameters are fixed. From this figure it is observed that for small H, Ra_c is independent of γ and is close to the LTE case. Since small value of H, there is no significant transfer of heat between the phases and the onset of criterion is not affected by the properties of the solid phase. For large values of H, through the stability criterion is independent of H, the condition for the onset of convection is based on the mean properties of the medium, and therefore the critical Rayleigh number is independent of γ.

The variation of critical Rayleigh number Ra_c with H for different values η_f as shown in the figure 5 We find that an increase in the value of η_f increases in the value of Ra_c indicating that the effect of increasing the thermal anisotropy parameter is to delay the onset of convection.

Figure 6 shows the variation of critical Rayleigh number with H for different values of thermal anisotropy parameter η_S. Its effect is found to be similar to that of η_f. However, for small values of H, Ra_c is found to be independent of η_S. For increasing values of parameter η_S the convection is stabilized. The effect of η_S at small values of the scaled inter-phase heat transfer coefficient is negligible.

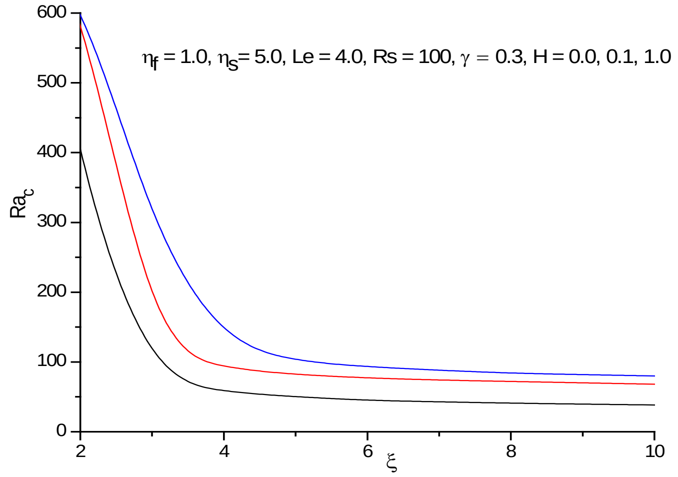

Figure 1: Variation of critical Rayleigh number Ra_C with ξ for various values

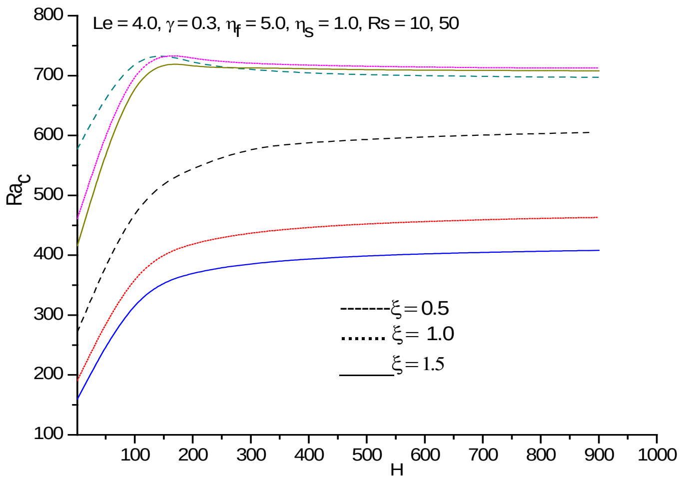

Figure 2: Variation of critical Rayleigh number Ra_c with H for different values

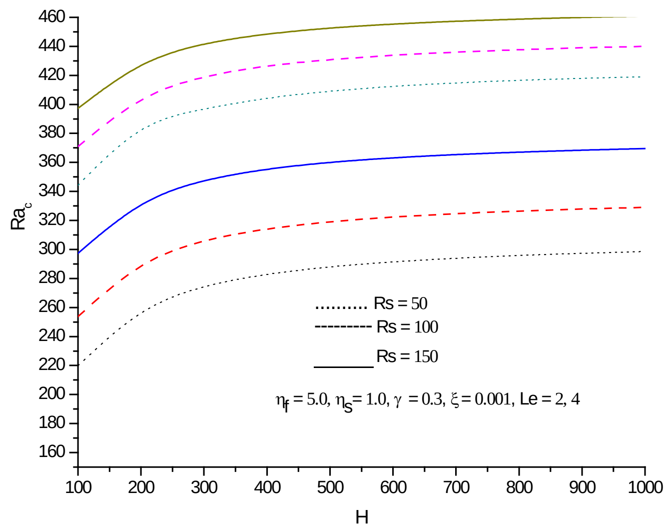

Figure 3: Variation of critical Rayleigh number Ra_C with H for different values

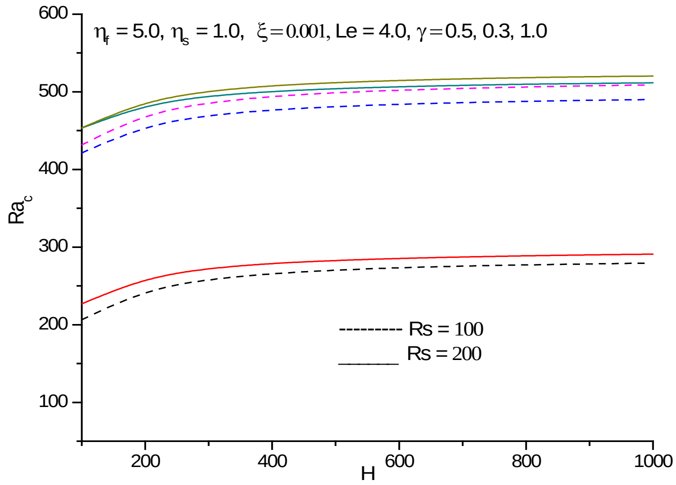

Figure 4. Variation of critical Rayleigh number Rac with H for different

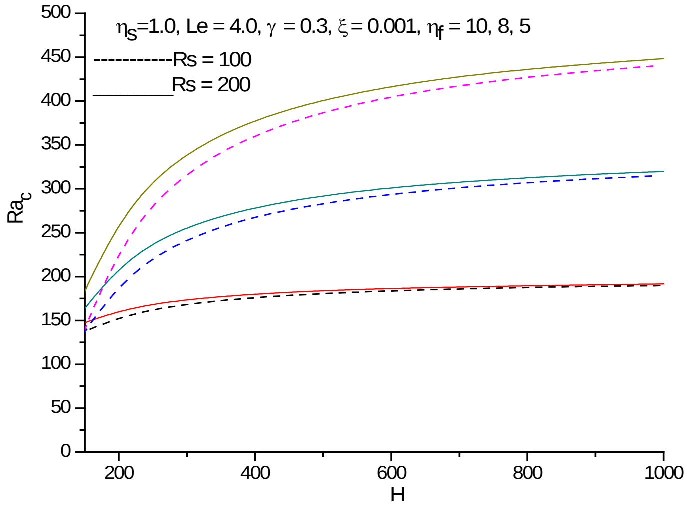

Figure 5: Variartion of critical Rayleigh number Ra_c with H for different

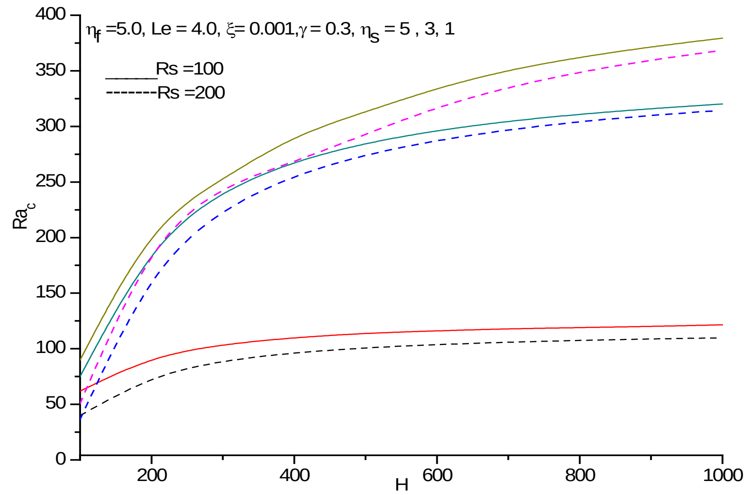

Figure 6: Variation of critical Rayleigh number Ra_C with H for various values

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