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NOTES ON I-FUZZY SUBBIGROUP OF A BIGROUP

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of I-fuzzy (interval valued fuzzy) subbigroup of a bigroup.

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Key Words: *Bigroup, fuzzy subset, I-fuzzy subset, fuzzy subbigroup, I-fuzzy subbigroup.*

INTRODUCTION

In 1965, the fuzzy subset was introduced by L.A.Zadeh [9], after that several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups was introduced by Azriel Rosenfeld [2]. Interval-valued fuzzy sets were introduced independently by Zadeh [10], Grattan-Guiness [3], Jahn [4], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function. Jun.Y.B and Kin.K.H [5] defined an interval valued fuzzy R-subgroups of nearrings. M.G.Somasundra Moorthy & K.Arjunan [7, 8] defined interval valued fuzzy subrings of a ring. In this paper, we introduce the some theorems in interval valued fuzzy (denoted as I-fuzzy) subbigroup of a bigroup.

1. PRELIRMINARIES

1.1 Definition: A set $(G, +, \bullet)$ with two binary operations + and $\bullet$ is called a bigroup if there exist two proper subsets G_1 and G_2 of G such that

(i) $G = G_1 \cup G_2$
(ii) $(G_1, +)$ is a group
(iii) $(G_2, \bullet)$ is a group.

1.2 Definition: Let X be a non–empty set. A **fuzzy subset A** of X is a function $A: X \rightarrow [0, 1]$.

1.3 Definition: Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup. Then a fuzzy set A of G is said to be a fuzzy subbigroup of G if there exist two fuzzy subsets A_1 of G_1 and A_2 of G_2 such that

(i) $A = A_1 \cup A_2$
(ii) A_1 is a fuzzy subgroup of $(G_1, +)$
(iii) A_2 is a fuzzy subgroup of $(G_2, \bullet)$.

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1.4 Definition: Let X be any nonempty set. A mapping [M]: X $\to$ D[0, 1] is called an interval valued fuzzy subset (I-fuzzy subset) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X. Thus $M^-(x)$ is an interval (a closed subset of [0, 1]) and not a number from the interval [0, 1] as in the case of fuzzy subset.

1.5 Definition: Let $[M] = \{\langle x, [M^-(x), M^+(x)] \rangle / x \in X\}$, $[N] = \{\langle x, [N^-(x), N^+(x)] \rangle / x \in X\}$ be any two interval valued fuzzy subsets of X. We define the following relations and operations:

(i) $[M] \subseteq [N]$ if and only if $M^-(x) \leq N^-(x)$ and $M^+(x) \leq N^+(x)$, for all x in X.

(ii) $[M] = [N]$ if and only if $M^-(x) = N^-(x)$ and $M^+(x) = N^+(x)$, for all x in X.

(iii) $[M] \cap [N] = \{\langle x, [\min\{M^-(x), N^-(x)\}, \min\{M^+(x), N^+(x)\}] \rangle / x \in X\}$.

(iv) $[M] \cup [N] = \{\langle x, [\max\{M^-(x), N^-(x)\}, \max\{M^+(x), N^+(x)\}] \rangle / x \in X\}$.

(v) $[M]^C = [1, 1] - [M] = \{\langle x, [1- M^+(x), 1- M^-(x)]\rangle / x \in X\}$.

1.6 Definition: Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup. The I-fuzzy subset $[A]: G \to D[0, 1]$ of G is said to be a I-fuzzy subbigroup of G if there exist two I-fuzzy subsets $[A_1]: G_1 \to D[0, 1]$ of G_1 and $[A_2]: G_2 \to D[0, 1]$ of G_2 such that

(i) $[A] = [A_1] \cup [A_2]$

(ii) $[A_1]$ is a I-fuzzy subgroup of $(G_1, +)$

(iii) $[A_2]$ is a I-fuzzy subgroup of $(G_2, \bullet)$.

2. PROPERTIES

2.1 Theorem: If $[A] = [M] \cup [N]$ is a I-fuzzy subbigroup of a bigroup $G = E \cup F$, then $\mu_{[M]}(-x) = \mu_{[M]}(x)$, $\mu_{[M]}(x) \leq \mu_{[M]}(e)$, $\mu_{[N]}(x^{-1}) = \mu_{[N]}(x)$, $\mu_{[N]}(x) \leq \mu_{[N]}(e')$ for all x, e in E and x, e' in F.

Proof: Let x, e in E and x, e' in F. Now $\mu_{[M]}(x) = \mu_{[M]}((-(-x))) \geq \mu_{[M]}(-x) \geq \mu_{[M]}(x)$. Therefore $\mu_{[M]}(-x) = \mu_{[M]}(x)$ for all x in E. And $\mu_{[M]}(e) = \mu_{[M]}(x-x) \geq \text{rmin}\{\mu_{[M]}(x), \mu_{[M]}(x)\}= \mu_{[M]}(x)$. Therefore $\mu_{[M]}(e) \geq \mu_{[M]}(x)$ for all x, e in E. Also $\mu_{[N]}(x) = \mu_{[N]}((x^{-1})^{-1}) \geq \mu_{[N]}(x^{-1}) \geq \mu_{[N]}(x)$. Therefore $\mu_{[N]}(x^{-1}) = \mu_{[N]}(x)$ for all x in F. And $\mu_{[N]}(e')= \mu_{[N]}(xx^{-1}) \geq \text{rmin}\{ \mu_{[N]}(x), \mu_{[N]}(x^{-1})\}= \mu_{[N]}(x)$. Therefore $\mu_{[N]}(e') \geq \mu_{[N]}(x)$ for all x, e' in F.

2.2 Theorem: If $[A] = [M] \cup [N]$ is a I-fuzzy subbigroup of a bigroup $G = E \cup F$, then

(i) $\mu_{[M]}(x-y) = \mu_{[M]}(e)$ gives $\mu_{[M]}(x) = \mu_{[M]}(y)$ for all x, y and e in E

(ii) $\mu_{[N]}(xy^{-1}) = \mu_{[N]}(e')$ gives $\mu_{[N]}(x) = \mu_{[N]}(y)$ for all x, y and e' in F.

Proof:

(i) Let x, y and e in E. Then $\mu_{[M]}(x) = \mu_{[M]}(x-y+y) \geq \text{rmin}\{\mu_{[M]}(x-y), \mu_{[M]}(y)\} = \text{rmin}\{\mu_{[M]}(e), \mu_{[M]}(y)\}= \mu_{[M]}(y) = \mu_{[M]}(y-x+x) \geq \text{rmin}\{\mu_{[M]}(y-x), \mu_{[M]}(x)\}= \text{rmin} \{\mu_{[M]}(e), \mu_{[M]}(x)\}= \mu_{[M]}(x)$. Therefore $\mu_{[M]}(x) = \mu_{[M]}(y)$ for all x and y in E.

(ii) Let x, y and e' in F. Then $\mu_{[N]}(x) = \mu_{[N]}(xy^{-1}y) \geq \text{rmin}\{\mu_{[N]}(xy^{-1}), \mu_{[N]}(y)\}=\text{rmin}\{\mu_{[N]}(e'), \mu_{[N]}(y)\}= \mu_{[N]}(y) = \mu_{[N]}(yx^{-1}x) \geq \text{rmin}\{\mu_{[N]}(yx^{-1}), \mu_{[N]}(x)\}= \text{rmin}\{\mu_{[N]}(e'), \mu_{[N]}(x)\}= \mu_{[N]}(x)$. Therefore $\mu_{[N]}(x) = \mu_{[N]}(y)$ for all x and y in F.

2.3 Theorem: If $[A] = [M] \cup [N]$ is a I-fuzzy subbigroup of a bigroup $G = E \cup F$, then

(i) $H_1= \{x / x \in E \text{ and } \mu_{[M]}(x) = [1, 1]\}$ is either empty or a subgroup of E.

(ii) $H_2 = \{x / x \in F \text{ and } \mu_{[N]}(x) = [1, 1]\}$ is either empty or a subgroup of F.

(iii) $K = H_1 \cup H_2$ is either empty or a subbigroup of G.

Proof: If no element satisfies this condition, then H_1 and H_2 are empty. Also $K = H_1 \cup H_2$ is empty.

(i) If x and y in H_1, then $\mu_{[M]}(x-y) \geq \text{rmin}\{\mu_{[M]}(x), \mu_{[M]}(y)\} \geq \text{rmin}\{[1, 1], [1, 1]\} = [1, 1]$.
Therefore $\mu_{[M]}(x-y) = [1, 1]$. We get x–y in H_1. Hence H_1 is a subgroup of G_1.

(ii) If x and y in H_2, then $\mu_{[N]}(xy^{-1}) \geq \text{rmin} \{ \mu_{[N]}(x), \mu_{[N]}(y)\} = \text{rmin} \{[1, 1], [1, 1]\} = [1, 1]$.
Therefore $\mu_{[N]}(xy^{-1}) = [1, 1]$. We get xy^{-1} in H_2. Hence H_2 is a subgroup of G_2.

(iii) From (i) and (ii) we get $K = H_1 \cup H_2$ is a subbigroup of G.

2.4 Theorem: If $[A] = [M] \cup [N]$ is a I-fuzzy subbigroup of a bigroup $G = E \cup F$, then

(i) $H_1= \{x / x \in E \text{ and } \mu_{[M]}(x) = \mu_{[M]}(e)\}$ is a subgroup of E

(ii) $H_2 = \{x / x \in F \text{ and } \mu_{[N]}(x) = \mu_{[N]}(e')\}$ is a subgroup of F

(iii) $K = H_1 \cup H_2$ is a subbigroup of G.

Proof:

(i) Clearly e in H_1 so H_1 is a non empty. Let x and y be in H_1. Then $\mu_{[M]}(x–y) \geq rmin\{\mu_{[M]}(x), \mu_{[M]}(y)\}= rmin\{\mu_{[M]}(e), \mu_{[M]}(e)\}= \mu_{[M]}(e)$. Therefore $\mu_{[M]}(x–y) \geq \mu_{[M]}(e)$ for all x and y in H_1. We get $\mu_{[M]}(x–y) = \mu_{[M]}(e)$ for all x and y in H_1. Therefore x–y in H_1. Hence H_1 is a subgroup of E.

(ii) Clearly e' in H_2 so H_2 is a non empty. Let x and y be in H_2. Then $\mu_{[N]}(xy^{-1}) \geq rmin\{\mu_{[N]}(x), \mu_{[N]}(y)\}= rmin\{\mu_{[N]}(e'), \mu_{[N]}(e')\} = \mu_{[N]}(e')$. Therefore $\mu_{[N]}(xy^{-1}) \geq \mu_{[N]}(e')$ for all x and y in H_2. We get $\mu_{[N]}(xy^{-1}) = \mu_{[N]}(e')$ for all x and y in H_2. Therefore xy^{-1} in H_2. Hence H_2 is a subgroup of F.

(iii) From (i) and (ii) we get $K = H_1 \cup H_2$ is a subbigroup of G.

2.5 Theorem: Let $[A] = [M]\cup[N]$ be a I-fuzzy subbigroup of a bigroup $G = E\cup F$.

(i) If $\mu_{[M]}(x–y) = [1, 1]$, then $\mu_{[M]}(x) = \mu_{[M]}(y)$ for all x and y in E.

(ii) If $\mu_{[N]}(xy^{-1}) = [1, 1]$, then $\mu_{[N]}(x) = \mu_{[N]}(y)$ for all x and y in F.

Proof:

(i) Let x and y belongs to E. Then $\mu_{[M]}(x) = \mu_{[M]}(x–y+y) \geq rmin\{\mu_{[M]}(x–y), \mu_{[M]}(y)\} = rmin\{[1, 1], \mu_{[M]}(y)\} = \mu_{[M]}(y) = \mu_{[M]}(–y) = \mu_{[M]}(–x+x–y) \geq rmin\{\mu_{[M]}(–x), \mu_{[M]}(x–y)\} = rmin\{\mu_{[M]}(–x), [1, 1]\} = \mu_{[M]}(–x) = \mu_{[M]}(x)$. Therefore $\mu_{[M]}(x) = \mu_{[M]}(y)$ for all x and y in E.

(ii) Let x and y belongs to F. Then $\mu_{[N]}(x) = \mu_{[N]}(xy^{-1}y) \geq rmin\{\mu_{[N]}(xy^{-1}), \mu_{[N]}(y)\} = rmin\{[1, 1], \mu_{[N]}(y)\}= \mu_{[N]}(y) = \mu_{[N]}(y^{-1}) = \mu_{[N]}(x^{-1}xy^{-1}) \geq rmin\{\mu_{[N]}(x^{-1}), \mu_{[N]}(xy^{-1})\} = rmin\{\mu_{[N]}(x^{-1}), [1, 1]\} = \mu_{[N]}(x^{-1}) = \mu_{[N]}(x)$. Therefore $\mu_{[N]}(x) = \mu_{[N]}(y)$ for all x and y in F.

2.6 Theorem: If $[A] = [M]\cup[N]$ is a I-fuzzy subbigroup of a bigroup $G = E\cup F$, then

(i) $\mu_{[M]}(x+y) = rmin\{\mu_{[M]}(x), \mu_{[M]}(y)\}$ for each x and y in E with $\mu_{[M]}(x) \neq \mu_{[M]}(y)$

(ii) $\mu_{[N]}(xy) = rmin\{\mu_{[N]}(x), \mu_{[N]}(y)\}$ for each x and y in F with $\mu_{[N]}(x) \neq \mu_{[N]}(y)$.

Proof:

(i) Let x and y belongs to E. Assume that $\mu_{[M]}(x)>\mu_{[M]}(y)$, then $\mu_{[M]}(y)=\mu_{[M]}(–x+x+y) \geq rmin\{\mu_{[M]}(–x), \mu_{[M]}(x+y)\} \geq rmin\{\mu_{[M]}(x), \mu_{[M]}(x+y)\} = \mu_{[M]}(x+y) \geq rmin\{\mu_{[M]}(x), \mu_{[M]}(y)\}= \mu_{[M]}(y)$.
Therefore $\mu_{[M]}(x+y) = \mu_{[M]}(y) = rmin\{\mu_{[M]}(x), \mu_{[M]}(y)\}$ for x and y in E.

(ii) Let x and y belongs to F. Assume that $\mu_{[N]}(x) > \mu_{[N]}(y)$, then $\mu_{[N]}(y) = \mu_{[N]}(x^{-1}xy) \geq rmin\{\mu_{[N]}(x^{-1}), \mu_{[N]}(xy)\}\geq rmin\{\mu_{[N]}(x), \mu_{[N]}(xy)\} = \mu_{[N]}(xy) \geq rmin\{\mu_{[N]}(x), \mu_{[N]}(y)\}= \mu_{[N]=}y)$.
Therefore $\mu_{[N]}(xy) = \mu_{[N]}(y) = rmin\{\mu_{[N]}(x), \mu_{[N]}(y)\}$ for x and y in F.

2.7 Theorem: If $[A] = [M]\cup[N]$ and $[B] = [O]\cup[P]$ are two I-fuzzy subbigroups of a bigroup $G = E\cup F$, then their intersection $[A]\cap[B]$ is a I-fuzzy subbigroup of G.

Proof: Let $[A] = [M]\cup[N] = \{\langle x, \mu_{[A]}(x)\rangle / x\in G\}$ where $[M] = \{\langle x, \mu_{[M]}(x)\rangle / x\in E\}$ and $[N] = \{\langle x, \mu_{[N]}(x)\rangle / x\in F\}$ and $[B] = [O]\cup[P] = \{\langle x, \mu_{[B]}(x)\rangle / x\in G\}$ where $[O] = \{\langle x, \mu_{[O]}(x)\rangle / x\in E\}$ and $[P] = \{\langle x, \mu_{[P]}(x)\rangle / x\in F\}$. Let $[C] = [A]\cap[B] = [R]\cup[S]$ where $[C] = \{\langle x, \mu_{C]}(x)\rangle / x\in G\}$, $[R] = [M]\cap[O] = \{\langle x, \mu_{R]}(x)\rangle / x\in E\}$ and $[S] = [N]\cap[P] =\{\langle x, \mu_{[S]}(x)\rangle/ x\in F\}$. Let x and y belong to E. Then $\mu_{[R]}(x–y) = rmin\{\mu_{[M]}(x–y), \mu_{[O]}(x–y)\}\geq rmin\{rmin\{\mu_{[M]}(x), \mu_{[M]}(y)\}, rmin\{\mu_{[O]}(x), \mu_{[O]}(y)\}\}\geq rmin\{rmin\{\mu_{[M]}(x), \mu_{[O]}(x)\}, rmin\{\mu_{[M]}(y), \mu_{[O]}(y)\}\}=rmin\{\mu_{[R]}(x), \mu_{[R]}(y)\}$. Therefore $\mu_{[R]}(x–y) \geq rmin\{\mu_{[R]}(x), \mu_{[R]}(y)\}$ for all x and y in E. Let x and y belong to F. Then $\mu_{[S]}(xy^{-1}) = rmin\{\mu_{[N]}(xy^{-1}), \mu_{[P]}(xy^{-1})\}\geq rmin\{rmin\{\mu_{[N]}(x), \mu_{[N]}(y)\}, rmin\{\mu_{[P]}(x), \mu_{[P]}(y)\}\}\geq rmin\{rmin\{\mu_{[N]}(x), \mu_{[P]}(x)\}, rmin\{\mu_{[N]}(y), \mu_{[P]}(y)\}\}= rmin\{\mu_{[S]}(x), \mu_{[S]}(y)\}$. Therefore $\mu_{[S]}(xy^{-1}) \geq rmin\{\mu_{[S]}(x), \mu_{[S]}(y)\}$ for all x and y in F. Hence $[A]\cap[B]$ is a I-fuzzy subbigroup of G.

2.8 Theorem: The intersection of a family of I-fuzzy subbigroups of a bigroup G is a I-fuzzy subbigroup of G.

Proof: It is trivial.

2.9 Theorem: If $[A] = [M]\cup[N]$ is a I-fuzzy subbigroup of a bigroup $G = E\cup F$, then

(i) $\mu_{[M]}(x+y) = \mu_{[M]}(y+x)$ if and only if $\mu_{[M]}(x) = \mu_{[M]}(–y+x+y)$ for all x and y in E

(ii) $\mu_{[N]}(xy) = \mu_{[N]}(yx)$ if and only if $\mu_{[N]}(x) = \mu_{[N]}(y^{-1}xy)$ for all x and y in F.

Proof:

(i) Let x and y be in E. Assume that $\mu_{[M]}(x+y) = \mu_{[M]}(y+x)$, then $\mu_{[M]}(–y+x+y) = \mu_{[M]}(–y+y+x) = \mu_{[M]}(e_1+x) = \mu_{[M]}(x)$. Therefore $\mu_{[M]}(x) = \mu_{[M]}(–y+x+y)$ for all x and y in E. Conversely, assume that $\mu_{[M]}(x) = \mu_{[M]}(–y+x+y)$, then $\mu_{[M]}(x+y) = \mu_{[M]}(x+y–x+x) = \mu_{[M]}(y+x)$. Therefore $\mu_{[M]}(x+y) = \mu_{[M]}(y+x)$ for all x and y in E.

(ii) Let x and y be in F. Assume that $\mu_{[N]}(x+y) = \mu_{[N]}(y+x)$, then $\mu_{[N]}(y^{-1}xy) = \mu_{[N]}(y^{-1}yx) = \mu_{[N]}(e_2x) = \mu_{[N]}(x)$. Therefore $\mu_{[N]}(x) = \mu_{[N]}(y^{-1}xy)$ for all x and y in F. Conversely, assume that $\mu_{[N]}(x) = \mu_{[N]}(y^{-1}xy)$, then $\mu_{[N]}(xy) = \mu_{[N]}(xyxx^{-1}) = \mu_{[N]}(yx)$. Therefore $\mu_{[N]}(xy) = \mu_{[N]}(yx)$ for all x and y in F.

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