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WEAKLY $I_{\hat{g}}$- CLOSED SETS AND WEAKLY I_{*g}-CLOSED SETS IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we define a new class of closed sets namely weakly closed sets in ideal topological spaces. Also, we study some characterizations and properties of weakly closed sets with respect to an ideal topological space.

Key Words: $wI_{\hat{g}}$-closed set, $wI_{\hat{g}}$-open set, wI_{*g}-closed set, wI_{*g}-open set.

1. INTRODUCTION

One of the important tools in General Topology is the ideals. Newcomb (1967) [9], Rancin(1972), Samuals(1975), Hamlet and JanKovic [4] (1990,1992,1990) motivated the research in general topology. A generalized closed set in topological space was introduced by Levine [7] (1967) in 1970. The notion of ideal topological space was studied by Kurotowski [6] (1933) and Vaidyanathaswamy [11] (1945). Jafari and Rajesh introduced I_g-closed set with respect to an ideal. In this paper, we introduced and study a new class of closed sets in ideal topological spaces called $wI_{\hat{g}}$ and wI_{*g} closed sets with respect to an ideal which is the weaker form of $I_{\hat{g}}$ and I_{*g} closed sets in ideal topological spaces.

2. PRELIMINARIES

Throughout the present paper (X, τ) always means a topological space. Let A be a subset of topological space (X, τ). The closure (resp. interior) of A are denoted by cl(A) (resp. Int(A)). An ideal (Kuratowski, 1933) [6] on a set X is a nonempty collection of subsets of X with heredity property and finite additivity property that is it satisfies the following two conditions:

1. $A \in I$ and $B \subseteq A$ then $B \in I$ (heredity)
2. $A \in I$ and $B \in I$ implies $A \cup B \in I$ (finite additivity)

Definition 2.1 [4]: A topological space (X,τ) with an ideal I on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^*$: $\wp(X) \to \wp(X)$, called a local functionof A with respect to τ and I is defined as follows: for $A \subseteq X$, $A^*(I, \tau) = \{x \in X \mid U \cap A \notin I \text{ for every } U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau \mid x \in U\}$.

Definition 2.2 [5]: A subset A of an ideal space (X, τ, I) is said to be$*$ - closedif $A^* \subseteq A$.

Definition 2.3 [1]: A subset A of a space (X, τ) is said to be $\hat{g}$-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.

Definition 2.4: A subset A of a space (X, τ) is said to be $\hat{g}$-open if its complement is $\hat{g}$-closed.

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Definition 2.5 [1]: A subset A of an ideal space (X, τ, I) is said to be $I_{\hat{g}}$-closed if A*⊆U whenever A⊆U and U is semi-open.

Definition 2.6 [10]: A subset A of a space (X, τ) is said to be *g-closed if A*⊆U whenever A ⊆ U and U is ĝ-open.

3. Weakly $I_{\hat{g}}$ closed sets and weakly I_{*g} -closed sets with respect to an Ideal

Definition 3.1: A subset A of an ideal space (X, τ, I) is said to be w$I_{\hat{g}}$-closed if int(A*) ⊆U whenever A ⊆U and U is semi-open.

Definition 3.2: A subset A of an ideal space (X, τ, I) is said to be w$I_{\hat{g}}$-open if X – A is w$I_{\hat{g}}$-closed.

Theorem 3.3: If (X, τ, I) is an ideal space, then every $I_{\hat{g}}$-closed set is w$I_{\hat{g}}$-closed but not conversely.

Proof: Let A be an $I_{\hat{g}}$-closed set. Let A ⊆ U and U is semi-open. Since A is $I_{\hat{g}}$-closed, A* ⊆ U whenever A ⊆ U and U is semi-open. We have int(A*) ⊆ A* ⊆U whenever A ⊆U and U is semi-open. Therefore, A is w$I_{\hat{g}}$-closed.

Example 3.4: Let X = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and I = {φ, {c}}. It is clear that {a} is w$I_{\hat{g}}$-closed but it is not $I_{\hat{g}}$-closed.

Theorem 3.5: Every ∗-closed set is w$I_{\hat{g}}$-closed but not conversely.

Proof: Let A be a ∗-closed, then A* ⊆ A. Let A ⊆ U where U is semi-open. Hence int(A*) ⊆ A* ⊆ U whenever A ⊆U and U is semi-open. Therefore, A is w$I_{\hat{g}}$-closed.

Example 3.6: Let X = {a, b, c}, τ = {φ, X, {b}, {c}, {b, c}} and I = {φ, {a}}. It is clear that {c} is w$I_{\hat{g}}$-closed but not *-closed.

Theorem 3.7: Let (X, τ, I) be an ideal space. For every A ∈ I, A is w$I_{\hat{g}}$-closed.

Proof: Let A ⊆ U where U is semi-open set. Since A* = φ for every A ∈ I, then cl*(A) = A ∪ A* = A, since A* = φ. int(A*) ⊆ cl*(A) = A ⊆U. Therefore, A is w$I_{\hat{g}}$-closed.

Theorem 3.8: If (X, τ, I) is an ideal space, then A* is always w$I_{\hat{g}}$-closed for every subset A of X.

Proof: Let A* ⊆ U where U is semi-open. Since int(A*)* ⊆ (A*)* ⊆ A*, we have (A*)* ⊆ U whenever A* ⊆ U and U is semi-open. Hence A* is -closed.

Theorem 3.9: Let (X, τ, I) be an ideal space. Then every ĝ-closed set is w$I_{\hat{g}}$-closed set but not conversely.

Proof: Let A be a ĝ-closed set. Then cl(A) ⊆ U whenever A ⊆ U and U is semi-open. We have int(A*) ⊆ cl*(A) ⊆ cl(A) ⊆ U whenever A ⊆ U and U is semi-open. Hence A is w$I_{\hat{g}}$-closed.

Example 3.10: Let X = {a, b, c}, τ = {φ, X, {b}, {c}, {b, c}} and I = {φ, {b}}. It is clear that {c} is w$I_{\hat{g}}$-closed set but not ĝ-closed set.

Theorem 3.11: Let (X, τ, I) be an ideal space and A ⊆ X. If A ⊆ B ⊆ int(A*), then A* = B* and B is ∗-dense in itself.

Proof: Since A ⊆ B, then A* ⊆ B* and since B ⊆int(A*), then B* ⊆ int(A*) ⊆ (A*)* ⊆ A*. Since A* ⊆ B* and B* ⊆ A*, A* = B*. Since B ⊆int(A*) ⊆ A* = B* . Therefore, B ⊆ B*. Hence B is ∗-dense in itself. Hence proved.

Theorem 3.12: Let (X, τ, I) be an ideal space. If every semi-open set is ∗-closed, then every subset of X is w$I_{\hat{g}}$-closed.

Proof: Let U be a semi-open set. Let A⊆ X be a subset of X. Let A ⊆ U where U is semi-open. If U is semi-open set such that A ⊆ U ⊆ X, then A* ⊆ U* ⊆ U. Therefore, int(A*) ⊆ A* ⊆ U whenever A ⊆ U and U is semi-open. Hence A is w$I_{\hat{g}}$-closed.

Theorem 3.13: Let (X, τ, I) be an ideal space. Then every I_g-closed, open set is w$I_{\hat{g}}$-closed.

Proof: Let A be an $I_{\hat{g}}$-closed, open set. Let A⊆ A where A is open. Since A is open and $I_{\hat{g}}$-closed, A* ⊆ A. Let A ⊆ U where U is semi-open. Since A* ⊆ A, int(A*) ⊆ A* ⊆ A ⊆ U which implies int(A*) ⊆ U whenever U is semi-open. Hence A is $wI_{\hat{g}}$-closed.

Theorem 3.14: Let (X, τ, I) be an ideal space. Then every g-closed set is $wI_{\hat{g}}$-closed but not conversely.

Proof: Let A ⊆ A where A is open. Since A is g-closed, cl(A) ⊆ A. We have A ⊆ cl(A) which implies cl(A) = A. Let A ⊆ U and U is semi-open.Therefore, int(A*) ⊆ cl*(A) ⊆ cl(A) ⊆ A ⊆ U. Hence A is$wI_{\hat{g}}$-closed.

Example 3.15: Let X = {a, b, c}, τ = {φ, X, {b}, {c}, {b, c}} and I = {φ, {b}}. It is clear that {c} is $wI_{\hat{g}}$-closed set but not g-closed set.

Definition 3.16: A subset A of an ideal space (X, τ, I) is said to be wI_{*g}-closed if int(A*) ⊆U whenever A ⊆U and U is ĝ-open.

Definition 3.17: A subset A of an ideal space (X, τ, I) is said to be wI_{*g}-open if X – A is wI_{*g}-closed.

Theorem 3.18: If (X, τ, I) is an ideal space, then everyI_{*g}-closed set is wI_{*g}-closed but not conversely.

Proof: Let A ⊆ U and U is ĝ-open. Since A is I_{*g}-closed, A* ⊆ U whenever A ⊆ U and U is ĝ-open. We have int(A*) ⊆ A* ⊆U whenever A ⊆U and U is ĝ-open. Therefore, A is wI_{*g}-closed.

Example 3.19: Let X = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and I = {φ, {c}}. It is clear that {a} is wI_{*g}-closed but it is not I_{*g}-closed.

Theorem 3.20: Every ∗-closed set is wI_{*g}-closed but not conversely.

Proof: Let A be a ∗-closed, then A* ⊆ A. Let A ⊆ U where U is $\hat{g}$-open. Therefore, int(A*) ⊆ A* ⊆ U. Hence int(A*) ⊆ U whenever A ⊆ U and U is $\hat{g}$-open. Therefore, A is wI_{*g}-closed.

Example 3.21: Let X = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and I = {φ, {c}}. It is clear that {b} is wI_{*g}-closed but not *-closed.

Theorem 3.22: Let (X, τ , I) be an ideal space. For every A ∈ I, A is wI_{*g}-closed.

Proof: Let A ⊆ U where U is $\hat{g}$-open set. Since A* = φ for every A ∈ I, then cl*(A) = A ∪ A* = A, since A* = φ. Therefore, int(A*) ⊆ cl*(A) = A ⊆ U. Hence int(A*) ⊆ U whenever A ⊆ U and U is $\hat{g}$-open. Therefore, A is wI_{*g}-closed.

Theorem 3.23: If (X, τ, I) is an ideal space, then A* is always wI_{*g}-closed for every subset A of X.

Proof: Let A* ⊆ U where U is $\hat{g}$-open. Since int(A*)* ⊆ (A*)* ⊆ A*, we have (A*)* ⊆ U whenever A* ⊆ U and U is $\hat{g}$-open. Hence A* is wI_{*g}-closed.

Remark 3.24: Every wI_{*g}-closed set is need not be a $\hat{g}$-closed set. This can be shown from the following example.

Example 3.25: Let X = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}} and I = {φ, {c}}. It is clear that {b} is wI_{*g}-closed set but not $\hat{g}$-closed set.

Theorem 3.26: Let (X, τ, I) be an ideal space and A ⊆ X. If A ⊆ B ⊆int(A*), then A* = B* and B is ∗-dense in itself.

Proof: Since A ⊆ B, then A* ⊆ B* and since B ⊆int(A*), then B* ⊆ int(A*) ⊆ (A*)* ⊆ A*. Since A* ⊆ B* and B* ⊆ A*, A* = B*. Since B ⊆int(A*) ⊆ A* = B*. Therefore, B ⊆ B*.Hence B is ∗-dense in itself. Hence proved.

Theorem 3.27: Let (X, τ, I) be an ideal space. If every $\hat{g}$-open set is ∗-closed, then every subset of X is wI_{*g}-closed.

Proof: Let U be a $\hat{g}$-open set. Let A⊆ X be a subset of X. Let A ⊆ U where U is $\hat{g}$-open. If U is $\hat{g}$-open set such that A ⊆ U ⊆ X, then A* ⊆ U* ⊆ U. Therefore, int(A*) ⊆ A* ⊆ U whenever A ⊆ U and U is $\hat{g}$-open. Hence A is wI_{*g}-closed.

Theorem 3.28: Let (X, τ, I) be an ideal space. Then every I_g-closed, $\hat{g}$-open set is wI_{*g}-closed.

Proof: Let A be an I_g-closed, open set. Let A⊆ A where A is open. Since A is open and I_g-closed, A* ⊆ A. Let A ⊆ U where U is ĝ-open. Since A* ⊆ A, int(A*) ⊆ A* ⊆ A ⊆ U which implies int(A*) ⊆ U whenever U is ĝ-open. Hence A is wI_{*g}-closed.

Theorem 3.29: Let (X, τ, I) be an ideal space. Then every g-closed set is wI_{*g}-closed but not conversely.

Proof: Let A ⊆ A where A is open. Since A is g-closed, cl(A) ⊆ A. We have A ⊆ cl(A) which implies cl(A) = A. Let A ⊆ U and U is ĝ-open.Therefore, int(A*) ⊆ cl*(A) ⊆ cl(A) ⊆ A ⊆ U. Hence A is wI_{*g}-closed.

Example 3.30: Let X = {a, b, c}, τ = { φ, X, {a}, {b}, {a, b}} and I = { φ, {c}}. It is clear that {a} is wI_{*g}-closed set but not g-closed set.

Theorem 3.31: Let (X, τ, I) be an ideal space. Then every *g-closed set is wI_{*g}-closed set but not conversely.

Proof: Let A be a *g-closed set. Then cl(A) ⊆ U whenever A ⊆ U and U is ĝ-open. We have
int(A*) ⊆ cl*(A) ⊆ cl(A) ⊆ U whenever A ⊆ U and U is ĝ-open. Hence A is wI_{*g}-closed.

Example 3.32: Let X = {a, b, c}, τ = { φ, X, {a}, {b}, {a, b}} and I = { φ, {c}}. It is clear that {b} is wI_{*g}-closed set but not *g-closed set.

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