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STUDY ON STRONGLY IRREGULAR FUZZY GRAPHS

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ABSTRACT

In this paper, Some results on Strongly Irregular Fuzzy Graphs and Strongly Total Irregular Fuzzy Graphs are established.

Keywords: *Degree of fuzzy graph, regular fuzzy graph, irregular fuzzy graph, highly irregular fuzzy graph, strongly irregular fuzzy graph and strongly total irregular fuzzy graph.*

INTRODUCTION

Rosenfeld [9] considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. Nagoor Gani and Radha[6] introduced regular fuzzy graphs,total degree and totally regular fuzzy graphs. Gnaana Bhragsam and Ayyaswamy[4] suggested a method to construct a neighbourly irregular graph of order n and also discussed some properties on neighbourly irregular graph. Yousef Alavi, *et al.*, [11] introduced k-path irregular graph and studied some properties on k-path irregular graphs. Nagoor Gani and Latha[7] introduced neighbourly irregular fuzzy graphs, neighbourly total irregular fuzzy graphs, highly irregular fuzzy graphs and highly total irregular fuzzy graphs. SP.Nandhini and E.Nandhini [8] introduced strongly irregular fuzzy graphs, strongly total irregular fuzzy graphs. In this paper, a comparative study between strongly irregular and strongly total irregular fuzzy graphs are made. Also some results on strongly irregular fuzzy graphs and strongly total irregular fuzzy graphs are studied. Throughout this paper only undirected fuzzy graphs are considered.

1. PRELIMINARIES

Definition 1.1: A fuzzy graph G = (σ, μ) is a pair of functions σ: V → [0, 1] and μ: V x V → [0, 1], where for all u, v ∈V, we have μ(u, v) ≤ σ(u) Λ σ(v).

Definition 1.2 The fuzzy subgraph H = (τ, ρ) is called a fuzzy subgraph of G = (σ, μ) if τ (u) ≤ σ (u) for all u ∈V and ρ(u, v) ≤ μ(u, v) for all u, v ∈V.

Definition 1.3: The underlying crisp graph of a fuzzy graph G = (σ, μ) is denoted by G* = (σ*, μ*), where σ* = {u ∈V / σ(u) > 0} and μ* = { (u, v) ∈V x V / μ(u, v) > 0}.

Definition 1.4: Let G = (σ, μ) be a fuzzy graph.

The degree of a vertex u is dG(u) = d(u) = $\sum_{u\neq v} \mu(u,v) = \sum_{uv\in E} \mu(u,v)$.

Definition 1.5: Let G = (σ, μ) be a fuzzy graph on G*. The total degree of a Vertex u∈V is defined by tdG(u) = $\sum_{u\neq v} \mu(u,v)$ + σ(u) = $\sum_{uv\in E} \mu(u,v)$ + σ (u) = dG(u) + σ(u).

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Definition 1.6: A path ρ in a fuzzy graph is a sequence of distinct vertices $u_0, u_1, u_2, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0$, $1 \le i \le n$. The path ρ is called a cycle if $u_0 = u_n$ and $n \ge 3$.

Definition 1.7: The complement of a fuzzy graph G = (σ, μ) is a fuzzy graph $G^C = (\sigma^C, \mu^C)$, where $\sigma^C = \sigma$ and $\mu^C(u, v) = \sigma(u) \Lambda \sigma(v) - \mu(u, v)$ for all u, v in V.

Example 1.8: In G= (σ, μ), σ(u) = 0.5, σ(v) = 0.4, σ(w) = 0.7, σ(x) = 0.3, σ(y) = 0.2And μ(u, v) = 0.3, μ(v, w) = 0.2, μ(w, x) = 0.3, μ(x, y) = 0.2, μ(u, y) = 0.1**,** d(u) = 0.4, d(v) = 0.5, d(w) = 0.5, d(x) = 0.5, d(y) = 0.3**.** In $G^C = (\sigma^C, \mu^C)$, σ(u) = 0.5, σ(v) = 0.4, σ(w) = 0.7, σ(x) = 0.3, σ(y) = 0.2And $\mu^C(u, v) = 0.1$, $\mu^C(v, w) = 0.2$, $\mu^C(u, w) = 0.5$, $\mu^C(u, x) = 0.3$, $\mu^C(u, y) = 0.1$ $\mu^C(y, w) = 0.2$, $\mu^C(v, y) = 0.2$, $\mu^C(v, x) = 0.3$ and d (u) = 1.0, d(v) = 0.8, d(w) = 0.9**,** d(x) = 0.6, d(y) = 0.5

Definition1.9: Let G= (σ, μ) be a fuzzy graph. Then G is irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

Example 1.10: G = (σ, μ) by σ(u) = 0.4 , σ(v) = 0.6, σ(w) = 0.4, σ(x) = 0.3,σ(y) = 0.5 and μ(u, v) = 0.2, μ(v, w) = 0.4, μ(w, x) = 0.3, μ(x, y) = 0.2, μ(u, y)=0.3. And d(u) = 0.5**,** d (v) = 0.6**,** d (w) = 0.7**,** d (x) = 0.5**,** d (y) = 0.5.

Definition 1.11: A fuzzy graph G = (σ, μ) is a complete fuzzy graph if μ(u, v) = σ(u) Λ σ(v) for all u, v ∈σ*.

Example 1.12: σ(u) = 0.3, σ(v) = 0.5, σ(w)= 0.7, σ(x) = 0.8, and μ(u, v) = 0.3, μ(v, w) = 0.5, μ(w, x) = 0.7, μ(x, u) = 0.3.

Definition 1.13: Let G = (σ, μ) be a fuzzy graph such that G* = (V, E) is a cycle. Then G is a fuzzy cycle if and only if there does not exist a unique edge (x, y) such that μ(x, y) = Λ {μ (u, v) / (u, v)> 0}.

Definition 1.14: Let G = (σ, μ) be a connected fuzzy graph. G is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of G have distinct degree.

Example 1.15: Define G = (σ, μ) by σ(u) = 0.7 , σ(v) = 0.8, σ(w) = 0.6, σ(x) = 0.5,and μ(u, v) = 0.7, μ(v, w) = 0.5, μ(w, x) = 0.5, μ(x, u) = 0.3. And d (u)=1,d(v) = 1.2,d(w) =1,d(x) =0.8

Definition 1.16: Let G = (σ, μ) be a fuzzy graph. Then G is totally irregular, if there is a vertex which is adjacent to vertices with distinct total degrees.

Example 1.17: Define G = (σ, μ) by σ(u) = 0.4 , σ(v) = 0.5, σ(w) = 0.3, σ(x) = 0.6,and μ(u, v) = 0.3, μ(v, w) = 0.2, μ(w, x) = 0.1, μ(x, u) = 0.4.And d (u) = 0.7,d(v) = 0.5,d(w) = 0.3, d(x) = 0.5,td (u) = 1.1,td(v) = 1.0,td(w) = 0.6**,** td(x) = 1.1

Definition 1.18: If every two adjacent vertices of a fuzzy graph G = (σ, μ) have distinct total degree, then G is said to be a neighbourly total irregular fuzzy graph.

Example 1.19: Define G = (σ, μ) by σ(u) = 0.4 , σ(v) = 0.5, σ(w) = 0.6, σ(x) = 0.2 and μ(u, v) = 0.3, μ(v, w) = 0.2, μ(w, x) = 0.1, μ(x, u) = 0.2.And d (u) = 0.5,d(v) = 0.5,d(w) = 0.3,d(x) = 0.3,td (u) = 0.9,td(v) = 1.0 , td(w) = 0.9, td(x) = 0.5

Definition 1.20: Let G = (σ, μ) be a connected fuzzy graph. G is said to be a highly irregular fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees.

Example 1.21: Define G = (σ, μ) by σ(u) = 0.7, σ(v) = 0.5, σ(w) = 0.9, σ(x) = 0.5 σ(y) = 0.8, and μ(u, v) = 0.3, μ(v, w) = 0.5, μ(w, x) = 0.5, μ(x, y) = 0.3, μ(y, u) =0.6,d(u) = 0.9,d(v) = 0.8,d(w) = 1.0,d(x) = 0.8,d(y) = 0.9

Definition 1.22: Let G = (σ, μ) be a connected fuzzy graph. G is said to be a highly total irregular fuzzy graph if every vertex of G is adjacent to vertices with distinct total degrees.

Example 1.23: Define G = (σ, μ) by σ(u) = 0.4 , σ(v) = 0.5, σ(w) = 0.5, σ(x) = 0.7,and μ(u, v) = 0.3, μ(v, w) = 0.2, μ(w, x) = 0.3, μ(x, u) = 0.4.And d (u) = 0.7,d(v) = 0.5,d(w) = 0.5,d(x) = 0.7,td (u) = 1.1,td(v) = 1.0**,** td(w) = 1.0, td(x) = 1.4

Definition 1.24: Let G = (σ, μ) be a connected fuzzy graph. G is said to be a strongly irregular fuzzy graph if every pair of vertices in G have distinct degrees.

Example 1.25: Define G = (σ, μ) by σ(u) = 0.8 , σ(v) = 0.5, σ(w) = 0.7, σ(x) = 0.6, and μ(u, v) = 0.3, μ(v, w) = 0.4, μ(w, x) = 0.6, μ(x, u) = 0.6.And d(u) = 0.9d(v) = 0.7, d(w) = 1.0, d(x) = 1.2.

Definition 1.26: Let G = (σ, μ) be a connected fuzzy graph. G is said to be a strongly total irregular fuzzy graph if every pair of vertex in G have distinct total degrees.

Example 1.27: Define G = (σ, μ) by σ(u) = 0.4, σ(v) = 0.5, σ(w) = 0.6, σ(x)=0.3, σ(y)=0.6, and μ(u, v) = 0.4, μ(v, w) = 0.1, μ(w, x) = 0.2, μ(x, y) = 0.3, μ(y, u) =0.3, d (u) = 0.7,d(v) = 0.5,d(w) = 0.3**,** d(x) = 0.5**,** d(y) =0.6, td (u) = 1.1**,** td(v) = 1.0, td(w) = 0.9**,** td(x) = 0.8 and td(y) = 1.2

2. PROPERTIES OF STRONGLY IRREGULAR FUZZY GRAPHS

Theorem 2.1: A fuzzy graph G = (σ, μ) where G* is a cycle with vertices 3 is strongly irregular if and only if the weights of the edges between every pair of vertices are all distinct.

Proof: For, if the weights of any edges are the same, it violates the definition of strongly irregular fuzzy graphs.

Conversely, the weights of edges between every pair of vertices are all distinct. Let u, v and w are the vertices of G

Suppose d(u) = d(v)

=> μ(u, v)+ μ(u, w)= μ(u, v)+ μ(v, w)

=> μ(u, w)= μ(v, w), a contradiction.

Therefore G is a Strongly irregular fuzzy graph.

Proposition 2.2: A fuzzy graph G = (σ, μ) where G* is a cycle with vertices 3. If G is a strongly irregular then G need not be a fuzzy cycle.

Example 2.3: Define G = (σ, μ) by σ(u) = 0.7, σ(v) = 0.5, σ(w) = 0.4 and μ(u, v) = 0.5, μ(v, w) = 0.3, μ(w, u) = 0.4, d(u) = 0.9, d(v) = 0.8, d(w) = 0.7.

There exist a unique edge μ(v, w) = 0.3 = Λ {μ (u, v) / (u, v) > 0}.

Proposition 2.4: If a fuzzy graph G = (σ, μ) is strongly irregular, then the fuzzy subgraph H = (τ, ρ) of G need not be a strongly irregular fuzzy graph.

Example 2.5: Define G = (σ, μ) by σ(u) = 0.8, σ(v) = 0.5, σ(w) = 0.7, σ(x) = 0.6, and μ(u, v)=0.3, μ(v, w) = 0.4, μ(x, w) = 0.6, μ(u, x) = 0.6, d(u) = 0.9, d(v) = 0.7, d(w) = 1.0, d(x) = 1.2 and define H= (τ, ρ) by τ(u)=0.7, τ (v) = 0.3, τ (w) = 0.7, τ (x) = 0.5, and ρ (u, v)=0.3, ρ (v, w) = 0.3, ρ (x, w) = 0.5, ρ (u, x) = 0.3, d(u) = 0.6, d(v) = 0.6, d(w) = 0.8, d(x) = 0.8.

Here d(u) = d(v) and d(w) = d(x) in H.

Proposition 2.6: Let G = (σ, μ) be a fuzzy cycle then G need not be a strongly irregular fuzzy graph.

Example 2.7: Define G = (σ, μ) by σ(u) = 0.8, σ(v) = 0.9, σ(w) = 0.7, σ(x) = 0.6, And μ(u, v)=0.5, μ(v, w) = 0.3, μ(x, u) = 0.3, μ(w, x) = 0.6, d(u) = 0.8, d(v) = 0.8, d(w) = 0.9, d(x) = 0.9.

Here d(u) = d(v) and d(w) = d(x).

Proposition 2.8: Let G = (σ, μ) be a complete fuzzy graph. If G is a fuzzy cycle then G need not be a strongly irregular fuzzy graph.

Example 2.9: Define G = (σ, μ) by σ(u) = 0.7, σ(v) = 0.9, σ(w) = 0.5, σ(x)=0.5 and μ(u, v) = 0.7, μ(u, x) = 0.5, μ(w, v) = 0.5, μ(x, w) = 0.5, d (u) = 1.2**,** d(v) = 1.2, d(w) = 1.0**,** d(x) = 1.0.

Theorem 2.10: Let G = (σ, μ) highly irregular fuzzy and neighbourly irregular fuzzy graph G = (σ, μ). If every pair of vertices in G is either adjacent or incident on the same vertex then G is strongly irregular.

Proof: Suppose every pair of vertices is either adjacent or incident on the same vertex.

Since G = (σ, μ) is both highly irregular and neighbourly irregular fuzzy graph, every vertices have distinct degrees.

Therefore G = (σ, μ) is strongly irregular fuzzy graph.

Theorem 2.11: Let G = (σ, μ) be a fuzzy graph, where G* is regular, σ is a constant function and μ(u, v) < σ (u) Λ σ (v) for all u, v∈ V(G).Then G is a strongly irregular fuzzy graph iff G^c is a strongly irregular fuzzy graph.

Proof: Let G= (σ, μ) be a strongly irregular fuzzy graph and σ(u) =c for all u ∈G. d(u) ≠ d(v) for all u, v∈ V(G).

$$\Leftrightarrow \sum \mu(u, x_i) \neq \sum \mu(v, y_j) \forall\, x_i \text{ incident on } u \text{ and } \forall\, y_j \text{ incident on } v$$

$$\Leftrightarrow \sum [c - \mu(u, x_i)] \neq \sum [c - \mu(v, y_j)] \forall\, x_i \text{ incident on } u \text{ and } \forall\, y_j \text{ incident on } v$$, since G* is regular.

$$\Leftrightarrow \sum [\sigma(u) \wedge \sigma(x_i) - \mu(u, x_i)] \neq \sum [\sigma(v) \wedge \sigma(y_j) - \mu(v, y_j)] \forall\, x_i \text{ incident on } u \text{ and } \forall\, y_j \text{ incident on } v.$$

$$\Leftrightarrow \sum \mu^c(u, x_i) \neq \sum \mu^c(v, y_j) \forall\, u, v \in G^c.$$

=> $G^c = (\sigma^c, \mu^c)$ is a strongly irregular fuzzy graph.

Theorem 2.12: Let G = (σ, μ) be a complete fuzzy graph. If G is a strongly irregular fuzzy graph then G^c is not a strongly irregular fuzzy graph.

Proof: Let G= (σ, μ) be complete fuzzy graph and strongly irregular fuzzy graph. d(u) ≠ d(v) for all u,v∈V(G) and $\sigma(u) \wedge \sigma(x_i) = \mu(u, x_i) \forall u \in G \,\&\, x_i \text{ incident on } u.$

$$\Rightarrow \sum \mu(u, x_i) \neq \sum \mu(v, y_j) \forall\, x_i \text{ incident on } u \text{ and } \forall\, y_j \text{ incident on } v.$$

=> $\sum [\sigma(u) \wedge \sigma(x_i) - \mu(u, x_i)] = 0 \;\forall\, u \in G^c$.

=> G^c is not a connected fuzzy graph.

=> G^c is not a strongly irregular fuzzy graph.

Theorem 2.13: The fuzzy subgraph H = (τ, ρ) of a strongly irregular fuzzy graph G = (σ, μ) with $\rho(u, v)) = \mu(u, v)$ $\forall\, u, v \in V$ is strongly irregular fuzzy graph.

Proof: Let G= (σ, μ) be a strongly irregular fuzzy graph.

=> d(u) ≠ d(v) for all u,v∈ V(G).

$$\Rightarrow \sum \mu(u, x_i) \neq \sum \mu(v, y_j) \forall\, x_i \text{ incident on } u \text{ and } \forall\, y_j \text{ incident on } v.$$

$$\Rightarrow \sum \rho(u, x_i) \neq \sum \rho(v, y_j) \forall\, x_i \text{ incident on } u \text{ and } \forall\, y_j \text{ incident on } v.$$

=> d(u) ≠ d(v) for all u,v∈ V(H).

Proposition 2.14: The fuzzy subgraph H = (τ, ρ) of a strongly irregular fuzzy graph G = (σ, μ) with $\rho(u, v) < \mu(u, v)$ $\forall\, u, v \in V$ need not be a strongly irregular fuzzy graph.

Example 2.15: Define G = (σ, μ) by σ(u) = 0.8, σ(v) = 0.5, σ(w) = 0.7, σ(x) = 0.6, and μ(u, v)=0.3, μ(v, w) = 0.4, μ(x, w) = 0.6, μ(u, x) = 0.6, d(u) = 0.9, d(v) = 0.7, d(w) = 1.0, d(x) = 1.2 and define H= (τ, ρ) by τ(u)=0.7, τ (v) = 0.3, τ (w) = 0.7, τ (x) = 0.5, and ρ (u, v)=0.2, ρ (v, w) = 0.3, ρ (x, w) = 0.5, ρ (u, x) = 0.3,d(u) = 0.5,d(v) = 0.5, d(w) = 0.8, d(x) = 0.8

Here d(u) = d(v) and d(w) = d(x) in H.

Theorem 2.16: The underlying crisp graph G* = (σ*,μ*)of a fuzzy graph G = (σ, μ) is complete then G is a neighbourly irregular fuzzy graph if and only if G is a strongly irregular fuzzy graph .

Proof: Let the underlying crisp graph G* = (σ*, μ*) of a fuzzy graph G = (σ, μ) is complete. Every two vertices are adjacent.

Suppose G is neighbourly irregular fuzzy graph.

⇔every two adjacent vertices have distinct degrees.

⇔every vertices of G have distinct degrees.

⇔G is a strongly irregular fuzzy graph.

Theorem 2.17: The underlying crisp graph G* = (σ*, μ*) of a fuzzy graph G = (σ, μ) is complete then G is a highly irregular fuzzy graph if and only if G is a strongly irregular fuzzy graph .

Proof: Let the underlying crisp graph G* = (σ*, μ*) of a fuzzy graph G = (σ, μ) is complete with n vertices. Every vertex of G is adjacent to remaining (n-1) vertices.

Suppose G is highly irregular fuzzy graph.

⇔Every vertex of G is adjacent to vertices with distinct degrees.

⇔Every vertices of G have distinct degrees.

⇔G is a strongly irregular fuzzy graph.

3. PROPERTIES OF STRONGLY TOTAL IRREGULAR FUZZY GRAPH

Proposition 3.1: A fuzzy graph G = (σ, μ) where G* is a cycle with vertices 3and G is strongly total irregular then the weights of the edges between every pair of vertices are all need not be distinct .

Example 3.2: Define G = (σ, μ) by σ(u) = 0.8 , σ(v) = 0.7, σ(w) = 0.6 and μ(u, v) = 0.2, μ(v, w) = 0.2, μ(w, u) = 0.4, td(u) = 1.4, td(v) = 1.1,td(w) = 1.2.

Proposition 3.3: A fuzzy graph G = (σ, μ) where G* is a cycle with vertices 3and G is strongly total irregular then G need not be a fuzzy cycle.

Example 3.4: Define G = (σ, μ) by σ(u) = 0.7, σ(v) = 0.5, σ(w) = 0.4 and μ(u, v) = 0.5, μ(v, w) = 0.3, μ(w, u) = 0.4, td(u) = 1.6, td(v) = 1.3,td(w) = 1.1.

There exist a unique edge μ(v, w)=0.3=Λ {μ (u, v) / (u, v)> 0}.

Theorem 3.5: A fuzzy graph where G* is a cycle with vertices 3 and σ is a constant function then G is strongly total irregular if and only if the weights of the edges between every pair of vertices are all distinct.

Proof: For, if the weights of any edges are the same, it violates the definition of strongly total irregular fuzzy graphs.

Conversely, the weights of edges between every pair of vertices are all distinct. Let u, v and w are the vertices of G and σ(u)=k for all u belongs to G.

Suppose td(u) = td(v)
=> σ (u)+μ(u, v)+ μ(u, w)= σ(v)+ μ(u, v)+ μ(v, w)

=>μ(u, w)= μ(v, w), a contradiction

Proposition 3.6: Let G = (σ, μ) be either a strongly irregular or a strongly total irregular fuzzy graph where G* is a cycle then G need not be a fuzzy cycle.

Example 3.7: Define G = (σ, μ) by σ(u) = 0.8 , σ(v) = 0.9, σ(w) = 0.3, σ(x) = 0.5, And μ(u, v)=0.6, μ(v, w) = 0.3, μ(x, u) = 0.5, μ(w, x) = 0.2, d(u) = 1.1d(v) = 0.9,d(w) = 0.5,d(x) = 0.7,t d(u) = 1.9,t d(v) = 1.8,t d(w) = 0.8,t d(x) = 1.2
But G is not a fuzzy cycle, since there exists a unique edge (u, v) such that μ(w, x) = 0.2= Λ {μ (u, v) / (u, v)> 0}.

Proposition 3.8: Let G = (σ, μ) be a fuzzy cycle then G need not be a strongly total irregular fuzzy graph.

Example 3.9: Define G = (σ, μ) by σ(u) = 0.8, σ(v) = 0.8, σ(w) = 0.7, σ(x) = 0.6, and μ(u, v)=0.5, μ(v, w) = 0.3, μ(x, u) = 0.3, μ(w, x) = 0.6, td(u) = 1.6, td(v) = 1.6, td(w) = 1.6, td(x) = 1.5.

Proposition 3.10: Let G = (σ, μ) be a fuzzy graph. If G is both strongly and strongly total irregular fuzzy graph then σ need not be a constant function.

Example 3.11: Define G = (σ, μ) by σ(u) = 0.8 , σ(v) = 0.9, σ(w) = 0.3, σ(x) = 0.5, And μ(u, v)=0.7, μ(v, w) = 0.3, μ(x, u) = 0.5, μ(w, x) = 0.2, d(u) = 1.2,d(v) = 1.0,d(w) = 0.5,d(x) = 0.7,td(u) = 2.0,td(v) = 1.9,td(w) = 0.8,td(x) = 1.2.

Theorem 3.12: If G= (σ, μ) is a strongly total irregular fuzzy graph then it is both highly total irregular fuzzy and neighbourly total irregular fuzzy graph.

Proof: If G is strongly irregular fuzzy graph, then every pair of vertices in G have distinct total degrees. Obviously every two adjacent vertices have distinct degrees and every vertex of G adjacent to vertices with distinct total degrees. Hence G is neighbourly total irregular and highly total irregular fuzzy graph.

Proposition 3.13: Let G= (σ, μ) highly total irregular fuzzy and neighbourly total irregular fuzzy graph then G need not be a strongly total irregular fuzzy graph.

Example 3.14: Define G = (σ, μ) by σ(u) = 0.4, σ(v) = 0.3, σ(w) = 0.5, σ(x) = 0.8, σ(y) = 0.7 and μ(u, v)=0.2, μ(v, w) = 0.1, μ(x, u) = 0.1, μ(v, y) = 0.2, td(x) = 0.9, td(u) = 0.7, td(v) = 0.8,td(w) = 0.6, td(y) = 0.9.

Theorem 3.15: Let G = (σ, μ) highly total irregular fuzzy and neighbourly total irregular fuzzy graph G = (σ, μ).

If every pair of vertices in G is either adjacent or incident on the same vertex then G is strongly total irregular.

Proof: Suppose every pair of vertices is either adjacent or incident on the same vertex.

Since G = (σ, μ) is both highly total irregular and neighbourly total irregular fuzzy graph, every vertices have distinct total degrees.

Therefore G = (σ, μ) is a strongly total irregular fuzzy graph.

Theorem 3.16: Let G = (σ, μ) be a fuzzy graph, where G* is regular, σ is a constant function and μ(u, v) < σ (u) Λ σ (v) for all u, v∈ V(G). Then G is a strongly total irregular fuzzy graph if and only if G^c is a strongly total irregular fuzzy graph.

Proof: Let G= (σ, μ) be a strongly total irregular fuzzy graph and σ(u) =c for all u ∈ G.

td(u) ≠ td(v) for all u, v∈ V(G) .

$$\Leftrightarrow \sigma(u)+\sum \mu(u,x_i) \neq \sigma(v)+\sum \mu(v,y_j) \forall\, x_i \textit{ incident on } u \textit{ and } \forall\, y_j \textit{ incident on } v.$$

$$\Leftrightarrow \sigma(u)+\sum [c-\mu(u,x_i)] \neq \sigma(v)+\sum [c-\mu(v,y_j)] \forall\, x_i \textit{ incident on } u \textit{ and } \forall\, y_j \textit{ incident on } v,$$ since G* is regular.

$$\Leftrightarrow \sigma(u)+\sum [\sigma(u)\wedge\sigma(x_i)-\mu(u,x_i)] \neq \sigma(v)+\sum [\sigma(v)\wedge\sigma(y_j)-\mu(v,y_j)] \forall\, x_i \textit{ incident on } u \textit{ and } \forall\, y_j \textit{ incident on } v.$$

$$\Leftrightarrow \sigma(u)+\sum \mu^c(u,x_i) \neq \sigma(v)+\sum \mu^c(v,y_j) \forall\, u,v \in G^c.$$

⇔ td(u) ≠ td(v) for all u,v∈ V(G^c) .

⇨ G^c= (σ^c, μ^c) is a strongly total irregular fuzzy graph.

Theorem 3.17: Let H = (τ, ρ) be a fuzzy subgraph of a fuzzy graph G = (σ, μ) where G* is regular. If τ is a constant function and $\rho(u,v)) = \mu(u,v)\ \forall\, u,v \in V$ then G is a strongly irregular fuzzy graph iff H is a strongly total irregular fuzzy graph .

Proof: Let G= (σ, μ) be a strongly irregular fuzzy graph.
<=> d(u) ≠ d(v) for all u, v∈ V(G).

$$\Leftrightarrow \sum \mu(u, x_i) \neq \sum \mu(v, y_j) \forall x_i \text{ incident on } u \text{ and } \forall y_j \text{ incident on } v.$$

$$\Leftrightarrow \sum \rho(u, x_i) \neq \sum \rho(v, y_j) \forall x_i \text{ incident on } u \text{ and } \forall y_j \text{ incident on } v.$$

$$\Leftrightarrow \tau(u) + \sum \rho(u, x_i) \neq \tau(v) + \sum \rho(v, y_j) \forall x_i \text{ incident on } u \text{ and } \forall y_j \text{ incident on } v,$$ since G* is regular.

< => td(u) ≠ td(v) for all u, v∈ V(H).

<=>The fuzzy subgraph H = (τ, ρ) is a strongly total irregular fuzzy graph.

Theorem 3.18: Let H = (τ, ρ)be a fuzzy subgraph of H = (τ, ρ) of G = (σ, μ)where G* is regular . τ and σ are constant functions and $\rho(u,v)) = \mu(u,v) \; \forall u, v \in V$. Then G is a strongly total irregular fuzzy graph if and only if H is a strongly total irregular fuzzy graph.

Proof: Let G= (σ, μ) be a strongly total irregular fuzzy graph.
=> td(u) ≠ td(v) for all u, v∈ V(G).

$$\Leftrightarrow \sigma(u) + \sum \mu(u, x_i) \neq \sigma(v) + \sum \mu(v, y_j) \forall x_i \text{ incident on } u \text{ and } \forall y_j \text{ incident on } v.$$

$$\Leftrightarrow \sum \mu(u, x_i) \neq \sum \mu(v, y_j) \forall x_i \text{ incident on } u \text{ and } \forall y_j \text{ incident on } v$$

$$\Leftrightarrow \sum \rho(u, x_i) \neq \sum \rho(v, y_j) \forall x_i \text{ incident on } u \text{ and } \forall y_j \text{ incident on } v.$$

$$\Leftrightarrow \tau(u) + \sum \rho(u, x_i) \neq \tau(v) + \sum \rho(v, y_j) \forall x_i \text{ incident on } u \text{ and } \forall y_j \text{ incident on } v,$$ since G* is regular.

<=> td(u) ≠ td(v) for all u, v∈ V(H).

<=>The fuzzy subgraph H = (τ, ρ) is a strongly total irregular fuzzy graph.

Theorem 3.19: The underlying crisp graph G* = (σ*, μ*)of a fuzzy graph G = (σ, μ) is complete then G is a neighbourly total fuzzy graph if and only if G is a strongly total irregular fuzzy graph .

Proof: Let the underlying crisp graph G* = (σ*, μ*) of a fuzzy graph G = (σ, μ) is complete. Every two vertices are adjacent.

Suppose G is neighbourly total irregular fuzzy graph.
⇔ Every two adjacent vertices have distinct total degrees.

⇔ Every vertices of G have distinct total degrees.

⇔ G is a strongly irregular total fuzzy graph.

Theorem 3.20: The underlying crisp graph G* = (σ*, μ*)of a fuzzy graph G = (σ, μ) is complete then G is highly total irregular fuzzy graph if and only if G is a strongly total irregular fuzzy graph .

Proof: Let the underlying crisp graph G* = (σ*, μ*) of a fuzzy graph G = (σ, μ) is complete with n vertices. Every vertex of G is adjacent to remaining (n-1) vertices.

Suppose G is highly total irregular fuzzy graph.

⇔Every vertex of G is adjacent to vertices with distinct total degrees.

⇔Every vertices of G have distinct total degrees.

⇔G is a strongly total irregular fuzzy graph.

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