International Journal of Mathematical Archive-6(7), 2015, 15-20
IJMA Available online through www.ijma.info ISSN 2229 – 5046

1-NEAR MEAN CORDIAL LABELING OF GRAPHS

K. PALANI*1, J. REJILA JEYA SURYA2

Department of Mathematics,
A. P. C. Mahalaxmi College, Thoothukudi, Tamilnadu, India.

(Received On: 28-05-15; Revised & Accepted On: 14-07-15)

ABSTRACT

Let G = (V, E) be a simple graph. A surjective function f: V(G)→{0,1,2} is said to be a 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv) = \begin{cases} 0 & if \ \frac{f(u)+f(v)}{2} \ \text{is an integer} \\ 1 & \text{otherwise} \end{cases}$$

satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ *where* $e_f(0)$ *is the number of edges with 0 label and* $e_f(1)$ *is the number of edges with 1 label.*

G is said to be a 1-Near Mean Cordial Graph if it has a 1-Near Mean Cordial Labeling. In this paper, we proved that Paths, Combs, Fans and Crowns are 1-Near Mean Cordial Graphs.

Keywords: *1-Near Mean Cordial Labeling, 1-Near Mean Cordial Graph.*

1. INTRODUCTION

The graphs considered here are finite, undirected and simple. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. The cardinality of V(G) and E(G) are respectively called order and size of G. Labeling of graphs has a wide application in various practical problems involved in circuit designing, communication network, astronomy etc. [1]. The concept of mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram in the year 2012 in [3, 4, 5, 6]. Let f be a function from V(G) to {0,1,2}. For each edge uv of G, assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a mean cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0,1,2\}$ where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x (x = 0,1,2) respectively. A graph with a mean cordial labeling is called mean cordial graph. Here, we introduce a new concept called 1-Near Mean Cordial Labeling and investigate the 1-Near Mean Cordial Labeling behavior of some standard graphs. Terms not defined here are used in the sense of Harary [2].

2. PRELIMINARIES

We define the concept of 1-Near Mean Cordial Labeling as follows.

Let G = (V, E) be a simple graph. A surjective function f: V(G)→{0,1,2} is said to be a 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv) = \begin{cases} 0 & if \ \frac{f(u)+f(v)}{2} \ \text{is an integer} \\ 1 & \text{otherwise} \end{cases}$$

satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with zero label and $e_f(1)$ is the number of edges with one label.

G is said to be a 1-Near Mean Cordial Graph if it has a 1-Near Mean Cordial Labeling. We proved that Paths, Combs, Fans and Crowns are 1-Near Mean Cordial Graphs.

2.1 Definition: (Path) If all the vertices in a walk are distinct, then it is called a *path* and a path of length k is denoted by P_{k+1} .

*Corresponding Author: K. Palani*1, Department of Mathematics,*
A. P. C. Mahalaxmi College, Thoothukudi, Tamilnadu, India.

2.2 Definition: (Fan) The join G_1+G_2 of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining V_1 with V_2 as vertex set $V(G_1) \cup V(G_2)$ and edge set $E[G_1 \cup G_2] = E(G_1) \cup E(G_2) \cup [uv : u \in V(G_1)$ and $v \in V(G_2)]$. The graph P_n+K_1 is called a *fan*.

2.3 Definition: (Crown) The *crown* ($C_n \odot K_1$) is obtained by joining a pendant edge to each vertex of C_n.

2.4 Definition: (Comb) The *corona* $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which has p_1 points) and p_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2 . The graph $P_n \odot K_1$ is called a *comb*.

3. MAIN RESULTS ON 1-NEAR MEAN CORDIAL GRAPH

3. 1 Theorem: A path P_n is a 1- Near Mean Cordial Graph.

Proof: Let G = (V, E) be a simple graph

Let G be P_n

Let $V[G] = \{u_i : 1 \le i \le n\}$ and $E[G] = \{(u_i u_{i+1}) : 1 \le i \le n-1\}$

Define f: $V[G] \rightarrow \{0,1,2\}$ by

$$f(u_i) = \begin{cases} 0 & if \;\; i \equiv 1 \; mod \; 4 \\ 2 & if \;\; i \equiv 2 \; mod \; 4 \\ 1 & if \;\; i \equiv 3 \; mod \; 4 \;\; (or) \; i \equiv 0 \; mod \; 4 \end{cases}, \; 1 \le i \le n$$

The induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} 0 & if \;\; i \equiv 1 \; mod \; 2 \\ 1 & if \;\; i \equiv 0 \; mod \; 2 \end{cases}, \; 1 \le i \le n-1$$

$$\text{Here, } e_f(0) = \begin{cases} \frac{n}{2} & if \;\; n \; is \; even \\ \frac{n-1}{2} & if \;\; n \; is \; odd \end{cases}$$

$$e_f(1) = \begin{cases} \frac{n}{2} - 1 & if \;\; n \; is \; even \\ \frac{n-1}{2} & if \;\; n \; is \; odd \end{cases}$$

Hence, the graph satisfies the condition $|\, e_f(0) - e_f(1) \,| \le 1$

Therefore, a path P_n is a 1-Near Mean Cordial Graph (1-NMCG).

Illustration:

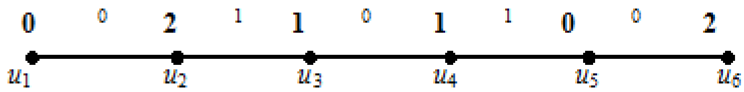

Fig.-3.1(a): P_6

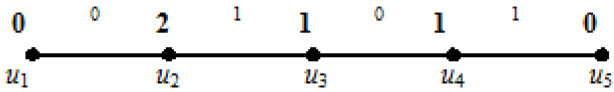

Fig.-3.1(b): P_5

3.2 Theorem: The graph $P_n + K_1$ is a 1- Near Mean Cordial Graph.

Proof: Let G be $P_n + K_1$

Let $V[G] = \{u_i : 1 \le i \le n, v\}$

Let $E[G] = \{[(u_i u_{i+1}) : 1 \le i \le n-1] \cup [(u_i v) : 1 \le i \le n]\}$

Define f: V[G]→{0,1,2} by

$$f(u_i) = \begin{cases} 0 & if\ \ i \equiv 1 mod 2 \\ 2 & if\ \ i \equiv 0 mod 2 \end{cases},\ 1 \le i \le n$$

$f(v) = 1$

The induced edge labeling are
$f^*(u_iu_{i+1}) = 0,\ 1\le i \le n-1$
$f^*(u_iv) = 1,\ 1\le i \le n$

Here, $e_f(0) = n-1$
$e_f(1) = n$

Hence, the graph satisfies the condition $|e_f(0) - e_f(1)| \le 1$

Therefore, the graph $P_n + K_1$ is a 1- Near Mean Cordial Graph.

Illustration:

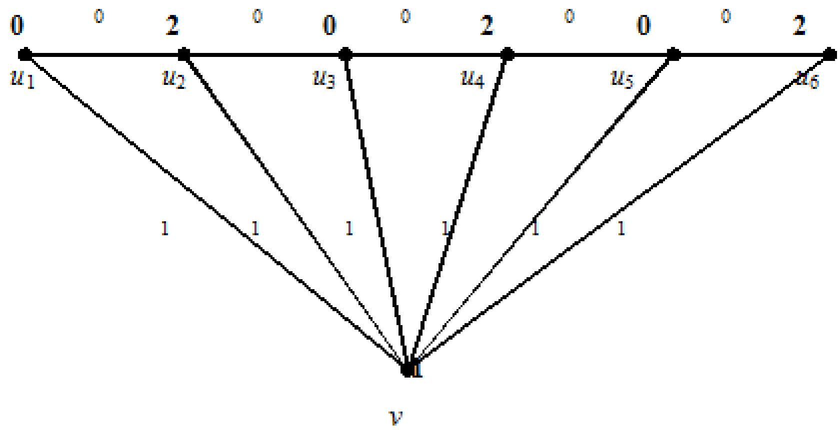

Fig.3.2: P_6+K_1

3.3 Theorem: The graph crown ($C_n \odot K_1$) is a 1- Near Mean Cordial Graph.

Proof: Let G = (V, E) be a simple graph

Let G be $C_n \odot K_1$

Case-(i): when n is odd

Let $V[G] = \{(u_i, u_{i1}) : 1\le i \le n\}$

Let $E[G] = \{[(u_iu_{i+1}):1\le i \le n-1] \cup [(u_iu_{i1}):1\le i \le n] \cup (u_1u_n)\}$

Define f: V[G]→{0,1,2} by

$$f(u_i) = \begin{cases} 0 & if\ \ i \equiv 1 mod 2 \\ 1 & if\ \ i \equiv 0 mod 2 \end{cases},\ 1 \le i \le n$$

$f(u_{11}) = 1$

$$f(u_{i1}) = \begin{cases} 1 & if\ \ i \equiv 0 mod 2 \\ 2 & if\ \ i \equiv 1 mod 2 \end{cases},\ 2 \le i \le n$$

The induced edge labeling are
$f^*(u_iu_{i+1}) = 1,\ 1\le i \le n-1$
$f^*(u_1u_n) = 0$
$f^*(u_1u_{11}) = 1$
$f^*(u_iu_{i1}) = 0,\ 2\le i \le n$

Here, $e_f(0) = n$
$e_f(1) = n$

Hence the graph satisfies the condition $|e_f(0)- e_f(1)| \leq 1$

Therefore, the graph $C_n \odot K_1$ (n : odd) is a 1-Near Mean Cordial Graph.

Case-(ii): when n is even

Let $V[G] = \{u_i : 1\leq i \leq n, u_{i1} : 1\leq i \leq n\}$

Let $E[G] = \{[(u_iu_{i+1}) : 1\leq i \leq n\text{-}1]\cup[(u_iu_{i1}) : 1\leq i \leq n]\cup(u_1u_n)\}$

Define f: $V[G]\rightarrow\{0,1,2\}$ by

$$f(u_i) = \begin{cases} 0 & if \;\; i \equiv 1mod2 \\ 2 & if \;\; i \equiv 0mod2 \end{cases}, 1 \leq i \leq n$$

$f(u_{i1}) = 1, 1 \leq i \leq n$

The induced edge labeling are
$f^*(u_iu_{i+1}) = 0, 1 \leq i \leq n\text{-}1$
$f^*(u_1u_n) = 0$
$f^*(u_iu_{i1}) = 1, 1 \leq i \leq n$

Here, $e_f(0) = n$
$e_f(1) = n$

Hence, the graph satisfies the condition $|e_f(0) – e_f(1)| \leq 1$

Therefore, the graph $C_n \odot K_1$ (n: even) is a 1- Near Mean Cordial Graph.

Illustration:

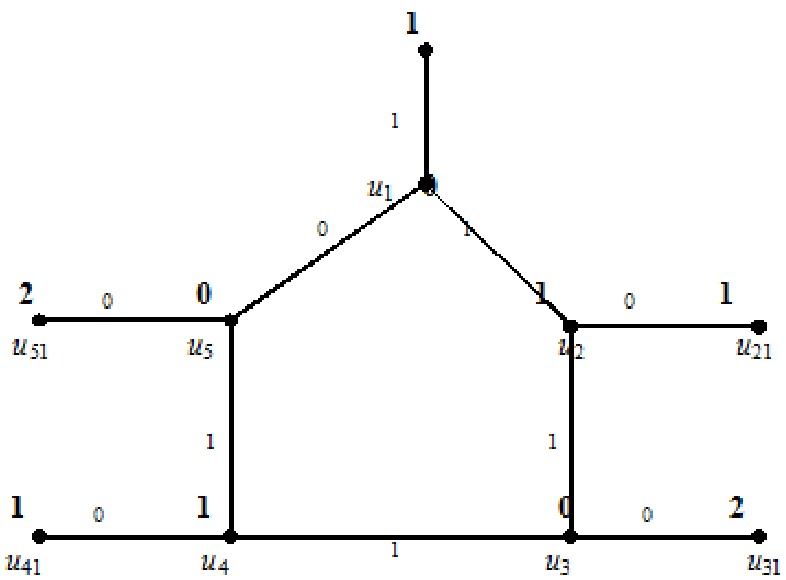

Fig.-3.3(a): $C_5 \odot K_1$

Illustration:

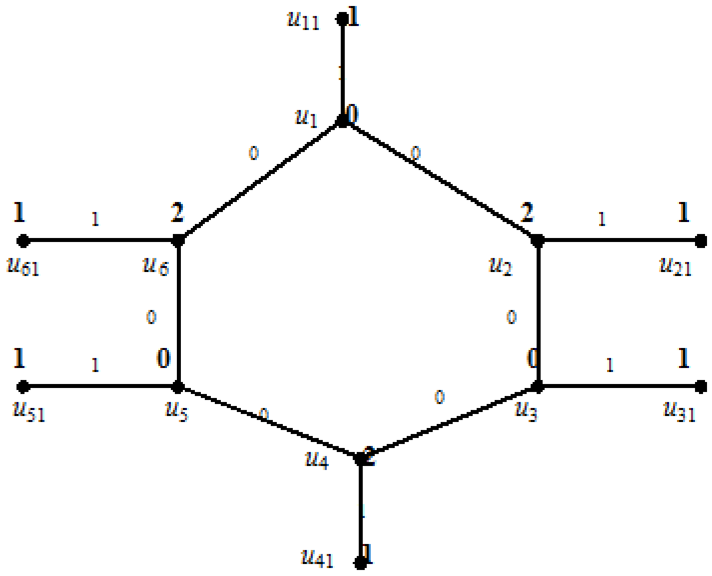

Fig.-3.3(b): $C_6 \odot K_1$

3.4 Theorem: The comb ($P_n \odot K_1$) is a 1- NMCG.

Proof: Let G = (V, E) be a simple graph

Let G be $P_n \odot K_1$

Let V[G] = $\{(u_i, v_i) : 1 \le i \le n\}$

Let E[G] = $\{[(u_i u_{i+1}) : 1 \le i \le n\text{-}1] \cup [(u_i v_i) : 1 \le i \le n]\}$

Define f: V[G] $\rightarrow \{0,1,2\}$ by

$$f(u_i) = \begin{cases} 1 & if\ i \equiv 1 mod 2 \\ 2 & if\ i \equiv 0 mod 2 \end{cases}, 1 \le i \le n$$

$$f(v_i) = 1 \text{ for all } i$$

The induced edge labeling are

$f^*(u_i u_{i+1}) = 0,\ 1 \le i \le n\text{-}1$

$f^*(u_i v_i) = 1,\ 1 \le i \le n$

Here, $e_f(0) = n\text{-}1$

$e_f(1) = n$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \le 1$

Therefore, the graph comb ($P_n \odot K_1$) is a 1- NMCG.

Illustration:

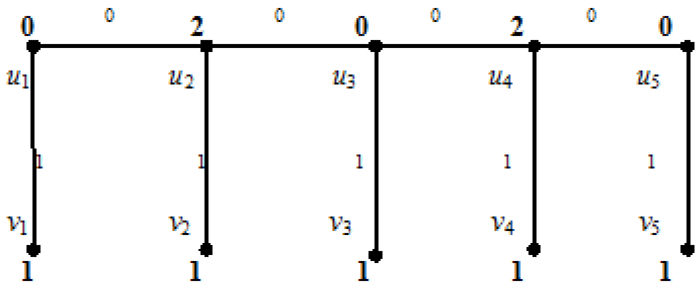

Fig.-3.4: $P_5 \odot K_1$

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Source of support: Nil, Conflict of interest: None Declared