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ORDER LEVEL INVENTORY MODEL FOR POWER PATTERNDEMAND WITH INVENTORY RETURNS AND SPECIAL SALES

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ABSTRACT

In this paper we considered the concept of Power pattern demand with inventory returns and special sales using power pattern demand function. This paper explains us how the stockiest will feel that there is large demand for the product and tempt to store large quantity in the inventory.

1. INTRODUCTION

The problem of returning or of selling the inventory excess to the stock level arises in any business organization when the stock level of inventory system is smaller than the amount on hand. On the other hand, if in any wholesale or retail business, the demand of particular item deplenishes due to launching of new product which is superior and cheaper. The other reason could be the effect of new budget such as price increase due to the product becomes more than the optimal stock level. In such an instance the optimum amount to be returned or sold, if any should be determined by balancing the losses due to various costs involved. Naddor [1], has considered this problem for the EOQ in inventory system. Dave U [2], has reconsidered this problem for an order level inventory system. I in all these models, the demand 0during in an inventory cycle occur at a constant rate, so that the on hand inventory at any time can be represented by linear function of time. This assumption does not hold good in many real world problems and in fact the on hand inventory, behaves as a non-linear function. Naddor [3], has specified a general class of demand pattem in terms of the on hand inventory Q(t) shown as

$$Q(t) = S - R(t/T)^{1/n}, \quad 0 \leq t \leq T \qquad (1)$$

Where 'S' is the stock on hand at beginning of the period (called the order level)
'T' is the length of the inventory cycle,
'R' is the demand size during T, and
'n' is constant, called pattern index.

The type of demand that leads to the above form of on hand inventory is known as power pattern demand. If a large portion of the demand occurs at the beginning of the period, we use n>l, and if it occurs at the end of the period, we use o<n<l. The case of n=l corresponds to the constant rate of the demand and n =∞ instantaneous demand. The quantum of reported research in this direction appears to be meager. Aggarwal and Goel [4] had derived order level inventory models with power pattern demand assuming that the items in stock deteriorate over time. K.S. Rao [5] has reconsidered the concept of power pattern demand and extensively discussed in the context of two levels of storage, for deteriorating items. The concept of power pattem demand (PPD) has an interesting application in the context of inventory return. The proposed model combines the aspect of PPD with inventory returns for an infinite horizonmodel, and we study the effect of 'pattern index' on the policy variables.

2 ASSUMPTIONS &NOTATIONS

The mathematical model is developed under the following assumptions.

(i) T is the fixed length of one period.
(ii) The demand size 'R' during 'T' is known deterministically.
(iii) Replenishment rate is infinite and its size is constant. Lead time is zero.

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(iv) The fixed lot size ‘q’ rises the inventory at the beginning of the scheduling period to the order ‘S’. Shortages if any are to be backlogged.
(v) The system starts with an amount of Q units on hand out of which only P units are retained after retuming or selling the rest. The problem is to determine the optimal value of P.
(vi) The inventory holding C_1 cost per unit per unit time. The shortage cost Rs. C_2per unit per time and the returning or selling costC_4perunit are known and constant during the period under consideration.

3. MODEL DESCRIPTION AND ANALYSIS

Consider the period $T^1$with an initial inventory level of Q units, and final inventory level of zero. Since (Q-P) units are retumed or sold, the P units are to be consumed by the time, say, $t_1(t_1 \le T)^1$

If Q (t) denotes the inventory level at time t ($0 \le t \le t_1$)

Then the differential equation governing the system during r, is given by

$$d\,Q_1(t)/dt + R\,t^{1/n-1}/nT^{1/n} = 0 \quad (2)$$

Using the boundary conditions Q_1 (O) = P and $Q_1(t_1)$ = 0, the solution of equation (3). Since it gives general structure and considers various pattern of demand.

$$Q_1(t) = P - R(t/T)^{1/n}(0 \le t \le t_1) \quad (3)$$

$\therefore t_1 = (P/R)^n T$...(4)

From (3.3.3) and (3.3.4) the total inventory carried duringt_1, is

$$I_1(P) = \int_0^{f_1} Q_1\,(t)dt = \frac{P^{n+1}}{(n+1)R^n}T \quad (5)$$

The total cost of the inventory system during the period T is given by

$$K(P) = C_4(Q-P) + C_1 I_1(P) + (T-t_1)K_1 \quad (6)$$

Where $K_1 = C_2 q[1-(C_1/C_1+C_2)^{1/n}]\frac{n}{(n+1)}$ (7)

is the total average inventory cost for the optimal order level inventory system with Power Pattern Demand (PPD) during ($T-t_1$) , the derivation can be had from Naddor [3]. Further, we note that S^0 of S in

$S^0 = q_p \sqrt[n]{\frac{C_2}{C_1+C_2}}$...(8)

Then using (3.3.4) and (3.3.5), (3.3.6) and (3.3.7) in (9) we get

$$K(P) = C_4(Q-P) + \frac{C_1 P^{n+1}}{(n+1)R^n}T + \frac{K_1 T}{R^n}(Q^n - P^n) \quad (9)$$

The value of P is the solution of$\frac{d}{dP}K(P) = 0$ and is given by the equation

$$C_1 P^n T - nK_1 T P^{n+1} = C_4 R^n \quad (10)$$

The above equation is a Polynomial in P of order n. It contains n roots, among which only admissible shall be considered. Here admissible roots means that a root which is having positive root with minimum cost when compared to all other positive roots. The value ofP^0 shall however be the positive root of the above expression. To ensure feasibility of the solution, the second derivative of K (P) with respect to P should be positive. This value of P° will give minimum cost solution. It can be seen from the theorem below that K (P) is convex.

Theorem: The cost function K (P) is convex if

$$C_1 \ge \frac{K_1(n-1)}{P}$$

WhereC_1, K_1, n and P as defined above

The second derivative of K (P) in (7) with respect to P is

$$\frac{C_1 Tn.P^{n+1}}{R^n} - nTK_1(n-1)P^{n-2} \ge 0$$

$$\Rightarrow C_1 \ge \frac{K_1(n-1)}{C_1}$$

Hence the proof when nil the demand pattern will be linear and equation reduces to

$$P^0 = \frac{RC_4}{C_1} + \frac{S_0}{2}$$

is the optimal quantity to be retained in the context of order level inventory model as given by Dave [23].

4: SOLUTION METHOD WITH n = 2

We now illustrate the solution method for the pattern index n = 2. The equation (3.3.9) reduces to

$$C_1TP^2 - K_1TnP - C_4R^2 = 0 \quad (11)$$

Which is a quadratic equation in P and the roots are given by

$$P° = \frac{K_1Tn \pm \Delta^{1/2}}{2C_1T} \quad (12)$$

Where $\Delta = (K_1Tn)^2 - 4(C_1T)(-C_4R^2)$ (13)

Interestingly it happened that, P° obtained in above equation turn out to be real roots only. This can be established in the following proposition.

Proposition

The equation defined in (10) will have real roots only.

Proof: For P° to be real if $\Delta \geq 0$where Δ is defined in (11) i.e., $nK_1T + 4C_1TC_4R^2$which is always positive. Hence the proof is complete. The equation (9) will have two real roots. The negative roots will be ignored. If there are two positive distinct roots we take a root which gives minimum cost as the optimum quantity to be retained. We therefore suggest the following sequence of steps.

Step-1: For the given hypothetical parametersC_1, C_2, C_4,T, D and patternindex n, compute P value from equation (12). Check $C_1 \geq \frac{k_1(n-1)}{P}$ If this inequality is not satisfied, stop. Otherwise go to next step.

Step-2: If the roots obtained in step l are real and distinct say P° and R consider the following cases.

(i) If two roots are less than 'Q' we compare the costs of P° and P| say C (P_0) and C (P_1). If C (P_0) <C (P_1),P_0 will be the optimum quantity to beretained. Otherwise R is the optimum quantity to be retained

(ii) IF both roots are greater than Q, the optimal quantity to be retained would be "Q" units

(iii) If one root of 'P' say P_0 greater than Q take P_0 = Q. Ifthe otherroot is less than Q sayP_1, we compare the cost of C (Q) and C (P_1), using equation (6). If C (Q) < C (P), we take 'Q' is the optimum inventory level to be retained. Otherwise P_1 will be taken as optimum quantity to beretained

We now illustrate above method numerically.

5. NUMERICALILLUSTRATION

As an illustration to the above developed model, consider a hypothetical system with the following parameter values.

C_1 = Re 0.56 per unit per month, C_2 = Rs 5.04 per unit per month.C_4 = Re 0.28 per unit, R=2400 units per month, Q: 4800 per unit per month and T = 0.5 month.

Now from equation (12) we get P_1 = 2797.75 and P°= -2058 79

Here we take P_1as the optimum quantity to be retained (P<Q) i.e., P° of P with minimum cost Rs. 1188.71

In table 3.1 the values of P° for different values of C_4 i.e special sales aresummarized to check the sensitivity of the model with respect to specialsales C_4

Table 3.1: Sensitivity of the Model to the Parameter C_4 i.e. Special Sales.

C_4	P_0	$P1$	K($P^0 = P_0$)
0.28	2797.75	-2058.79	1188.708
0.30	2881.04	-2142.08	1227.915
0.32	2961.66	-2222.69	1265.484
0.34	3039.84	-2300.87	1301.465
0.36	3115.79	-2376.83	1335.905
0.38	3189.70	-2450.74	1368.847
0.40	3261.73	-2522.76	1400.33
0.42	3332.00	-2593.03	1430.39
0.44	3400.64	-2661.68	1459.061
0.46	3467.77	-2728.80	1486.374

From the above table we note that P is an increasing function of C_4 i.e., special sales.

6. SENSITIVITY ANALYSIS WITH RESPECT TO PATTERNINDEX

It is interesting to note that as the pattern index is greater than 1, the stock clearance will be faster when compared to linear stock depletion. To carry out sensitivity analysis with respect to pattern index ‘n’ at first we must evaluate (10) to find optimum value of P° of P. To do so, we consider same parametric values given in section 3.5 with varying pattern index. The equation (10) is a polynomial in P and can be written as

$$\emptyset(P) = \frac{1}{C_1}\left[\frac{C_4 R^n}{TP^{n-1}} + nk_1\right] \tag{15}$$

Since in the above equation $\emptyset(P)$=P is a function of P, one can use iterative procedure by ensuring $\emptyset(x)$<1. The following results are obtained for various values of pattern index n.

Table 3.2: Behaviour of optimum quantity to be retained as a function of pattern index n

n	S^0	K(S)	P^0	K(P^0)
1.1	1090.394	289.3593	2921.64	1140.085
1.2	1099.132	277,2942	2904.368	1139.72
1.3	1106.581	266.1225	2888.12	1140.96
1.4	1113.006	255.764	2872.84	1143.71
1.5	1118.604	246.142	2858.45	1147.892
1.6	1127.89	237.19	2844.89	1153.44
1.7	1131.78	228.84	2832.09	1160.32
2.8	1131.78	221.05	2820.01	1168.50
1.9	1135.27	213.75	2808.58	1177.96
2.0	1138.42	206.91	2797.76	1188.71

The values in the table from column 2 to 5 are obtained using equation (7), (8), (9) & (10) respectively. The above table demonstrates that as pattern n index increases the order level S° and quantity to be maintained i.e. P° will be an increasing function of pattern index n

DISCUSSION

In this paper we have used the concept of Power pattern demand with inventory returns and special sales using power pattern demand function. We have obtained expressions for optimum quantity to be retained and the minimum cost. The model is sensitive with respect to special sales and pattern index. This gives a generalized model. It is general in the sense that it considers various patterns of index n. The case of n>1 is interesting since the large portion of demand occurs at the beginning of the period so that inventory returns will be realized quickly. Moreover, the stockiest will feel that there is large demand for the product and tempt to store large quantity in the inventory. This obviously leads to the situation of inventory returns. The case of inventory returns will be more interesting in the context of stochastic demand. With this we end the discussion on deterministic inventory models with inventory returns & special sales and proceed to the next chapter to discuss stochastic inventory models in the context of inventory returns and special sales.

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