International Journal of Mathematical Archive-6(6), 2015, 187-196
Available online through www.ijma.info ISSN 2229 – 5046

RADIO LABELING FOR BARYCENTRIC SUBDIVISION OF PATHS AND INCENTRIC SUBDIVISION OF SPOKEWHEEL GRAPHS

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(Received On: 04-06-15; Revised & Accepted On: 23-06-15)

ABSTRACT

A radio labeling of a graph G is an injective function $f: V(G) \to N \bigcup \{0\}$ such that for every $u, v \in V$, $|f(u) - f(v)| \geq diam(G) + 1 - d(u,v)$.

In this paper, we find the radio number of Barycentric Subdivision of path, Incentric Subdivision of Spokewheel graphs and Radio mean number of Incentric Subdivision of Spokewheel graphs.

Keywords: *Radio labeling, Radio number, Radio mean labeling, Radio mean number, Subdivision, Barycentric Subdivision, Incentric Subdivision, Spoke*

1. INTRODUCTION

Radio labelling, or multilevel distance labeling, is motivated by the channel assignment problem introduced by Hale (1980) [5]. Chartrand et al. investigated the upper bound for the radio number of path P_n. The exact value for the radio number of path was given by Liu and Zhu [6]. A wireless network is composed of a set of stations (or, transmitters) on which appropriate channels are assigned. The task is to assign a channel to each station such that the interference which is caused by the geographical distance between stations is avoided.

Let G= (V (G), E(G)) be a simple connected graphs. The distance between two vertices x and y of G is denoted by d(x, y) and diam(G) indicate the diameter of G.

Definition 1.1: [2] A radio labeling is an injective function $f: V(G) \to N \cup \{0\}$ such that $|f(x) - f(y)| \geq 1 + diam(G) - d(x, y)$ holds for every two distinct vertices x and y of G. The span of a labeling f is the greatest integer in the range of f. The minimum span taken over all radio labelings of the graph is called radio number of G, denoted by rn(G).

Definition 1.2: [1] Let G= (V, E) be a graph. Let e=uv be an edge of G and w is not a vertex of G. The edge e is subdivided when it is replaced by edges $e^1 = uw$ and $e^{11} = wv$

2. RADIO LABELING FOR BARYCENTRIC SUBDIVISION OF PATH

Definition 2.1 [1]: If every edge of graph G is subdivided then the resulting graph is called barycentric subdivision of graph G. In otherwords barycentric subdivision is the graph obtained by inserting a vertex of degree two into every edge of original graph.

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Consider barycentric subdivision of path and join each newly inserted vertices of incident edges by an edge. It is denoted by $BS(P_n)$

Existing Result 2.1 [4]

For $i = 1, 2, 3, ..., p-1, d(x_i, x_{i+1}) \leq \begin{cases} n^2 + 2n - 1 & if \quad n \quad is \quad odd \\ n^2 + 3n - 1 & if \quad n \quad is \quad even \end{cases}$

Theorem 2.1: Let $BS(P_n)$ be a barycentric subdivision of path P_n on n vertices .Then $rn(BS(P_n)) = n^2 + 1$ if n is odd.

Proof: Let f be an optimal radio labeling for $BS(P_n)$ where $0 = f(x_1) < f(x_2) < f(x_3) < ... < f(x_p)$

Define the radio labeling $f : V(BS(P_n)) \to N \cup \{0\}$ and satisfying the radio condition

$f(x_{i+1}) \geq f(x_i) + d + 1 - d(x_{i+1}, x_i)$ for all $1 \leq i \leq p-1$ (2.1)

Let $n = 2a + 1$ and $a = \left\lfloor \frac{n}{2} \right\rfloor, a \geq 1$

In this case diameter $d = 2a + 1$ and $p = 2a + 1$

Using the radio condition defined in (2.1) and summing up p-1 inequalities we get

$$\begin{aligned} rn(BS(P_n)) &= f(x_p) \\ &\geq (p-1)(d+1) - \sum_{i=1}^{p-1} d(x_i, x_{i+1}) \\ &= 2n(n+1) - (n^2 + 2n - 1) = n^2 + 1 \end{aligned}$$

$\therefore rn(BS(P_n)) \geq n^2 + 1$

Arrange the vertices of the graph $BS(P_n)$ as follows

Starting from $v_i^j, 1 \leq i \leq n, 1 \leq j \leq 2$ and ending with v_{n+1}^1. Then rename the vertices by

$$v_i^j = \begin{cases} v_{L(a+1-i),}^j & 1 \leq i \leq a, & 1 \leq j \leq 2 \\ v_c^j, & i = a+1, & 1 \leq j \leq 2 \\ v_c^{j+2}, & i = a+2, & j = 1 \\ v_{R(i-(a+1)),}^j & a+2 \leq i \leq 2a+1, & j = 2 \\ v_{R(i-(a+2)),}^1 & a+3 \leq i \leq 2a+2, & j = 1 \end{cases}$$

So for $BS(P_n)$, $V(BS(P_n)) = \{v_i^j, v_{n+1}^1 : 1 \leq i \leq n, 1 \leq j \leq 2\}$

Label the vertices $x_1, x_2, ..., x_p$ as in the following procedure

$v_c^2 \to v_{Ra}^2 \to v_{La}^2 \to v_c^3 \to v_{La}^1 \to v_{Ra}^1$

$v_{L1}^2 \to v_{R(a-1)}^2 \to v_{L1}^1 \to v_{R(a-1)}^1 \to v_{L2}^2$

$v_{R(a-2)}^2 \to v_{L2}^1 \to v_{R(a-2)}^1 \to ... \to$

$v_{L(a-1)}^2 \to v_{R1}^2 \to v_{L(a-1)}^1 \to v_{R1}^1 \to v_c^1$

Define a function $f: V(BS(P_n)) \to N \cup \{0\}$ by

$f(x_1) = 0$ and for all $1 \le i \le p-1$

$$f(x_{i+1}) = f(x_i) + d + 1 - d(x_i, x_{i+1}) \qquad (2.2)$$

Then the labeling $f: V(BS(P_n)) \to N \cup \{0\}$ as $V: x_1, x_2, ..., x_p$

$f(v): 0, 5, 6, ..., n^2+1$ is clearly a radio labeling satisfying (2.2) with span n^2+1.

$\therefore\ rn(BS(P_n)) \le n^2+1$

Hence $rn(BS(P_n)) = n^2+1$ if n is odd

Example 2.1: In Table 1, Figure 1 and Figure 2, an ordering of the vertices, renamed version and radio labeling of $BS(P_9)$ are shown

Table-1

$v_c^2 \to v_{R4}^2 \to v_{L4}^2 \to v_c^3 \to v_{L4}^1$

$v_{R4}^1 \to v_{L1}^2 \to v_{R3}^2 \to v_{L1}^1 \to v_{R3}^1$

$v_{L2}^2 \to v_{R2}^2 \to v_{L2}^1 \to v_{R2}^1 \to v_{L3}^2$

$v_{R1}^2 \to v_{L3}^1 \to v_{R1}^1 \to v_c^1$

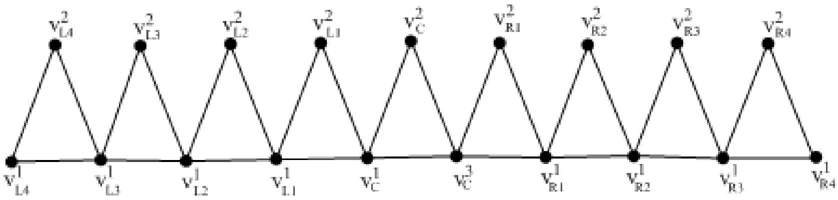

Figure-1

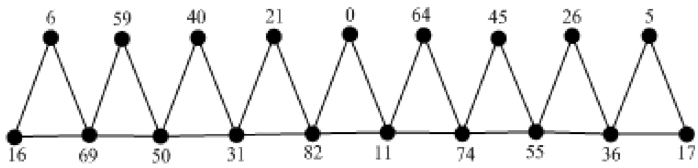

Figure-2:

$$rn(BS(P_9)) = 82$$

Theorem 2.2: Let $BS(P_n)$ be a barycentric subdivision of path P_n on n vertices .Then $rn(BS(P_n)) \ge n^2 - n + 1$ if n is even.

Proof: Let f be an optimal radio labeling for $BS(P_n)$, where $0 = f(x_1) < f(x_2) < f(x_3) < ... < f(x_p)$

Define the radio labeling $f: V(BS(P_n)) \to N \cup \{0\}$ and satisfying the radio condition

$$f(x_{i+1}) \ge f(x_i) + d + 1 - d(x_{i+1}, x_i) \text{ for all } 1 \le i \le p-1 \qquad (2.3)$$

Let $n = 2a$ and $a = \left\lfloor \frac{n}{2} \right\rfloor, a \ge 1$

In this case diameter $d = 2a$ and $p = 2n+1$

Using the radio condition defined in (2.3) and summing up $p-1$ inequalities we get

$$rn(BS(P_n)) = f(x_p)$$
$$\geq (p-1)(d+1) - \sum_{i=1}^{p-1} d(x_i, x_{i+1})$$
$$= 2n(n+1) - (n^2 + 3n - 1)$$

$\therefore\ rn(BS(P_n)) \geq n^2 - n + 1$ if n is even

Theorem 2.3: Let $BS(P_n)$ be a barycentric subdivision of path P_n on n vertices .Then $rn(BS(P_n)) \leq n^2 + 1$ if n is even.

Proof: For $BS(P_n)$ define $f : V(BS(P_n)) \to N \cup \{0\}$ by
$f(x_1) = 0$ and for all $1 \leq i \leq p-1$
$f(x_{i+1}) = f(x_i) + d + 1 - d(x_i, x_{i+1})$ as per following ordering of vertices:

Arrange the vertices of the graph $BS(P_n)$ as follows

Starting from $v_i^j, 1 \leq i \leq n, 1 \leq j \leq 2$ and ending with v_{n+1}^1

Then rename the vertices by $v_i^j = \begin{cases} v_{L(a+1-i)}^j, & 1 \leq i \leq a, & 1 \leq j \leq 2 \\ v_c^1, & i = a+1, & j = 1 \\ v_{R(i-a)}^j, & a+1 \leq i \leq 2a, & j = 2 \\ v_{R(i-(a+1))}^1, & a+2 \leq i \leq 2a+1, & j = 1 \end{cases}$

So for $BS(P_n), V(BS(P_n)) = \{v_i^j, v_{n+1}^1 : 1 \leq i \leq n, 1 \leq j \leq 2\}$ be the vertices $x_1, x_2, ..., x_p$ as in the following procedure

$v_{L1}^2 \to v_{Ra}^2 \to v_{L1}^1 \to v_{Ra}^1 \to v_{L2}^2$
$v_{R(a-1)}^2 \to v_{L2}^1 \to v_{R(a-1)}^1 \to \ldots \to$
$v_{La}^2 \to v_{R1}^2 \to v_{La}^1 \to v_{R1}^1 \to v_c^1$

Thus it is possible to assign labeling to the vertices of $BS(P_n)$ with span n more than the lower bound.

Hence $rn(BS(P_n)) \leq n^2 + 1$ if n is even

Example 2.2: In Table 2, Figure 3 and Figure 4 an ordering of the vertices, renamed version and optimal radio labeling of $BS(P_{10})$ are shown

Table-2

$v_{L1}^2 \to v_{R5}^2 \to v_{L1}^1 \to v_{R5}^1$
$v_{L2}^2 \to v_{R4}^2 \to v_{L2}^1 \to v_{R4}^1$
$v_{L3}^2 \to v_{R3}^2 \to v_{L3}^1 \to v_{R3}^1$
$v_{L4}^2 \to v_{R2}^2 \to v_{L4}^1 \to v_{R2}^1$
$v_{L5}^2 \to v_{R1}^2 \to v_{L5}^1 \to v_{R1}^1 \to v_c^1$

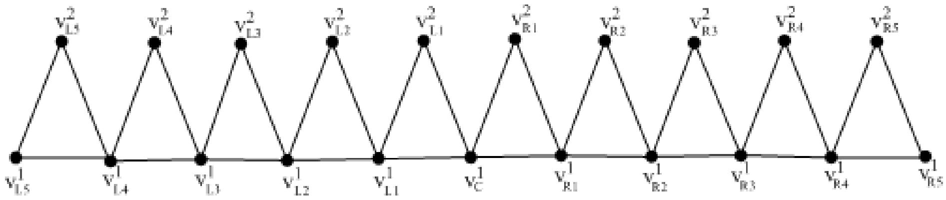

Figure-3

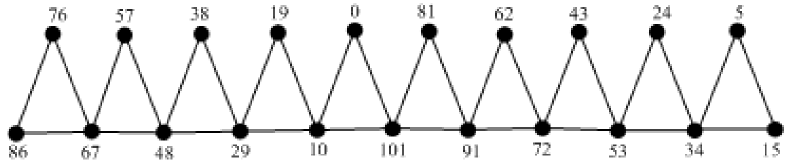

Figure-4:
$rn(BS(P_{10})) = 101$

3. RADIO NUMBER FOR INCENTRIC SUBDIVISION OF SPOKE WHEEL GRAPHS

Definition 3.1: A vertex of degree 3 is called rim vertex [1]. A vertex which is adjacent to all the rim vertices is called the central vertex.

Definition 3.2: The edges with one end incident with the rim and the other incident with the central vertex are called spokes [1].

Definition 3.1: If every edge inside a wheel graph is subdivided then the resulting graph is called in centric subdivision of spoke wheel graph. In other words, in centric subdivision is the graph obtained by inserting a vertex of degree two into every edge inside the original graph.

Definition 3.3: The graph $SS(W_n)$ [6] is obtained from the wheel W_n by subdividing each spokes by a vertex. Let $W_n = C_n + k_1$ where $C_n = v_1 v_2 .... v_n v_1$ and $v(k_1) = \{u\}$ and the spokes are subdivided by the vertices u_i $(1 \le i \le n)$.

Note that the diameter of $SS(W_n)$ is 4.

Thereom 3.1: $rn(SS(W_n)) = 4n + 2$ for $n \ge 8$

Proof: Let u be the centre vertex.

The cyclic vertices are $v_1, v_2, ..., v_n$ and the vertices of incentric subdivision of spokes are $u_1, u_2, ..., u_n$.

Clearly $SS(W_n)$ has $2n+1$ vertices and $diam(SS(W_n)) = 4$.

Define radio labeling $f : V \to N \cup \{0\}$ satisfying the condition
$d(u,v) + |f(u) - f(v)| \ge 1 + diam(SS(W_n)) = 5$, for every pair of distinct vertices u and v.

Case-1: Label u with 0.

That is $f(u) = 0$

Label the vertices of cycle and vertices of subdivision which are at distance 3, alternatively.

Then $f(v_i) = 4i$ and $f(u_{3+i}) = 4i + 2$, $i = 1, 2, ..., n$
Where $u_{n+1} = u_1$, $u_{n+2} = u_2$ and $u_{n+3} = u_3$

Hence $rn(f) = 4n + 2$

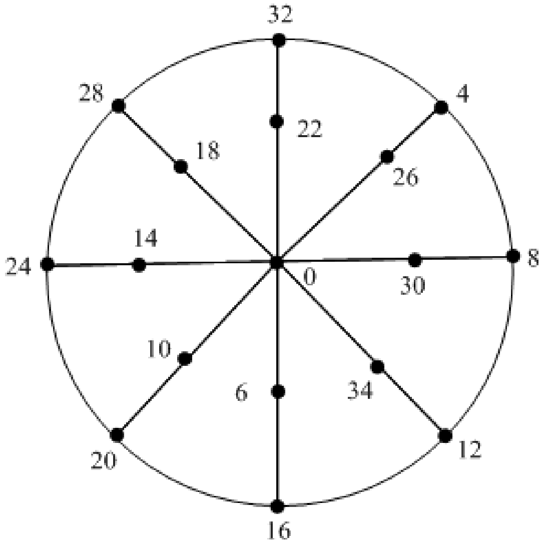

Figure-5: $rn(SS(W_8)) = 34$

Case-2: Label the cyclic vertices with 0

Let $f(v_1) = 0$

Then label the vertices of subdivision and cyclic vertices alternatively, which are at distance3

Then $f(v_i) = 4i - 3, i = 2, 3, ..., n$ and
$f(u_{3+i}) = 4i - 1, i = 1, 2, ..., n$
Where $u_{n+1} = u_1$, $u_{n+2} = u_2$ and $u_{n+3} = u_3$

Since $d(u_3, u) = 1$, $f(u) = 4n + 3$

Hence $rn(f) = 4n + 3$

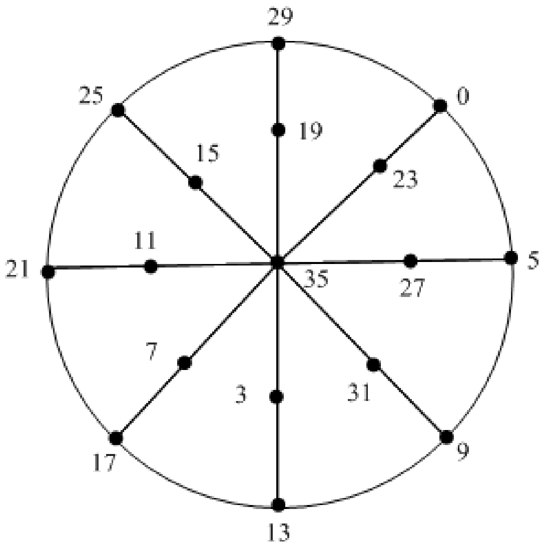

Figure-6: $rn(SS(W_8)) = 35$

Case-3: Label the vertices of subdivision with 0

Let $f(u_3) = 0$

Since $d(u_3,u)=1$ and $d(u,u_1)=1$, $f(u)=5$ and $f(v_1)=8$

Now we label the vertices of subdivision and cyclic vertices alternatively with distance 3.

Then $f(u_{3+i})=4i+6, i=1,2,...,n$ and $f(v_i)=4i+4, i=1,2,...,n$
where $u_{n+1}=u_1$, $u_{n+2}=u_2$ and $u_{n+3}=u_3$

Hence $rn(f)=4n+4$

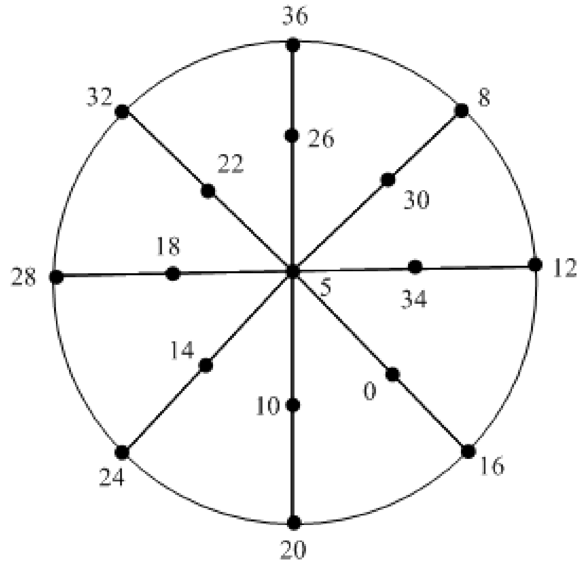

Figure-7: $rn(SS(W_8))=36$

Hence $rn(SS(W_n))=\min\{rn(f)\}$
$$=\min\{4n+2,4n+3,4n+4\}$$
$$=4n+2$$

Example 3.1: The radio number of some incentric subdivision of spokewheel graphs are given in Fig.8

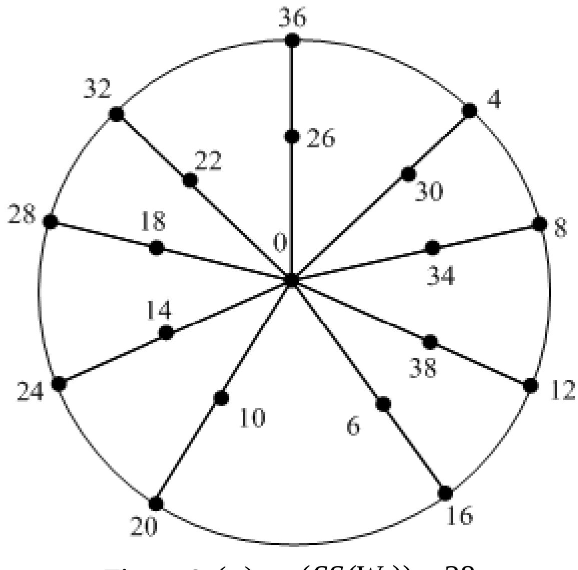

Figure-8: (a) $rn(SS(W_9))=38$

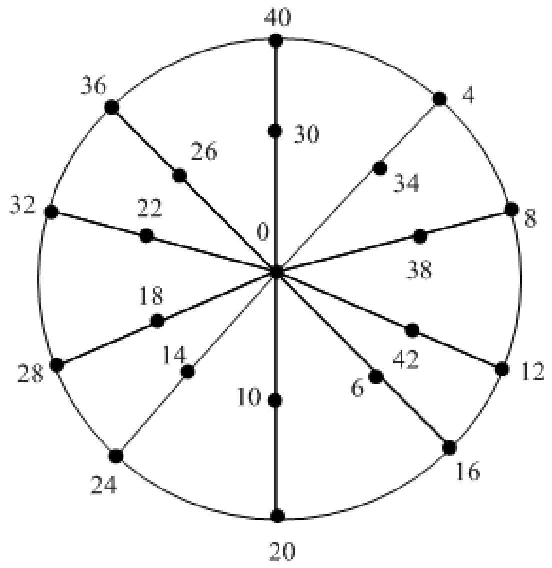

Figure-8: $(b)\ rn(SS(W_{10})) = 42$

4. RADIO MEAN NUMBER OF INCENTRIC SUBDIVISION OF SPOKE WHEEL GRAPHS

Definition 4.1: A radio mean labeling is a one to one mapping f from $V(G)$ to N Satisfying the condition $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam(G)$, for every $u, v \in V(G)$

The radio mean number of G is denoted by rmn(G)

Theorem 4.1: $rmn(SS(W_n)) = 2n+1, \quad n \geq 8$

Proof: Let u be the centre vertex

The cyclic vertices are $v_1, v_2, ..., v_n$ and the vertices of subdivision of spokes are $u_1, u_2, ..., u_n$.

Clearly $SS(W_n)$ has $2n+1$ vertices and $diam(SS(W_n)) = 4$

Define the radio mean labeling $f : V \to N$ satisfying the condition $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam(SS(W_n)) = 5$, for every pair of distinct vertices u and v.

Consider the rim vertices $v_i \quad (1 \leq i \leq n)$

Define the radio mean labeling $f : V \to N$ by

$$f(v_i) = \begin{cases} 1 & if \quad i = 1 \\ n-i & if \quad i = 2 \\ n-i+2 & if \quad i = 3 \\ 2 & if \quad i = 4 \\ n & if \quad i = 5 \\ i-3 & if \quad i = 6, 7, ...n \end{cases}$$

$f(u_i) = n+i, i = 1, 2, ..., n$ and $f(u) = 2n+1$

Now we check the radio mean condition for the above labeling f .

Case-1: Consider the pair (u_i, u_j)

Subcase-(a): Verify the pair (v_1, v_i) .

It is easy to verify that the pairs (v_1, v_i) , $i = 4, 6, 7, 8$

For $i \notin \{4, 6, 7, 8\}$ we have

$$d(v_1, v_i) + \left\lceil \frac{f(v_1) + f(v_i)}{2} \right\rceil \geq 1 + \left\lceil \frac{1+6}{2} \right\rceil \geq 5 .$$

Subcase-(b): Verify the pair (v_4, v_i).

The pairs $(v_4, v_6), (v_4, v_7)$ satisfy the radio mean condition.

For $i \neq 1, 6, 7$, $d(v_4, v_i) + \left\lceil \frac{f(v_4) + f(v_i)}{2} \right\rceil \geq 1 + \left\lceil \frac{2+5}{2} \right\rceil \geq 5$.

Subcase-(c): Check the pair (v_i, v_j) , $i, j \notin \{1, 4\}$

$$d(v_i, v_j) + \left\lceil \frac{f(v_i) + f(v_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{3+4}{2} \right\rceil \geq 5$$

Case-2: Check the pair (u_i, u_j)

$$d(u_i, u_j) + \left\lceil \frac{f(u_i) + f(u_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{n+1+n+2}{2} \right\rceil \geq 13$$

Case-3: Check the pair (v_i, u_j)

$$d(v_i, u_j) + \left\lceil \frac{f(v_i) + f(u_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{1+n+1}{2} \right\rceil \geq 7$$

Case-4: Examine the pair (u, u_i)

$$d(u, u_i) + \left\lceil \frac{f(u) + f(u_i)}{2} \right\rceil \geq 1 + \left\lceil \frac{2n+1+n+1}{2} \right\rceil \geq 16$$

These cases establish the radio mean condition.

Hence $rmn(SS(W_n)) \leq 2n+1$.

Since the number of vertices is $2n+1$ and the labels are unique, it follows that $rmn(SS(W_n)) \geq 2n+1$.

Hence $rmn(SS(W_n)) = 2n+1$

Example 4.1: The radio mean number of some incentric subdivision of spokewheel graphs are given in Fig.9

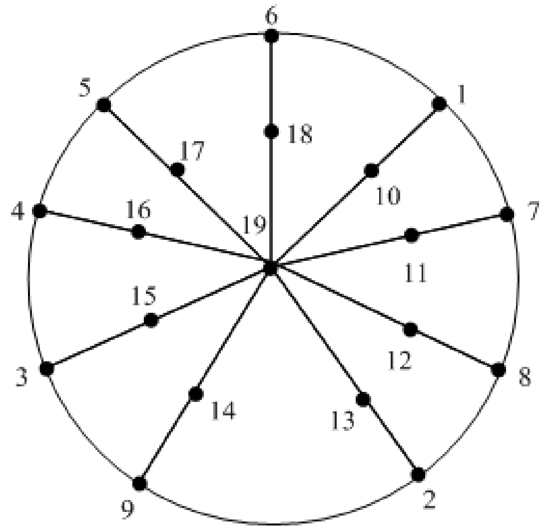

Figure-9: (a) $rmn(SS(W_9)) = 19$

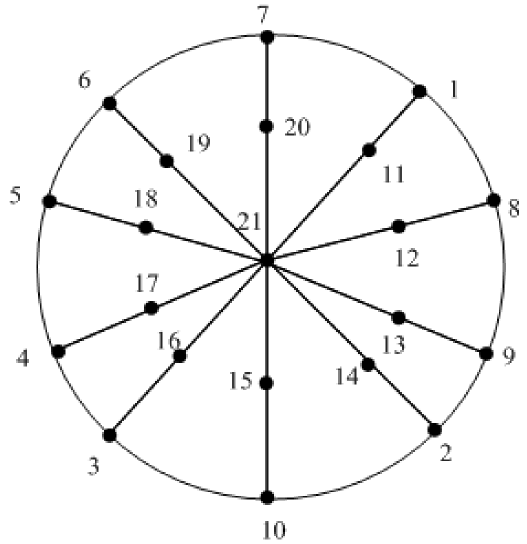

Figure-9: (b) $rmn(SS(W_{10})) = 21$

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Source of support: Nil, Conflict of interest: None Declared