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AN INVENTORY MODEL FOR DETERIORATING GOODS WITH TIME DEPENDENT QUADRATIC DEMAND WITH PARTIAL BACKLOGGING

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ABSTRACT

A deterministic inventory model is study in which demand is consider to be deterministic (quadratic demand rate) and in which shortages are also allowed. In this paper we try to minimizes the total inventory cost and maximizes the total profit. Holding cost and deterioration rate is considered to be constant.

Key Words: *deteriorating items, quadratic demand, partial backlogging.*

INTRODUCTION

The economic order quantity was firstly given by F.Harris in 1916. F.Harris work was generalized by Wilson (1934). Dave and Patel (1981) were first to study about deteriorating items with linearly increasing demand and in which shortages are not allowed. Deterioration is most realistic feature in today life. Due to deterioration or damage of product inventory faces a lot of problems. Problems of shortages or loss of good will. Some of recent researchers of management and mathematics are busy in this field. Some of them take deterioration as constant while some take demand as a function of selling price, linear demand, quadratic demand, ramp type demand, time and price dependent demand and so on.

Kumar N and Sharma A.K. (2001) developed an optimum ordering interval with known demand for items with variable rate of deterioration and shortages. Ouyang and cheng (2005) developed economic order quantity with exponential declining demand and partial backlogging. Pareek *et al.* (2009) developed an economic order quantity model with deteriorating items with salvage value and shortages. Shah, N.H. and Mishra, P. (2010) developed an inventory model for deteriorating items in which demand is consider to be stock dependent. Biswaranjan, mandal (2010) developed an inventory model for weibull distribution deteriorating items in which shortages are allowed and demand is consider to be ramp type demand. Yadav and yadav (2013) work on volume flexibility in production model with weibull deterioration with partial backlogging and demand is cubic demand rate. Milu acharya and smrutirekha debata (2014) developed an inventory model for deteriorating goods with time dependent demand under partial backlogging.

In this paper we develop a economic inventory model by considering demand as quadratic demand and developed an inventory model for deteriorating items where holding cost and deteriorating rate are constant. Shortages are allowed and are Partially backlogged.

ASSUMPTIONS AND NOTATIONS

The basic assumptions are as given by

- The demand rate is time dependent and demand is quadratic.
- Shortages are allowed and partially backlogged.
- Lead time is zero.
- The replenishment rate is instantaneous.
- The deterioration rate is constant.

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The basic notations are as given by

- A is the fixed ordering cost per order.
- θ is the constant deterioration rate.
- $Q(t)$ is the level of inventory at time t ($0\leq t \geq T$).
- t_1 is the time when inventory reaches to zero.
- Q is the ordering point quantity per cycle.
- C_1 is the cost of each deteriorated item.
- C_2 is the inventory holding cost per unit per unit of time.
- C_3 is the shortage cost per unit per unit of time.
- M is the maximum inventory level for the ordering cycle such that M = Q(0).

MATHEMATICAL MODEL

Here we consider the deteriorating inventory model with quadratic demand rate. Replenishment occur at time t=0 when the inventory level attains its maximum. From t=0 to t_1, the inventory level reduces due to demand and deterioration. At t_1, the inventory level reaches to zero, then shortages is allowed to occur during the time interval (t_1, T) is completely backlogged. The total number of backlogged items is replaced by the next replenishment. According to assumptations and notations given above, the behavior of inventory system can be described as below by the following differential equations :

$$\frac{dQ(t)}{dt} + \theta . Q(t) = -(a + bt + ct^2), \; 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dQ(t)}{dt} = -(a + bt + ct^2), \quad t_1 \leq t \leq T \tag{2}$$

With boundary conditions Q(0)=M ,Q(t_1) = 0

The solutions of equations (1) and (2) with boundary conditions are as follows.

$$Q(t) = \frac{-1}{\theta}\left(a - \frac{b}{\theta} + \frac{2c}{\theta^2}\right) + \frac{1}{\theta}\left(-bt + \frac{2ct}{\theta}\right) - \frac{ct^2}{\theta^2} + e^{\theta(t_1-t)}\left(\frac{1}{\theta}\left(a - \frac{b}{\theta} + \frac{2c}{\theta^2}\right) + \frac{1}{\theta}\left(-bt_1 + \frac{2ct_1}{\theta}\right) + \frac{ct_1^2}{\theta^2}\right) \tag{3}$$

$$Q(t) = \left[a(t_1 - T) + b\left(\frac{t_1^2}{2} - \frac{T^2}{2}\right) + c\left(\frac{t_1^3}{3} - \frac{T^3}{3}\right)\right] \qquad t_1 \leq t \leq T \tag{4}$$

The beginning inventory level can be computed as

$$M = Q(0) = \frac{a}{\theta}\left(e^{\theta t_1} - 1\right) - \frac{b}{\theta^2}\left(e^{\theta t_1} - 1\right) + \frac{2c}{\theta^3}\left(e^{\theta t_1} - 1\right) + \frac{bt_1e^{\theta t_1}}{\theta} - \frac{2ct_1e^{\theta t_1}}{\theta^2} + \frac{ct_1^2e^{\theta t_1}}{\theta} \tag{5}$$

The total number of items which perish in the interval [0,t_1]

$$\varphi_T = M - \int_0^{t_1}(a + bt + ct^2)dt$$

$$\frac{a}{\theta}\left(e^{\theta t_1} - 1\right) - \frac{b}{\theta^2}\left(e^{\theta t_1} - 1\right) + \frac{2c}{\theta^3}\left(e^{\theta t_1} - 1\right) - at_1 + b\left(\frac{t_1e^{\theta t_1}}{\theta} - \frac{t_1^2}{2}\right) + c\left(\frac{t_1^2}{\theta}e^{\theta t_1} - \frac{t_1^3}{3}\right) \tag{6}$$

The total number of inventory carried during the interval [0,t_1]

$$\varphi_L = \int_0^{t_1} Q(t)\,dt = \frac{a}{\theta}t_1 + \frac{b}{\theta^2}t_1 - \frac{2c}{\theta^3}t_1 - \frac{bt_1^2}{2\theta} + \frac{ct_1^2}{\theta^2} - \frac{ct_1^3}{3\theta} - \frac{a}{\theta^2}\left(e^{\theta t_1} - 1\right) + \frac{b}{\theta^3}\left(e^{\theta t_1} - 1\right) - \frac{2c}{\theta^4}\left(e^{\theta t_1} - 1\right)$$
$$- \frac{bt_1}{\theta^2}\left(e^{\theta t_1} - 1\right) - \frac{2ct_1}{\theta^3}\left(e^{\theta t_1} - 1\right) + \frac{ct_1^2}{\theta^2}\left(e^{\theta t_1} - 1\right) \tag{7}$$

The total shortages quantity during the interval [0,t_1]

$$\varphi_s = -\int_{t_1}^{T} Q(t)\,dt \;\; \frac{a}{2}(-2t_1T + T^2 + t_1^2) + \frac{b}{6}(3t_1^2T - T^3 - t_1^3) + \frac{c}{12}(4t_1^3T - T^4 - 3t_1^4) \tag{8}$$

Then the average total cost per unit time under the condition $t_1 \leq T$ is given by

$$TC = \frac{1}{T}(A + C_1\varphi_T + C_2\varphi_L + C_3\varphi_S)$$
$$= \frac{1}{T}(A + C_1(\frac{a}{\theta}\left(e^{\theta t_1} - 1\right) - \frac{b}{\theta^2}\left(e^{\theta t_1} - 1\right) + \frac{2c}{\theta^3}\left(e^{\theta t_1} - 1\right) - at_1 + b\left(\frac{t_1e^{\theta t_1}}{\theta} - \frac{t_1^2}{2}\right)$$
$$+ c(\frac{t_1^2}{\theta}e^{\theta t_1} - \frac{t_1^3}{3})) + C_2(\frac{a}{\theta}t_1 + \frac{b}{\theta^2}t_1 - \frac{2c}{\theta^3}t_1 - \frac{bt_1^2}{2\theta} + \frac{ct_1^2}{\theta^2} - \frac{ct_1^3}{3\theta} - \frac{a}{\theta^2}\left(e^{\theta t_1} - 1\right) + \frac{b}{\theta^3}\left(e^{\theta t_1} - 1\right)$$
$$- \frac{2c}{\theta^4}\left(e^{\theta t_1} - 1\right) - \frac{bt_1}{\theta^2}\left(e^{\theta t_1} - 1\right) - \frac{2ct_1}{\theta^3}\left(e^{\theta t_1} - 1\right) + \frac{ct_1^2}{\theta^2}\left(e^{\theta t_1} - 1\right)) + C_3\,\frac{a}{2}(-2t_1T + T^2 + t_1^2)$$
$$+ \frac{b}{6}(3t_1^2T - T^3 - t_1^3) + \frac{c}{12}(4t_1^3T - T^4 - 3t_1^4)) \tag{9}$$

The necessary condition for TC in (9) to be minimized is

$$\frac{\partial TC}{\partial t_1} = 0$$

5.0 SENSITIVITY ANALYSIS

Consider an inventory system with the following parameter in proper unit A=2000, a=25, b=20.0, c=15, theta=0.5, $C_1 = 0.1, C_2 = 0.1, C_3$=0.3, T=2.5. The computer output by using maple mathematical software is t1=0.1756 and TC=799.049 .The variation in the parameter is as follows:

Table 5.1: Variation in parameter A

A	t_1	TC
1000	0.1756	398.409
1500	0.1756	598.409
2000	0.1756	798.409
2500	0.1756	998.409

Table-5.2: Variation in parameter θ

θ	t_1	TC
0.25	0.009608	799.18238
0.5	0.1756	798.409
0.75	0.2807	797.9685
1	0.30	796.4562

Table- 5.3: Variation in parameter a

A	t_1	TC
25	0.1756	799.0488
30	0.21660	799.867
35	0.25491	801.255
40	0.29087	802.5837

Table- 5.4: Variation in parameter b

B	t_1	TC
20	0.1756	799.0488
25	0.16870	796.884
30	0.16229	795.3558
35	0.156320	793.8249

From table 5.1, 5.2, 5.3 and 5.4 we observed that the total cost increases if we increases the parameters a, A and we observed that total cost decreases if we increases the parameter b,θ .it's also observed that the parameter a and A is more sensitive than the parameter θ and b.

6.0 CONCLUDING REMARKS

In this paper, we developed a model for deteriorating item with time dependent demand and partial backlogging and give analytical solution of the model that minimize the total inventory cost. The deterioration factor taken into consideration in the present model, as almost all items undergo either direct spoilage (like fruits, vegetable etc) or physical decay (in case of radioactive substance etc.) in the course time, deterioration is natural feature in the inventory system. The model is very useful in the situation in which the demand rate is depending upon the time.

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